

Analysis of a Lunar Base Electrostatic Radiation Shield Concept

Charles Buhler, PI

John Lane, Co-PI

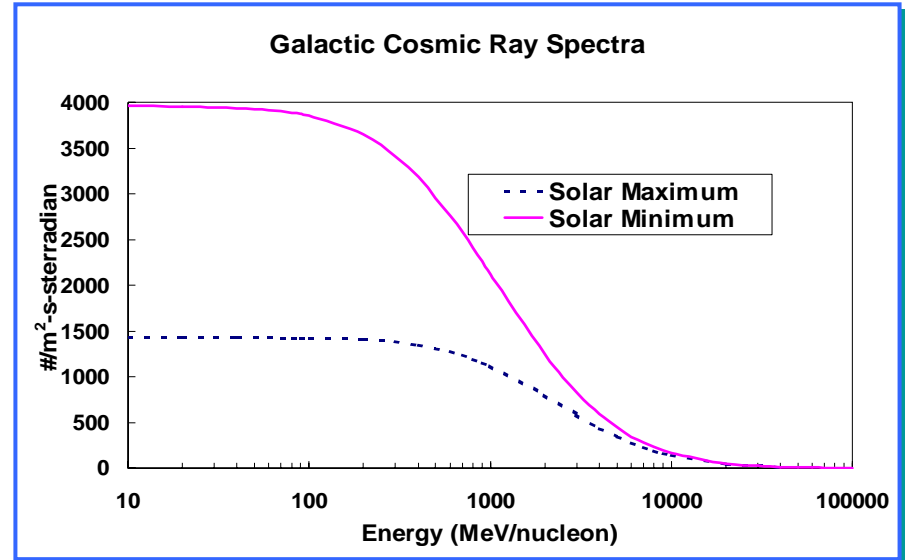
**ASRC Aerospace Corporation
Kennedy Space Center, Florida 32899**

INTERPLANETARY RADIATION ENVIRONMENT

Main Components: (Atomic Nuclei)

➤ Galactic Cosmic Rays (GCRs)

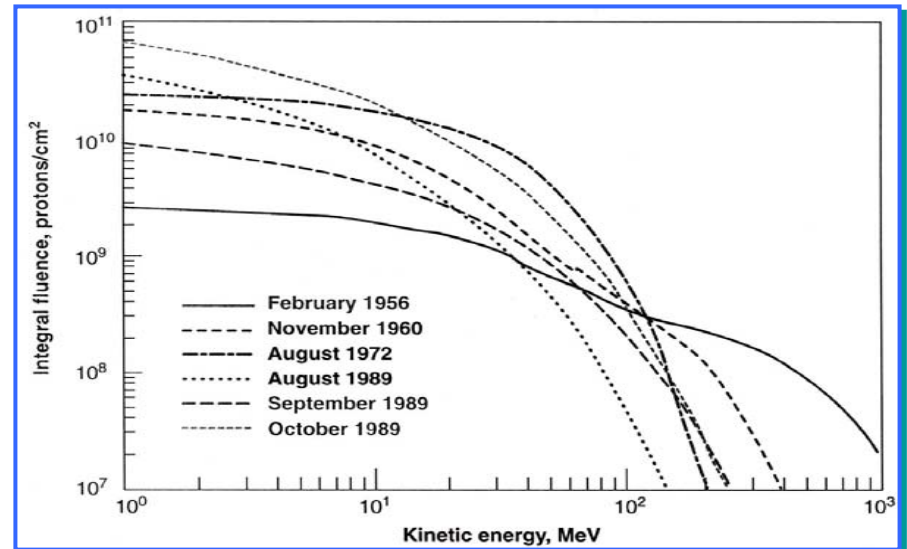
- Median energy ~ 1800 MeV/nuc
- Continuous flux, varies with the solar cycle



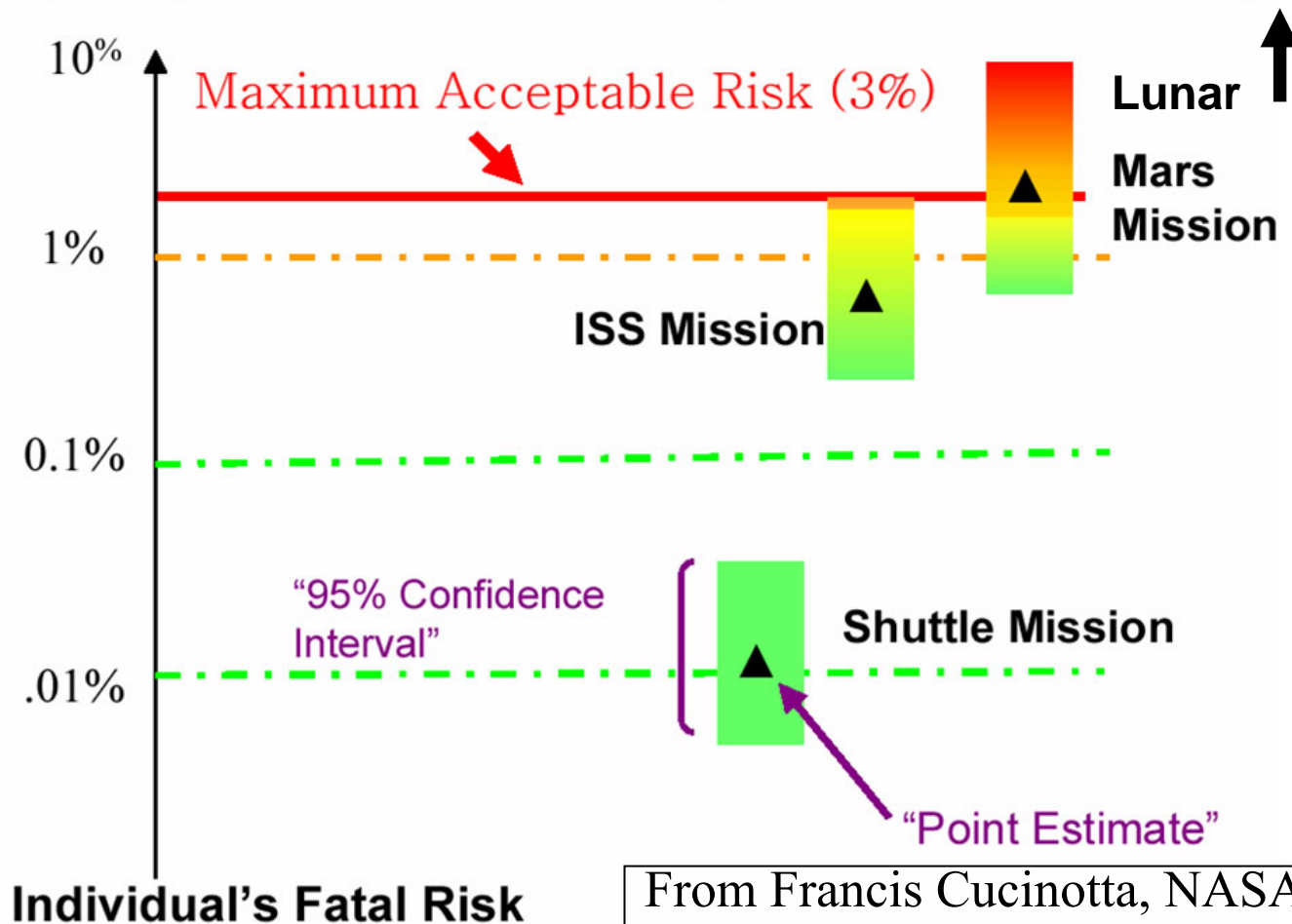
➤ Solar Energetic Particles (SEPs)

- Sporadic, lasting hours to days
- Soft spectra with highly variable composition

$n_i \sim n_e \sim 6 \text{ cm}^{-3}$



Uncertainties in Radiation Risk Projections



From Francis Cucinotta, NASA JSC
Space Radiation Health Project
(private communication)

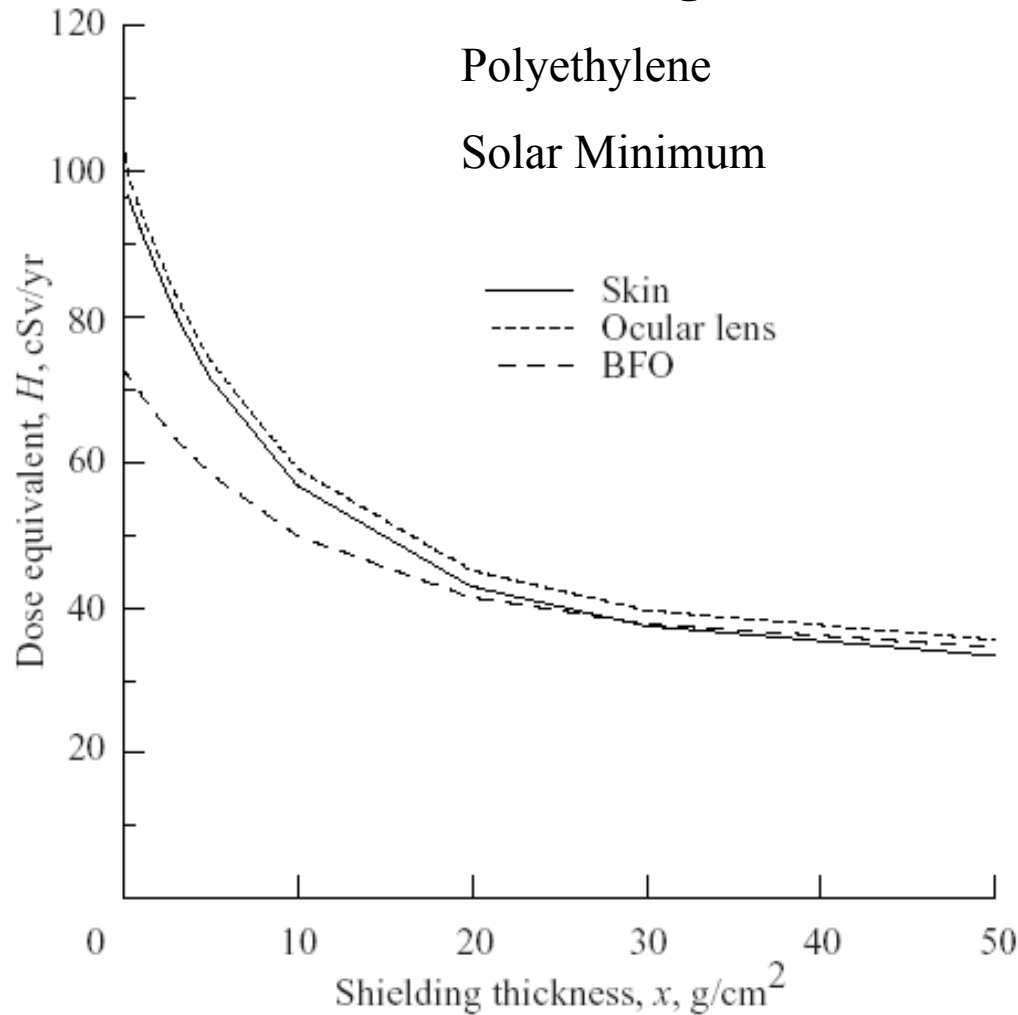
Shielding Solutions must

- Reduce radiation exposure
- Be lightweight
- Safe
- Practical
- Achievable in time for Moon Missions

There are two basic types: Active and Passive Shields

Spacecraft Limitations

Passive shielding can reduce the exposure by only about a factor of two due to weight constraints.



John W. Wilson *et al.*

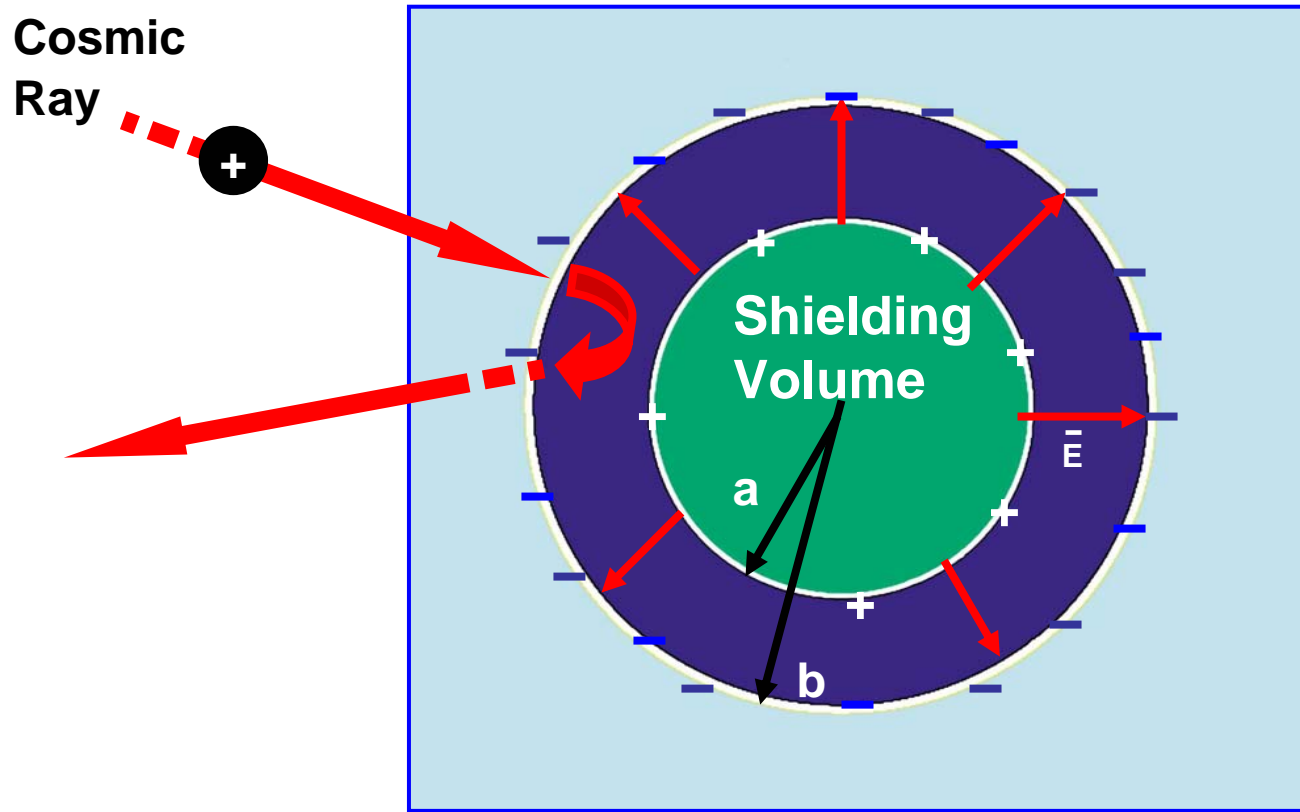
NASA Technical Paper 3682 (1997)

Active Shielding Solutions

- **Electromagnetic Shields**

- Magnets (>73 papers since 1961)
- Plasma (>18 papers since 1964)
- Electrostatic (>16 papers since 1962)

Traditional Spacecraft Design



Concentric, oppositely charged spherical electrodes for shielding against galactic heavy ions. Electrostatic generator keeps 'a' at a high voltage with respect to 'b'.

Why Not Electrostatics?

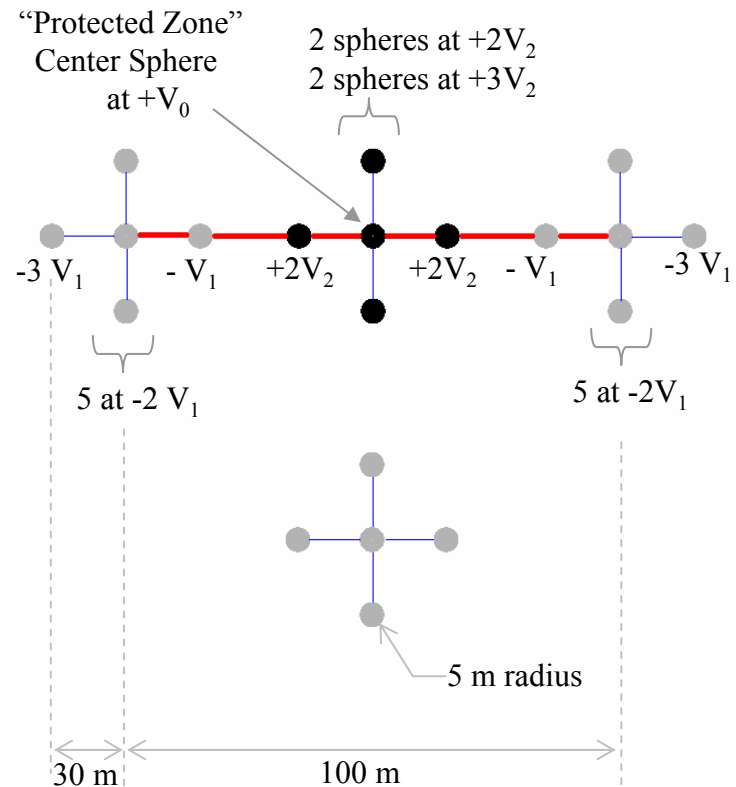
– Debye Length $\sim 11.5\text{m}$

- Assumption: The charge on a conductor is much lower than the charge in the surrounding volume.

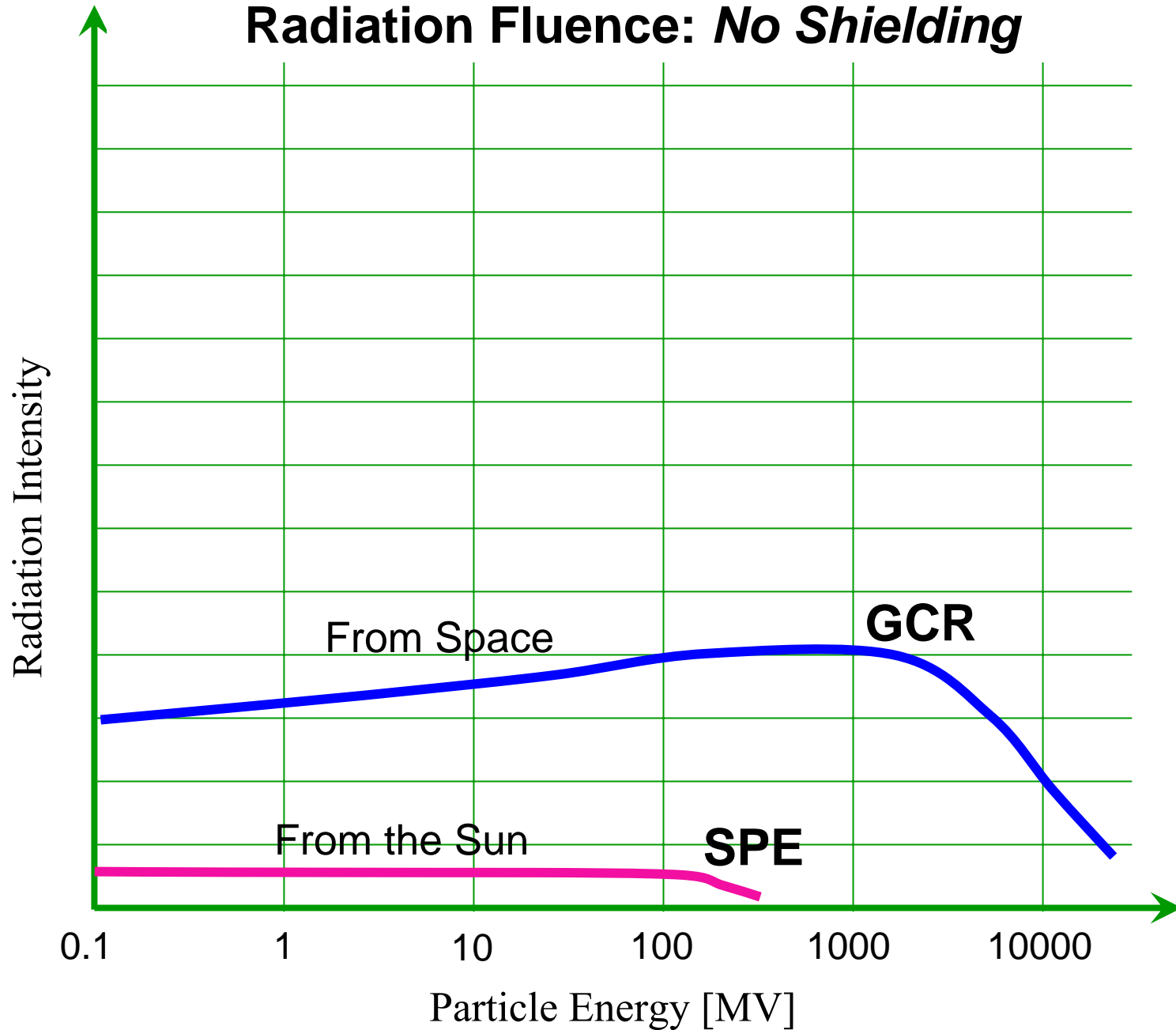
– Extreme voltages are required and surrounding a spacecraft or lunar base with a conducting shell is not realistic

- The same effect can be generated using an alternate geometry.

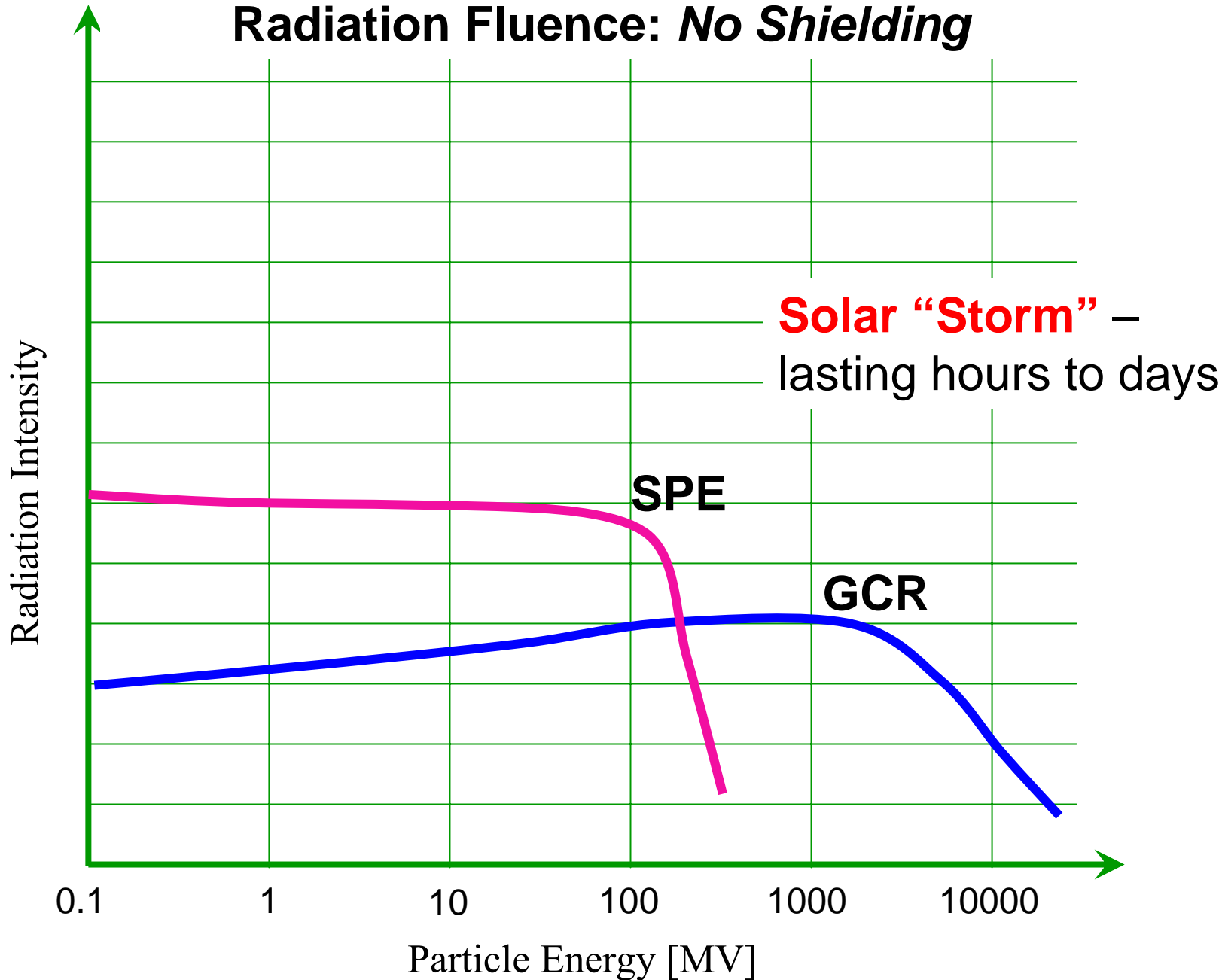
NASA Design



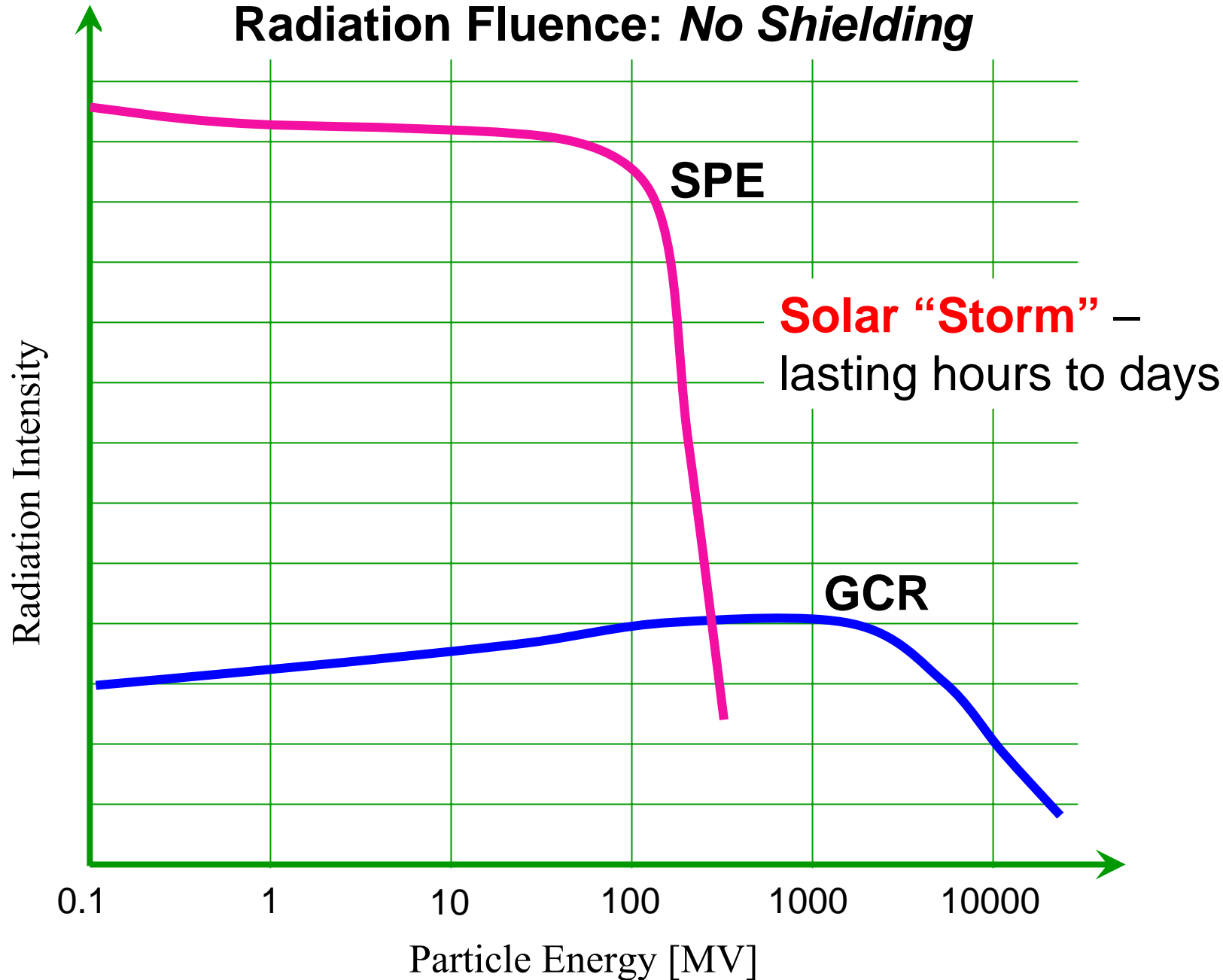
Radiation Fluence: *No Shielding*



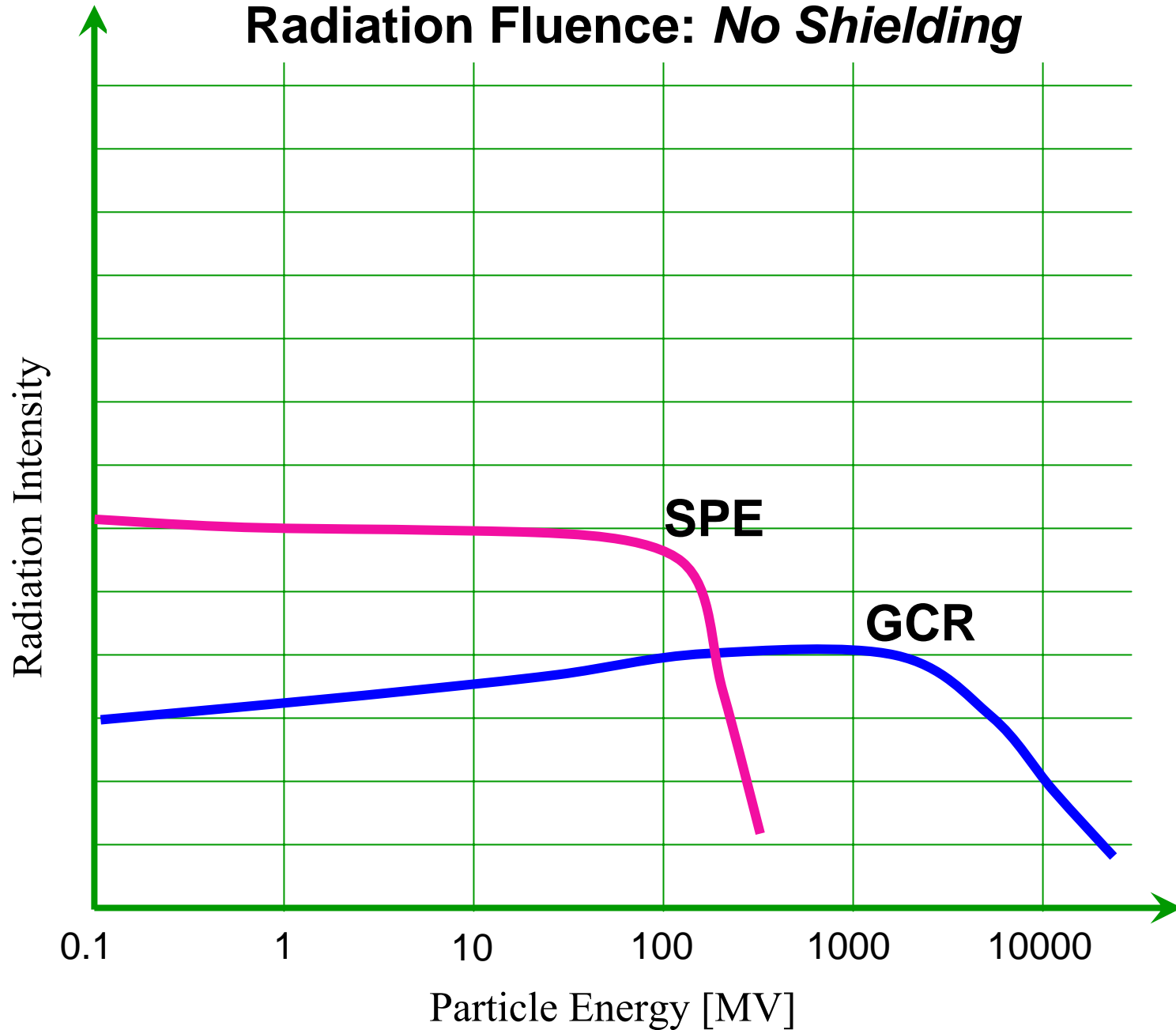
Radiation Fluence: *No Shielding*



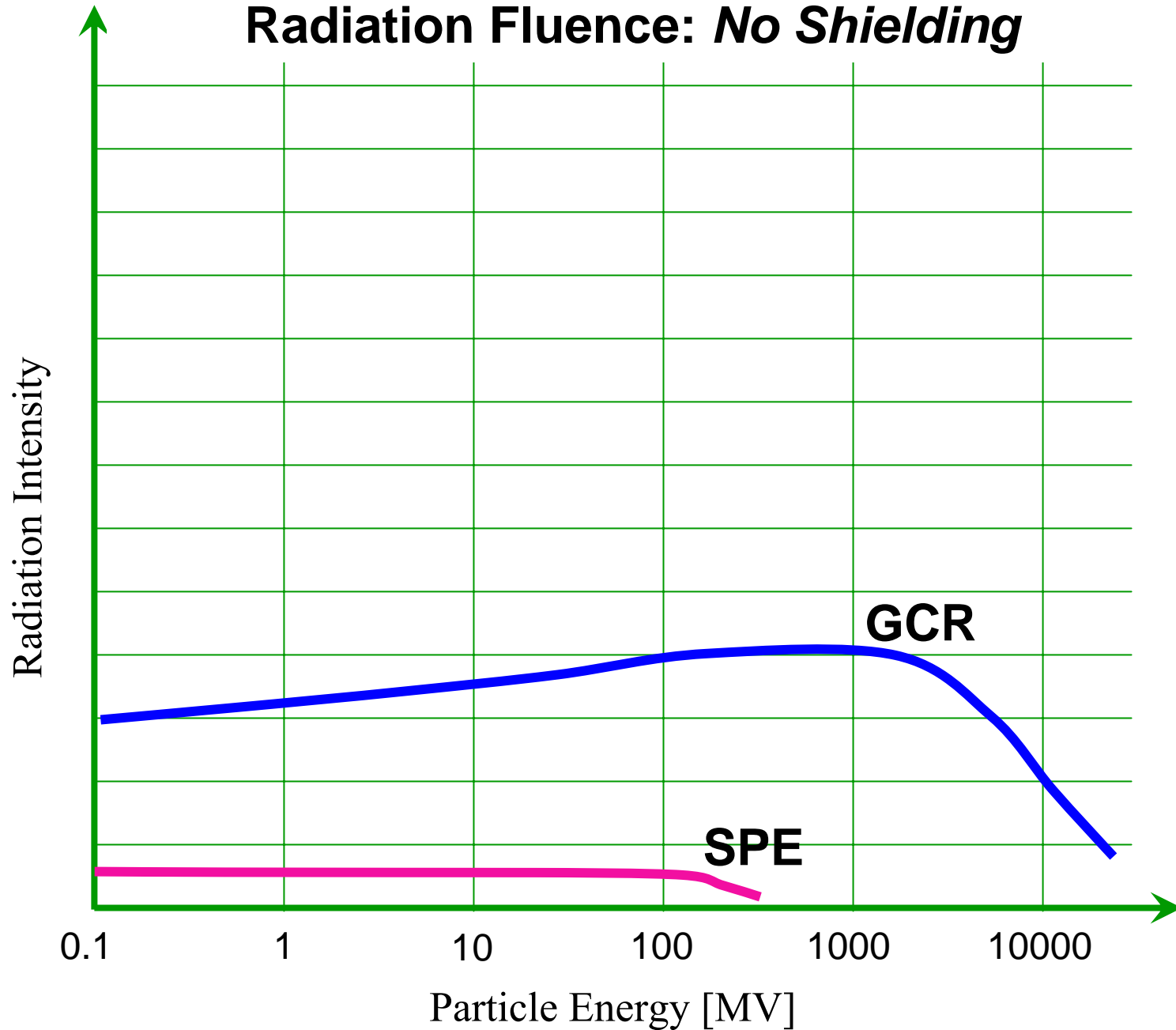
Radiation Fluence: *No Shielding*



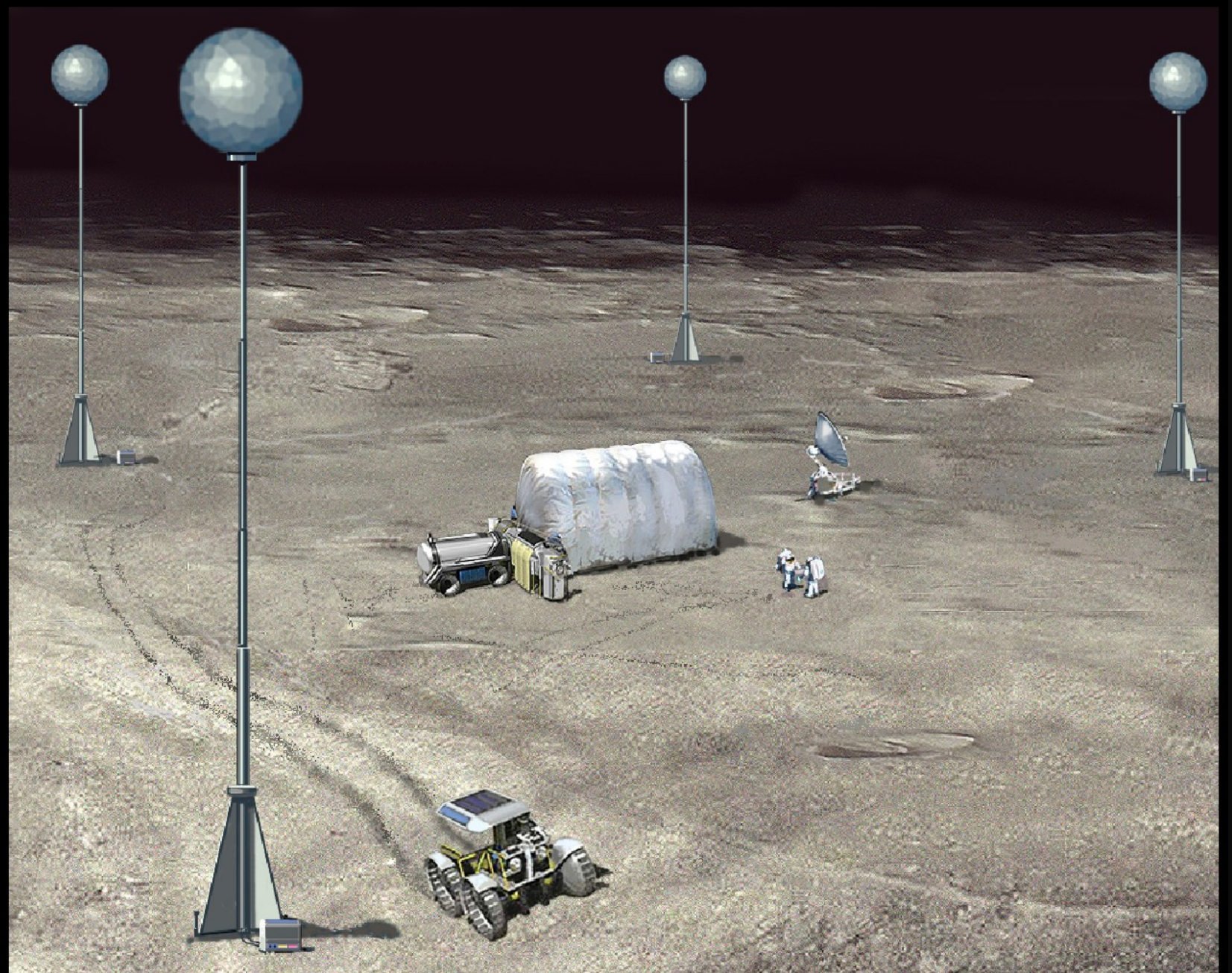
Radiation Fluence: *No Shielding*



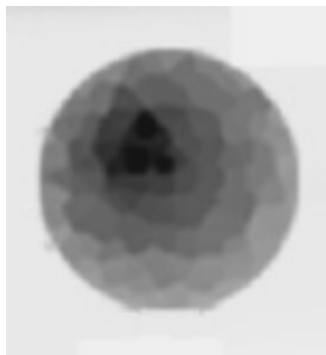
Radiation Fluence: *No Shielding*



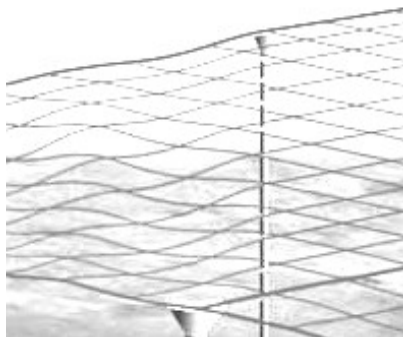
An Electrostatic Shield Design



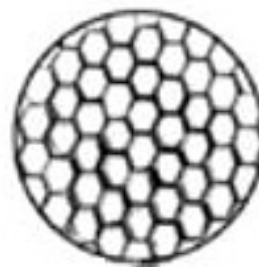
Some Possible Electrode Geometries



Thin Film Polymer
with Conductive
Inner Coating



Conductive
Mesh Screen

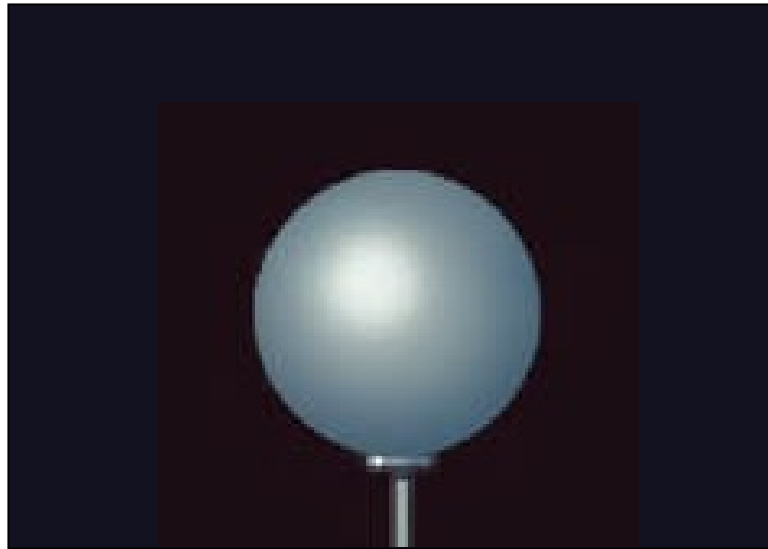


Conductive
Wire Mesh
Sphere



Arbitrary
Geometry

Electrode Geometry Used in ASRC *Lunar Electrostatic Shield Model (LESM)*



Thin Film Polymer Balloon with
Conductive Inner Coating

Theory of Design

Lorentz Force

$$\mathbf{F} = q\mathbf{E} + q\mathbf{v} \times \mathbf{B}$$

Theory of Design

Coulomb Force

$$\mathbf{F} = q\mathbf{E} + q\mathbf{v} \times \mathbf{B}$$

Theory of Design

Coulomb Force

$$\mathbf{F} = q\mathbf{E}$$

Electrostatic Shield: *Use only a time independent electric field, i.e., $\mathbf{E} = 0$*

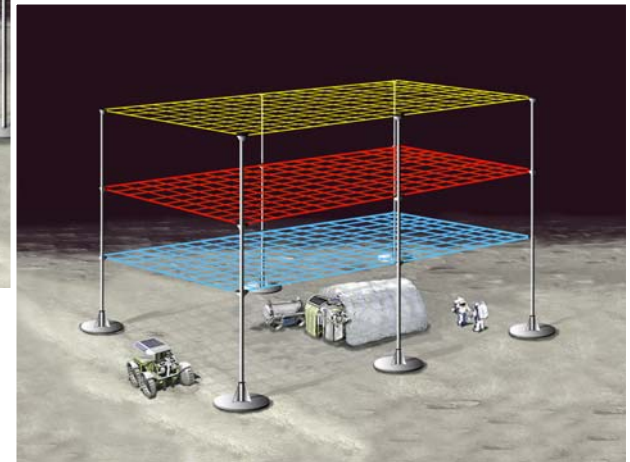
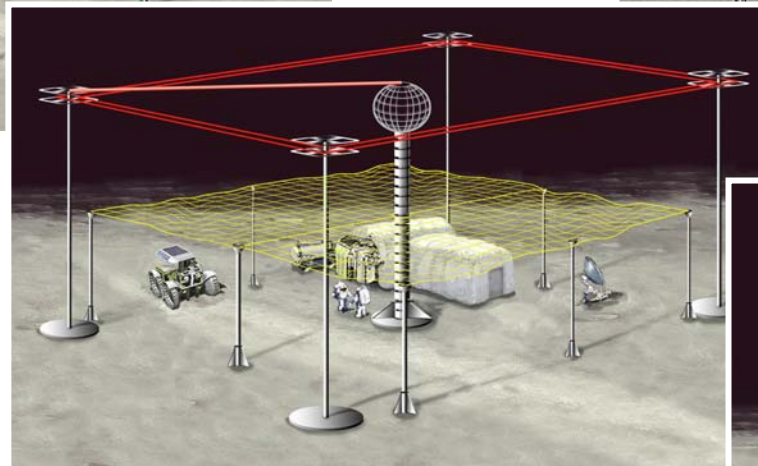
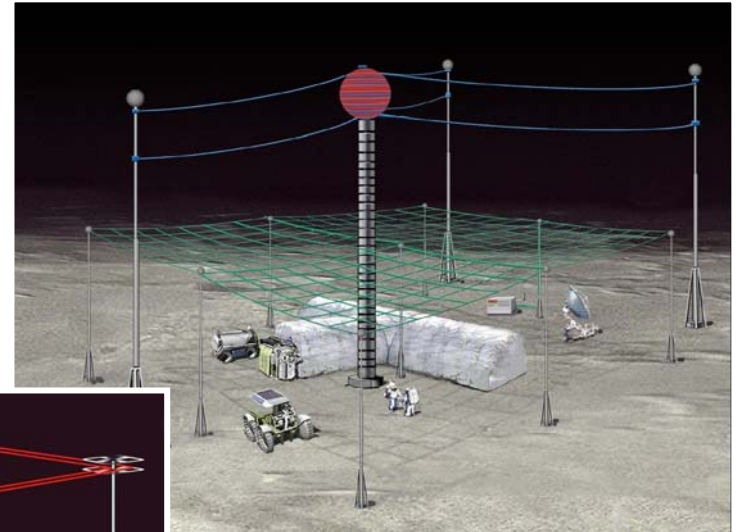
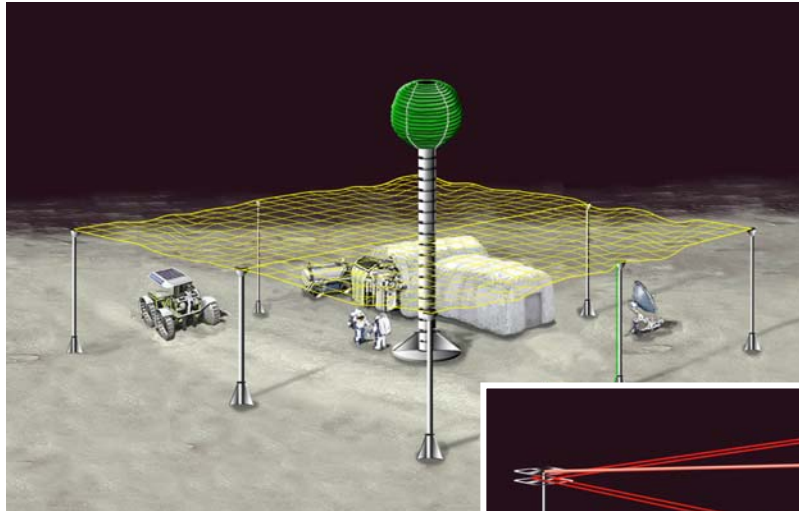
Design Strategy

STEP 1: Determine an Ideal Electric Field to Repel Charged Particle Radiation (primarily positive ions and electrons)



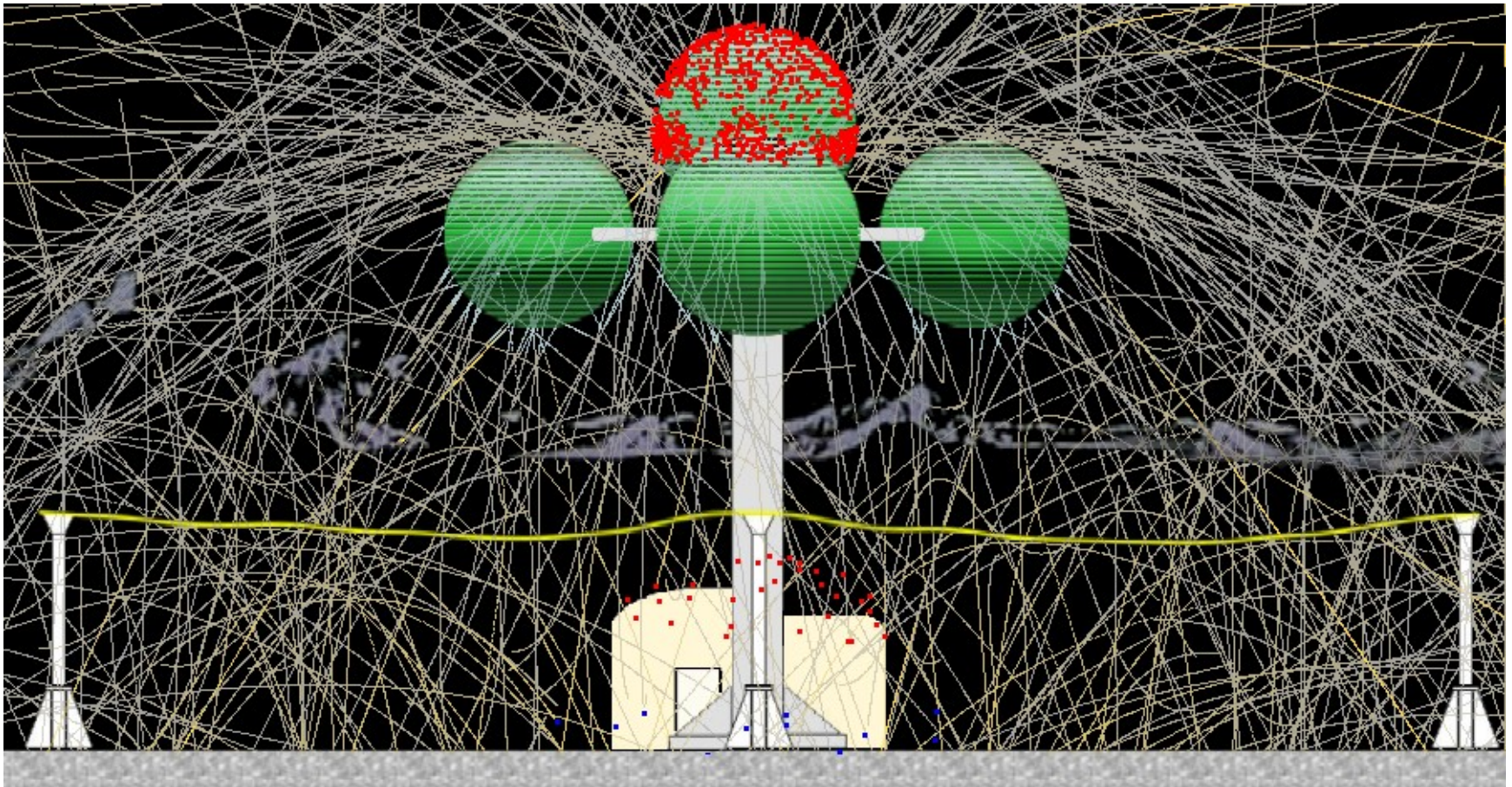
Design Strategy

STEP 2: Find a Way to Generate an Approximation of the Ideal Field



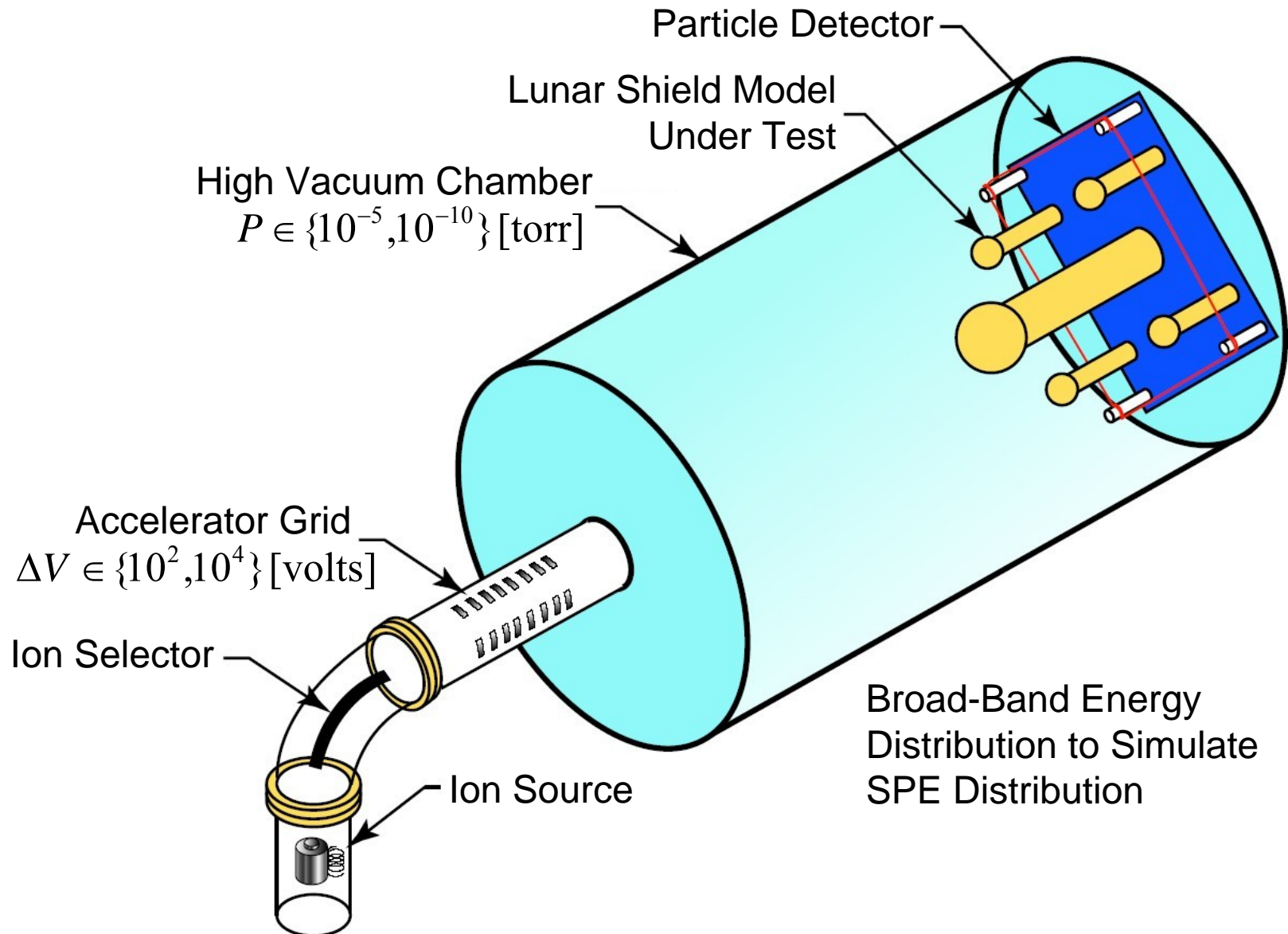
Design Strategy

STEP 3: Perform Mathematical Modeling and Computer Simulation of Proposed Configurations

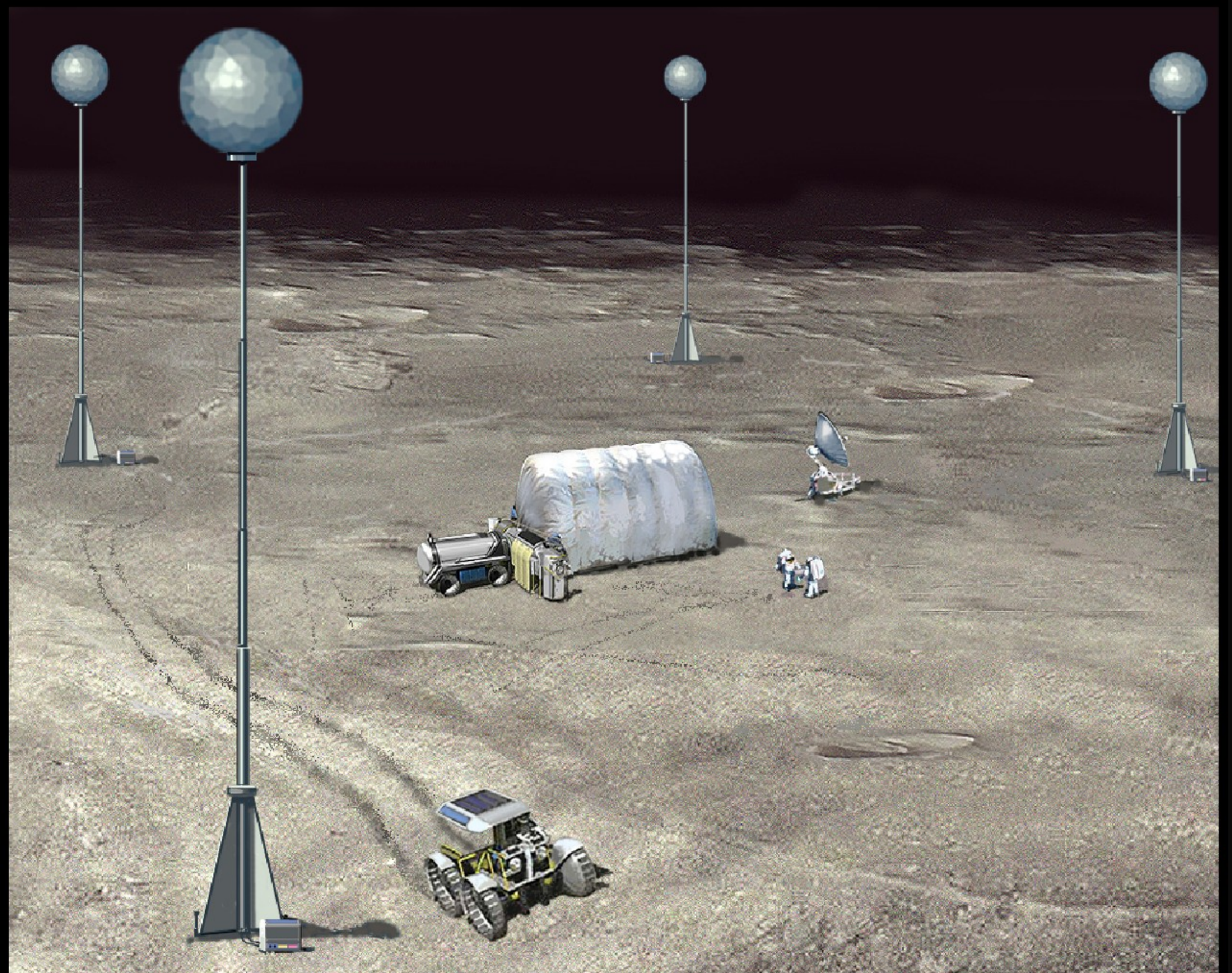


Design Strategy

STEP 4: Perform Experiments and Testing on a Scale Model



An Electrostatic Shield Design



Electrostatic Shield Design Constraints

Electrical

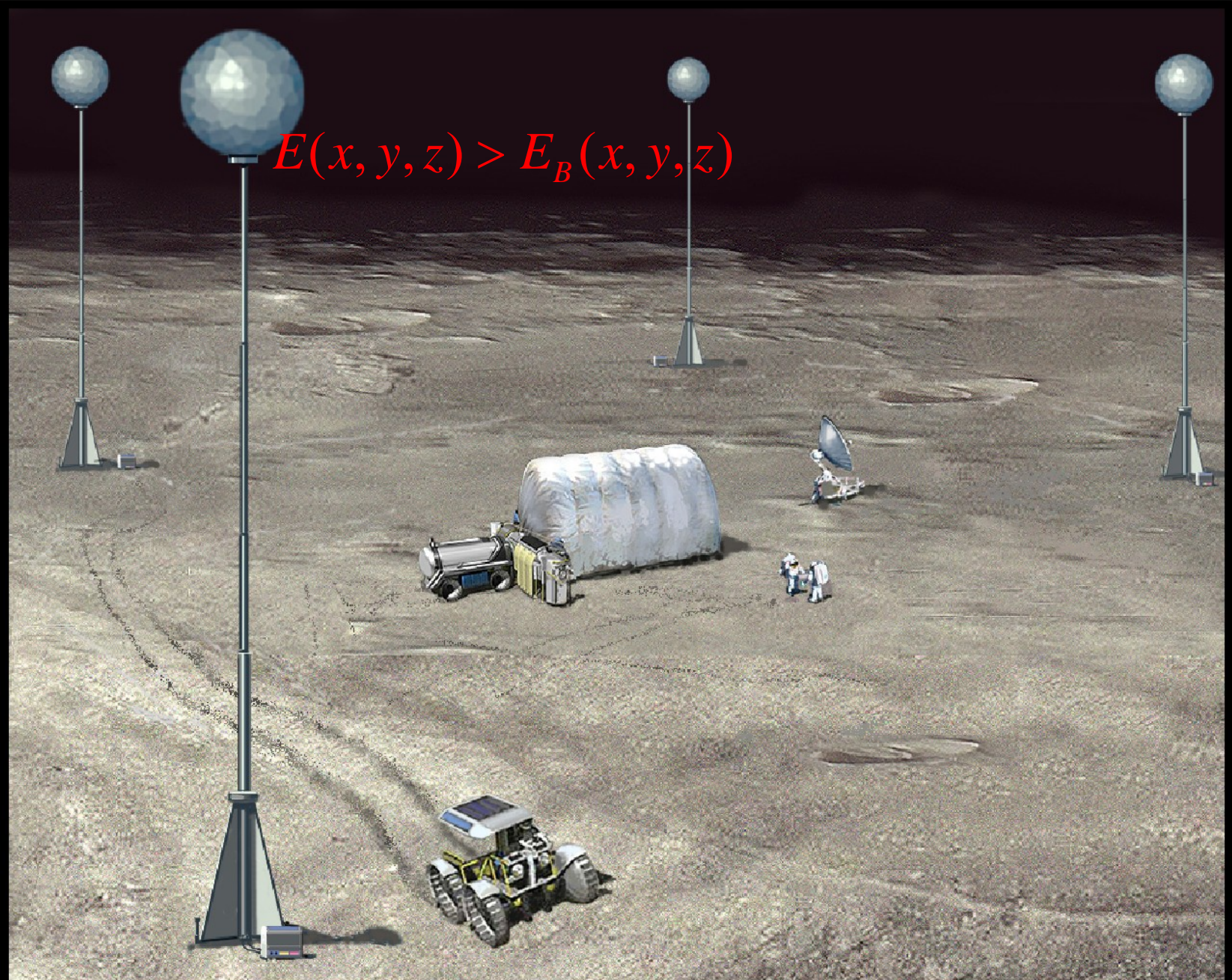
Non-Electrodes

- *Electric Field Strength* everywhere must remain well below a breakdown threshold value: $E(x, y, z) < E_B(x, y, z)$
In the case of non-conductors, $E_B(x, y, z)$ is related to the *Dielectric Strength* of the materials subjected to $E(x, y, z)$.

Electrodes

- *Surface Charge Distribution* must remain well below a threshold breakdown value: $\sigma(x, y, z) < \sigma_B(x, y, z)$
In the case of conductors, when $\sigma(x, y, z)$ exceeds $\sigma_B(x, y, z)$, the Coulomb force expels charge from the surface of the conductor.

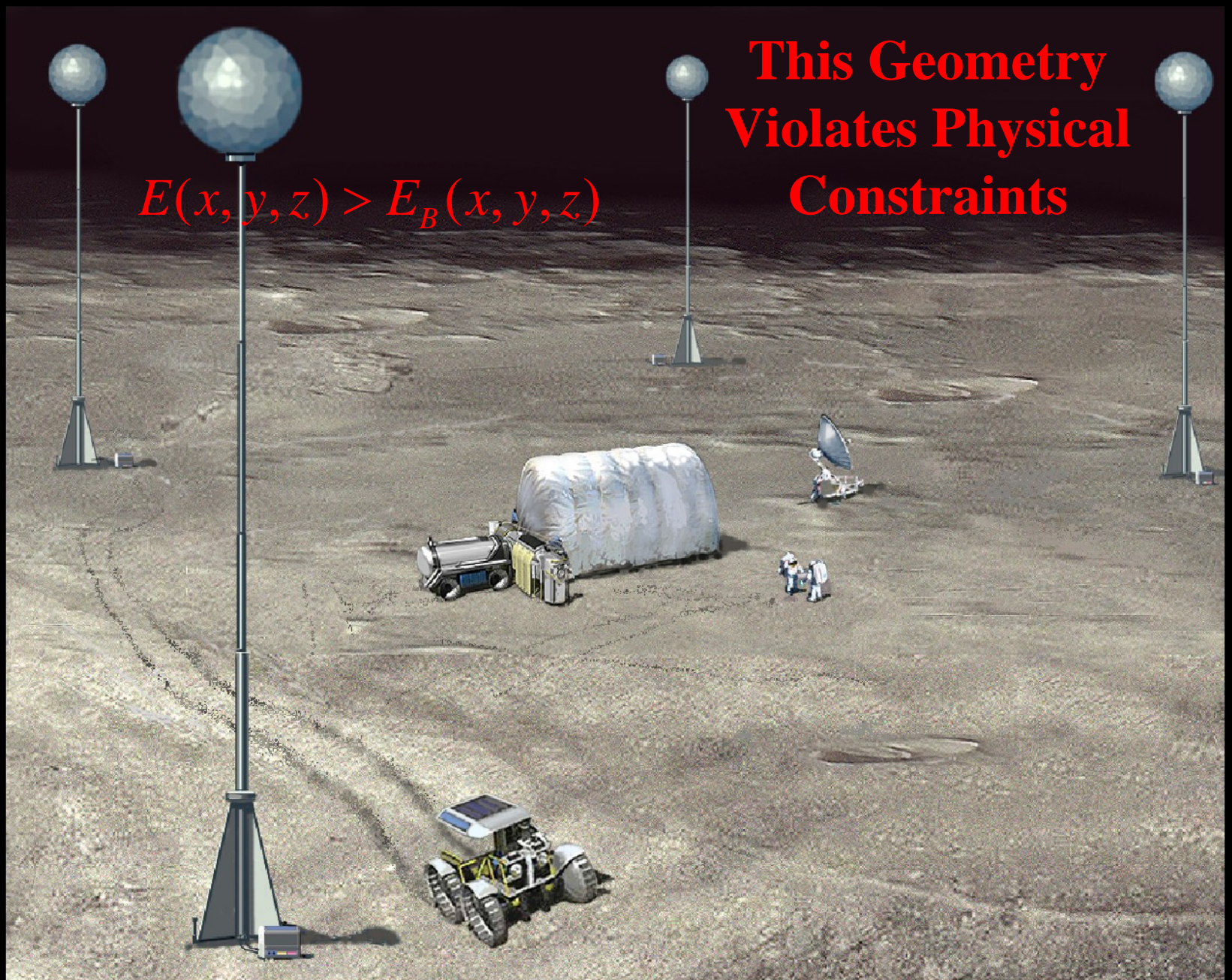
Electrostatic Shield Design Constraints



Electrostatic Shield Design Constraints

$$E(x, y, z) > E_B(x, y, z)$$

**This Geometry
Violates Physical
Constraints**



Electrostatic Shield Design Constraints

Mechanical

Forces

- The Coulomb forces between electrodes must not exceed the mechanical strength of the materials. In the case of thin film polymers, for example, the tensile strength can not be exceeded.

Size and Weight

- Size and weight are limited by considerations related to transportation to the lunar surface and by practical assembly and construction activities.

Electrostatic Shield Design Constraints

Power, Dust, and X-Rays

Power

- Collision of charged particles with electrodes leads to a current, which must be minimized in order to constrain power requirements.

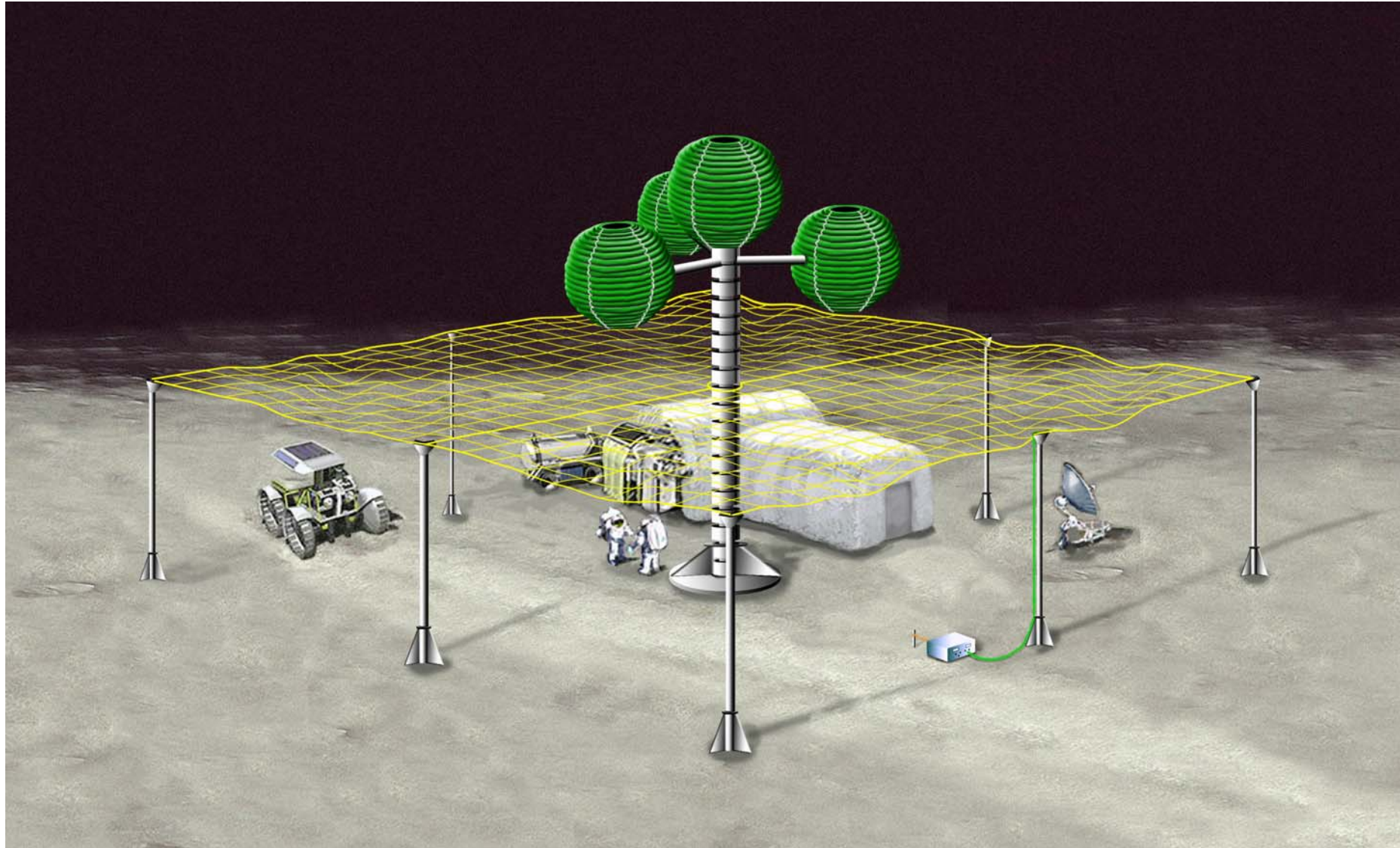
Surface Dust and Free Electrons

- The design must avoid attraction of surface dust and electrons to the high voltage electrodes.

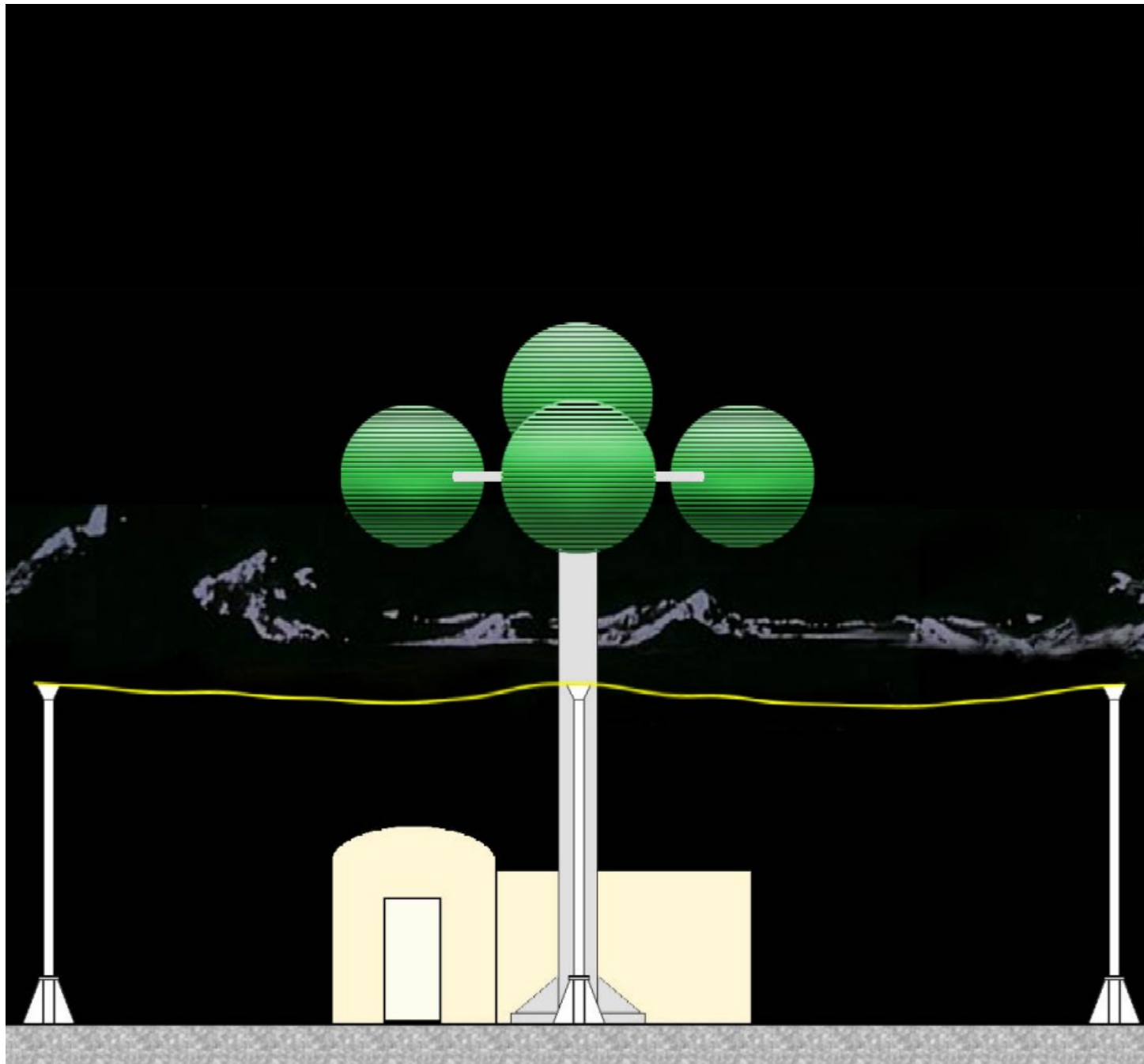
Bremsstrahlung X-Rays

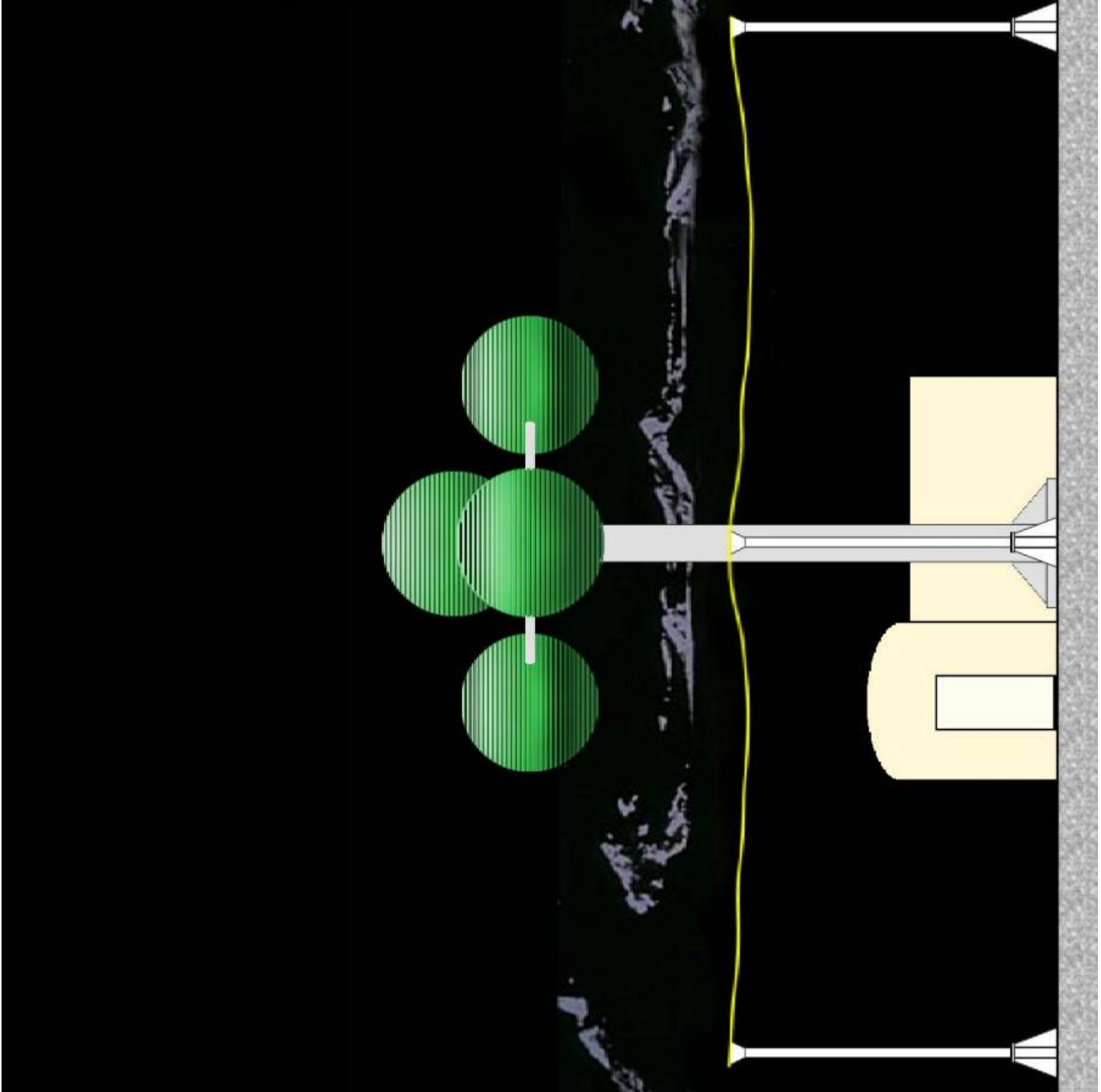
- Solar wind electrons accelerated by high voltage positive electrodes, must not be allowed to decelerate due to collisions with the electrodes.

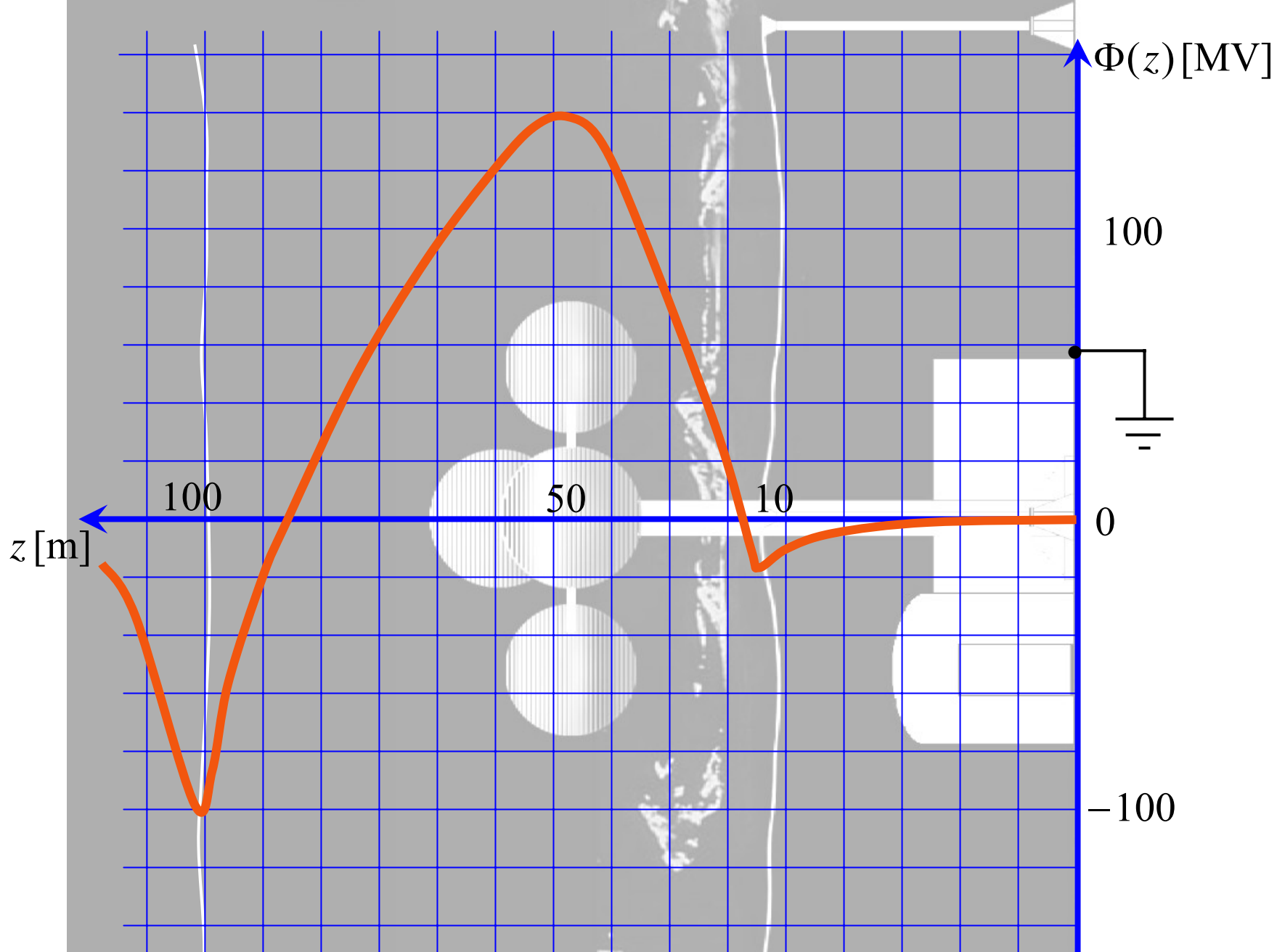
Four Positive Sphere Electrodes



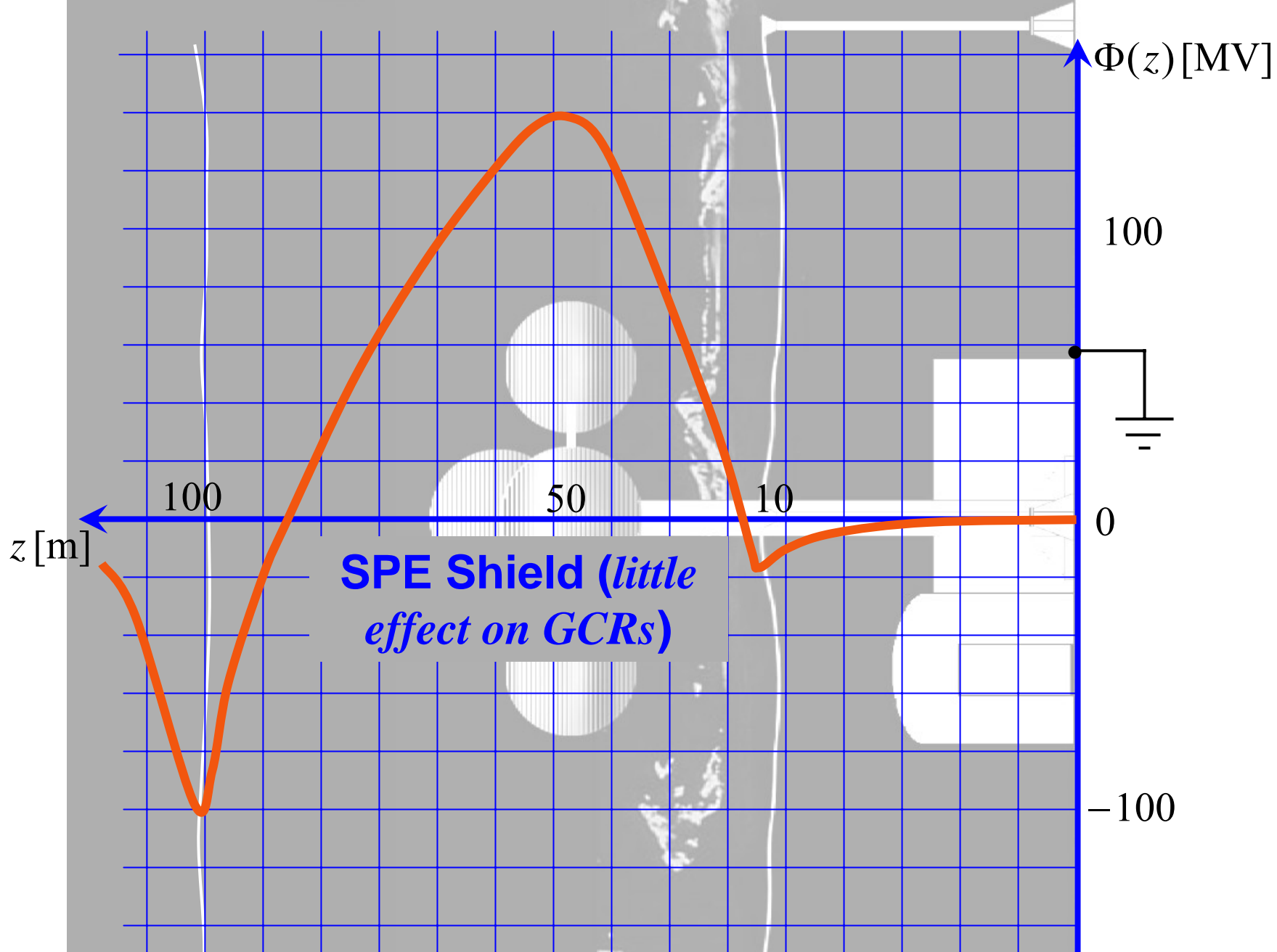
Front View of Lunar Habitat and Positive High Voltage Shield





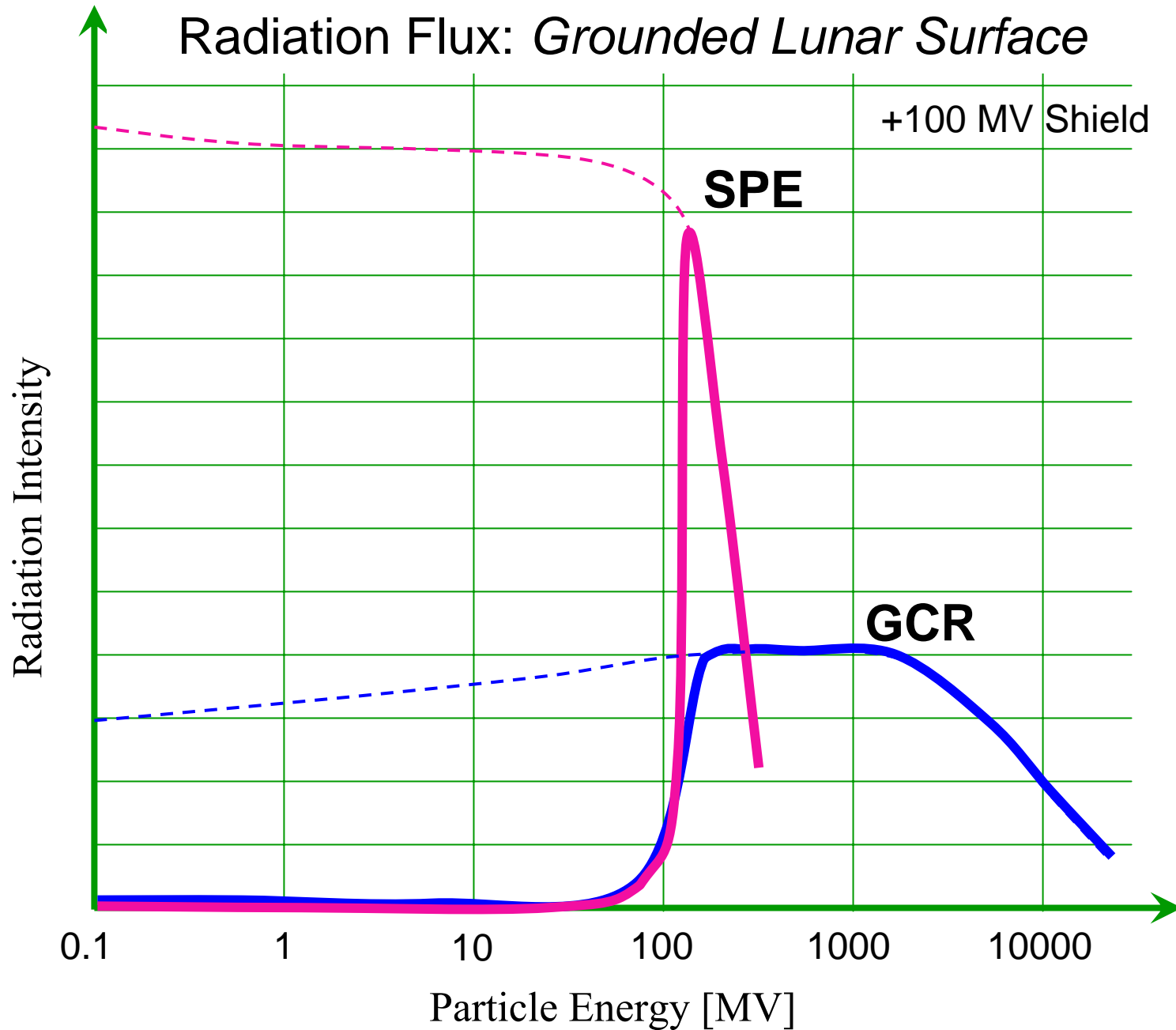


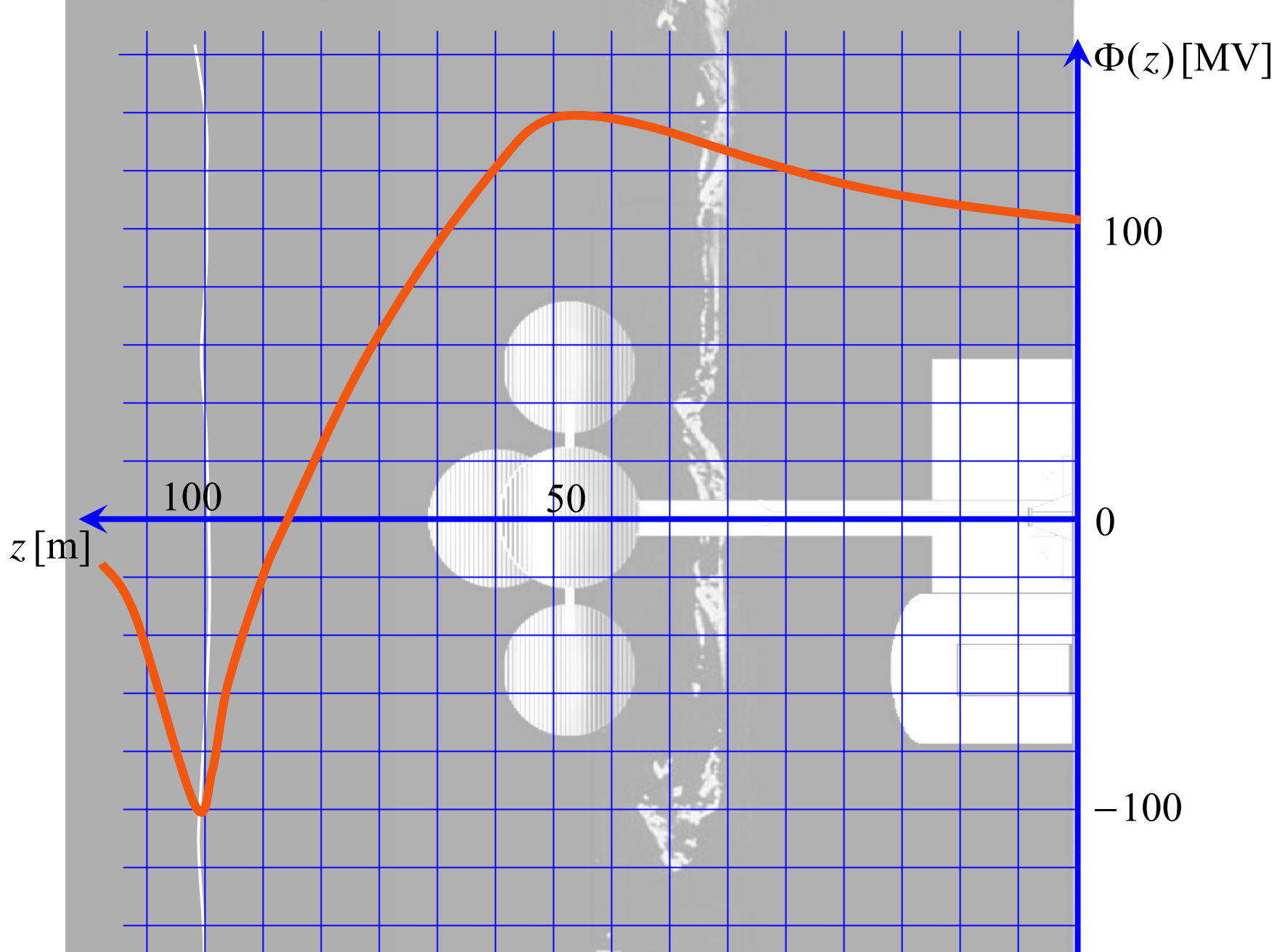
Voltage Potential Profile – Assume: *Grounded* Surface



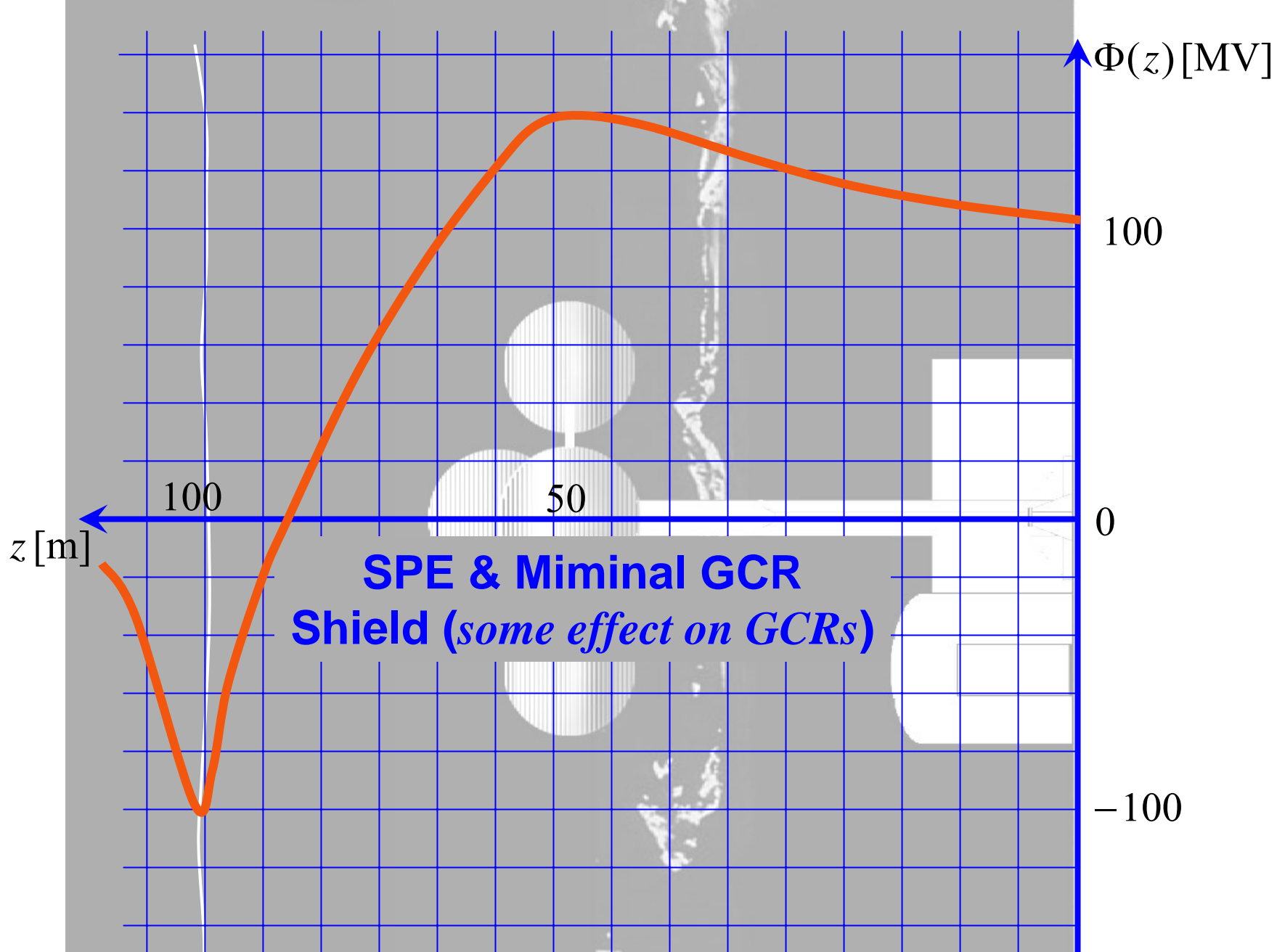
Voltage Potential Profile – Assume: *Grounded Surface*

Radiation Flux: *Grounded Lunar Surface*



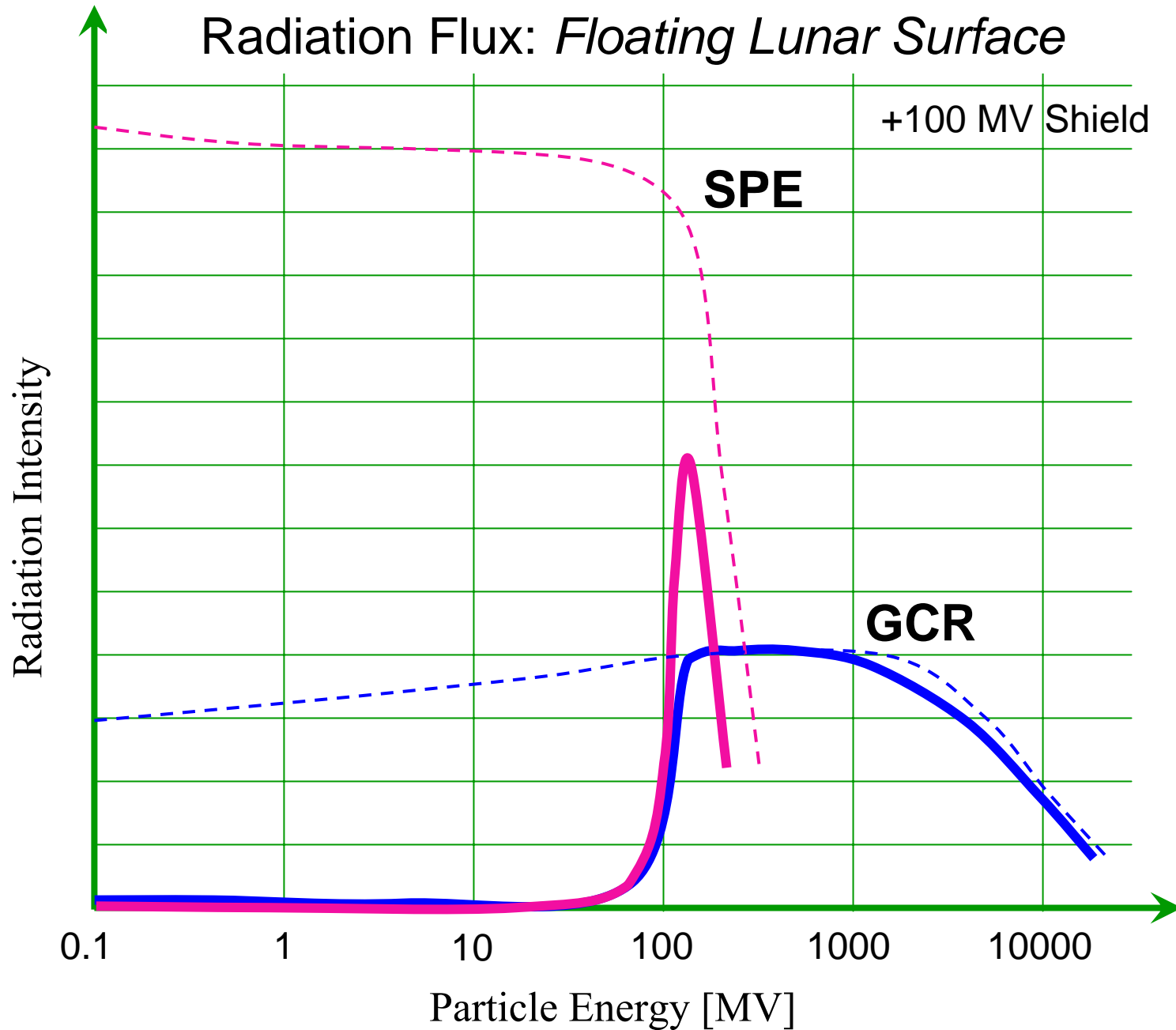


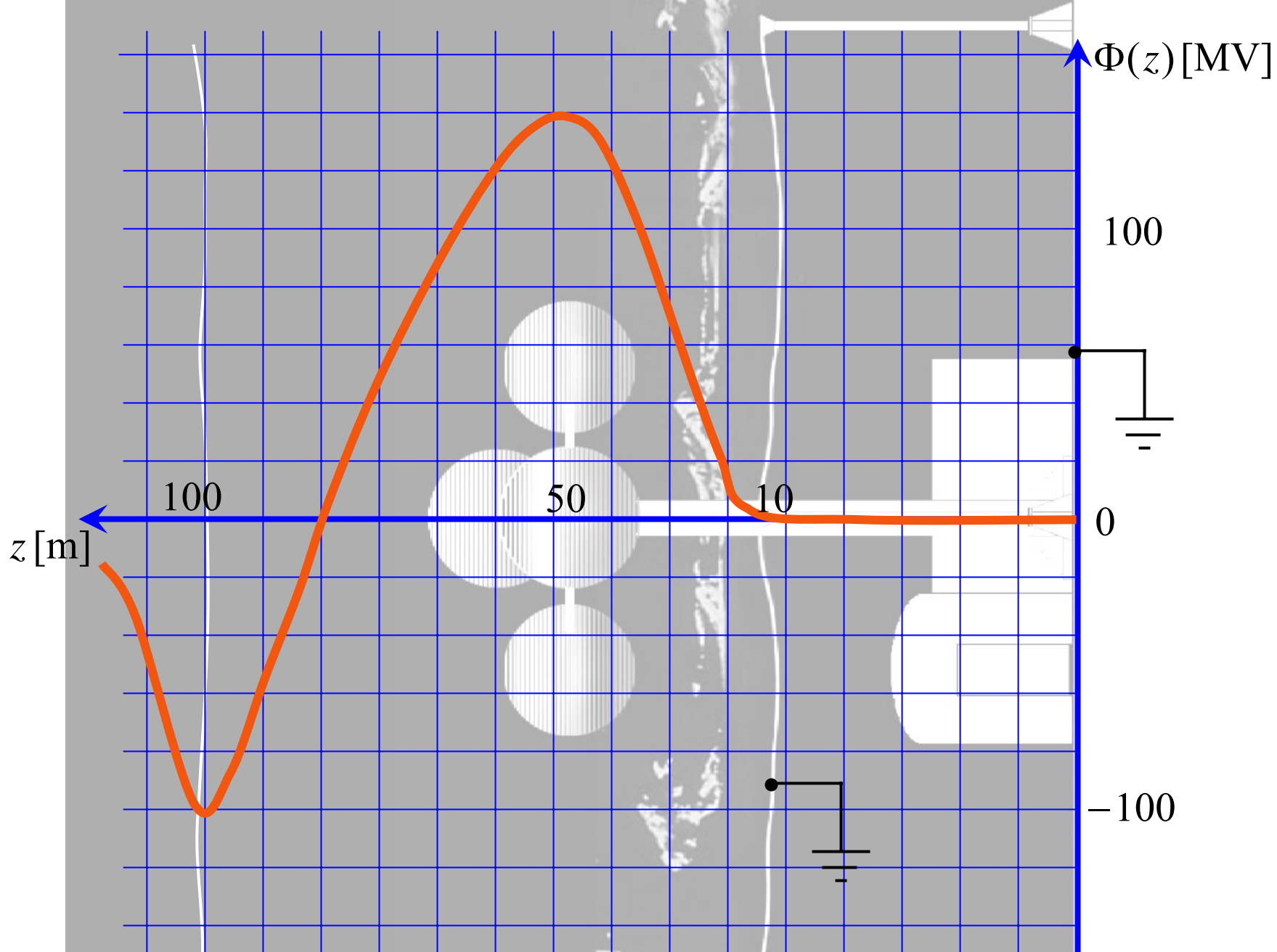
Voltage Potential Profile – Assume: *Floating Surface*



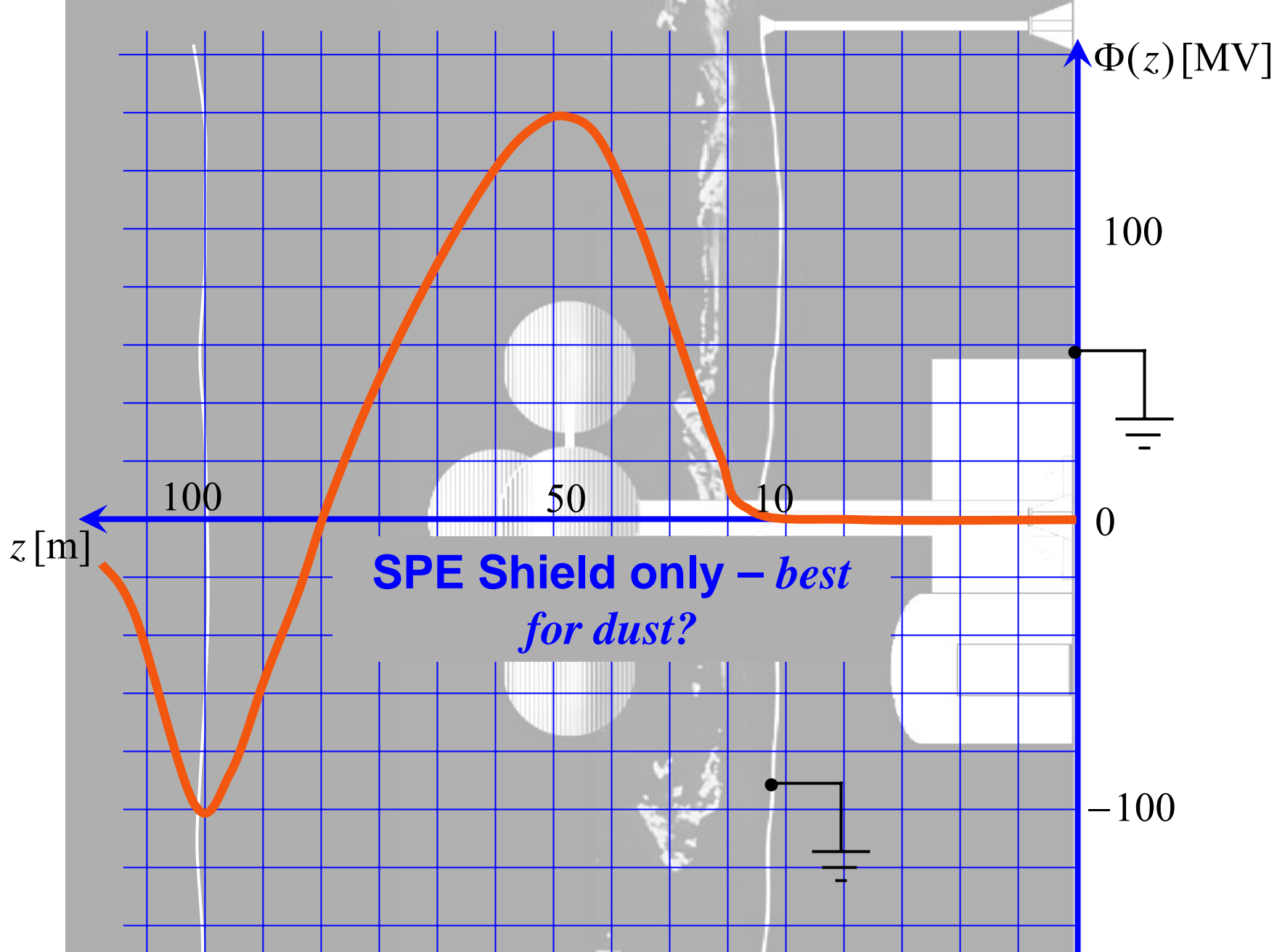
Voltage Potential Profile – Assume: *Floating* Surface

Radiation Flux: *Floating Lunar Surface*

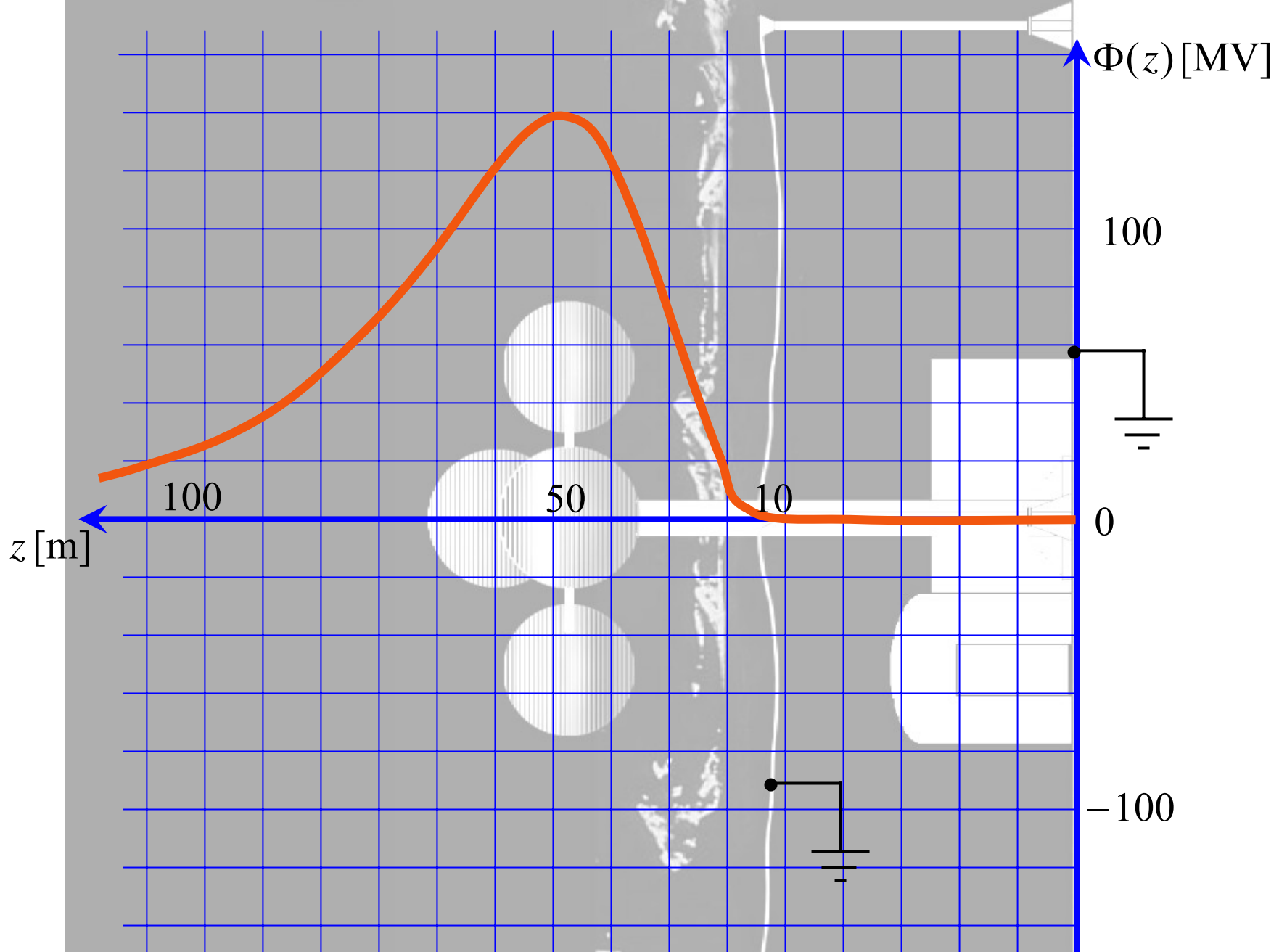




Voltage Potential Profile – Ground *Ground Shield*

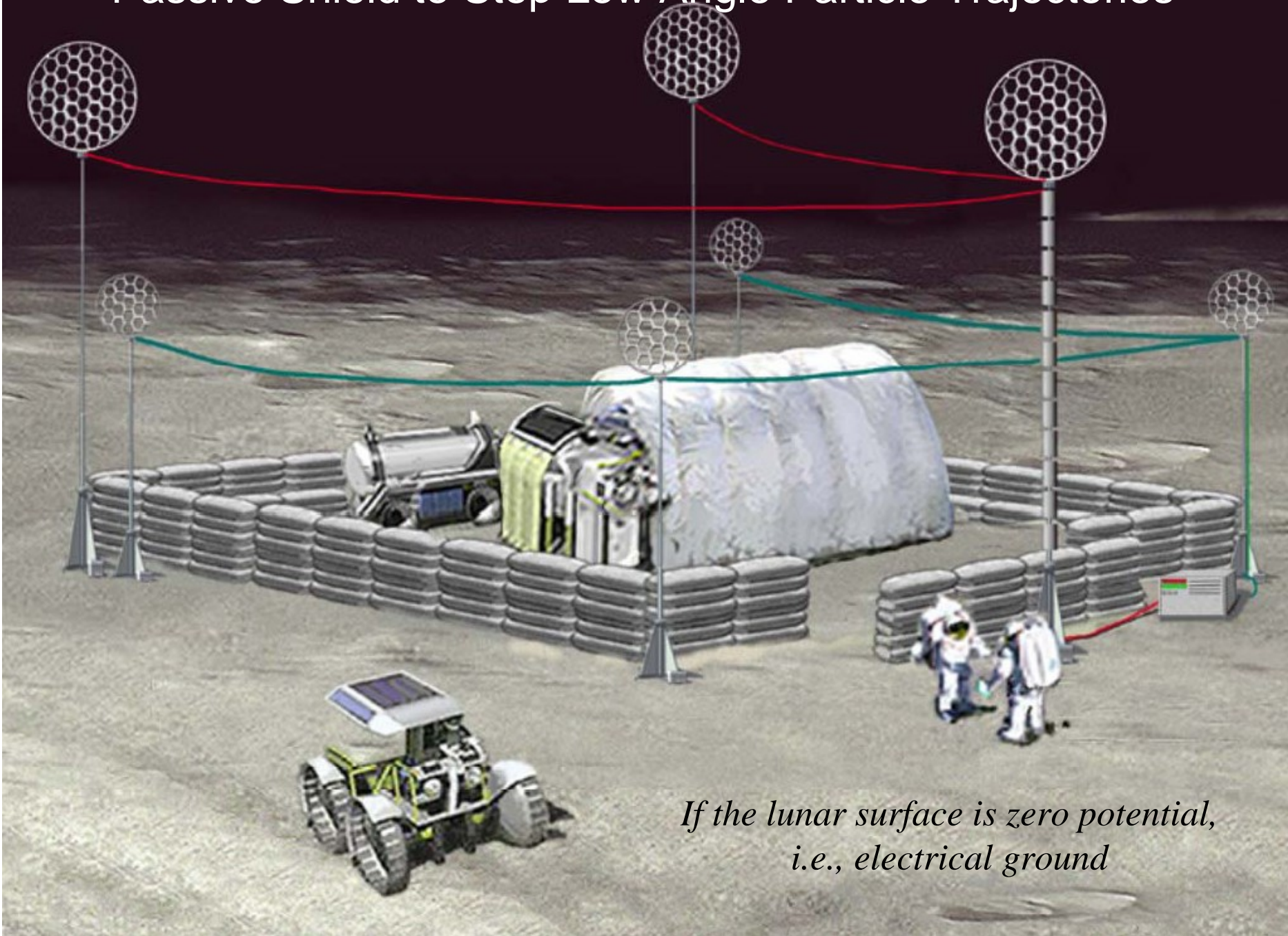


Voltage Potential Profile – Ground *Ground Shield*



Voltage Potential Profile – Use **B** Field to Shield Electrons

Passive Shield to Stop Low Angle Particle Trajectories



*If the lunar surface is zero potential,
i.e., electrical ground*

Radiation Fluence, Shield Transmission, Biological Response, and Dosage

SPE

Radiation Intensity

0.1

1

10

100

1000

10000

Particle Energy [MV]

$\xi(E)$

Shield Efficiency

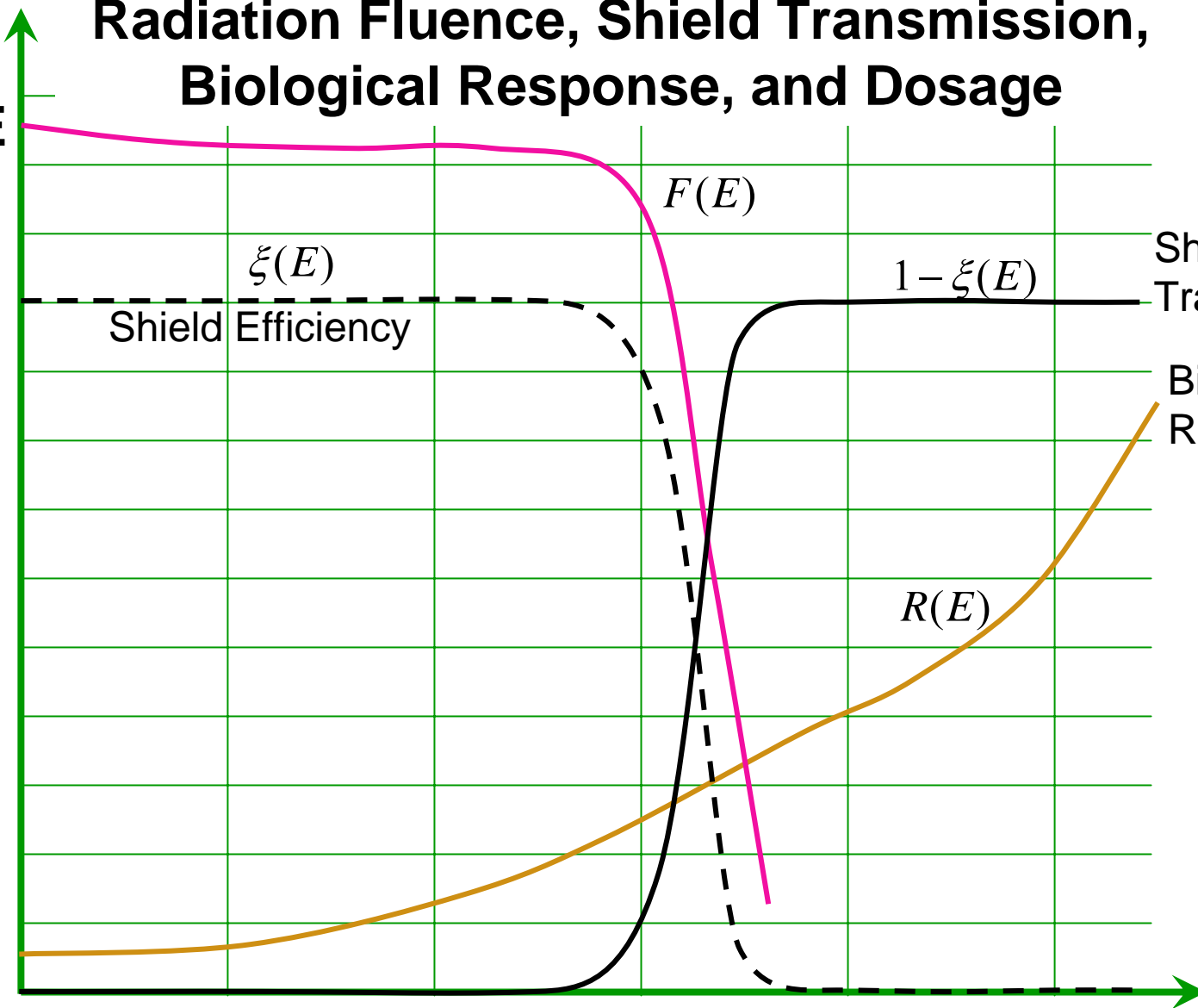
$F(E)$

$1 - \xi(E)$

Shield Transmission

Biological Response

$R(E)$



Radiation Fluence, Shield Transmission, Biological Response, and Dosage

SPE

Radiation Intensity

Shield Efficiency

Dosage (without Shielding):

$$D_0 \approx \int F(E)R(E)dE$$

Biological Response

$R(E)$

$F(E)$

0.1

1

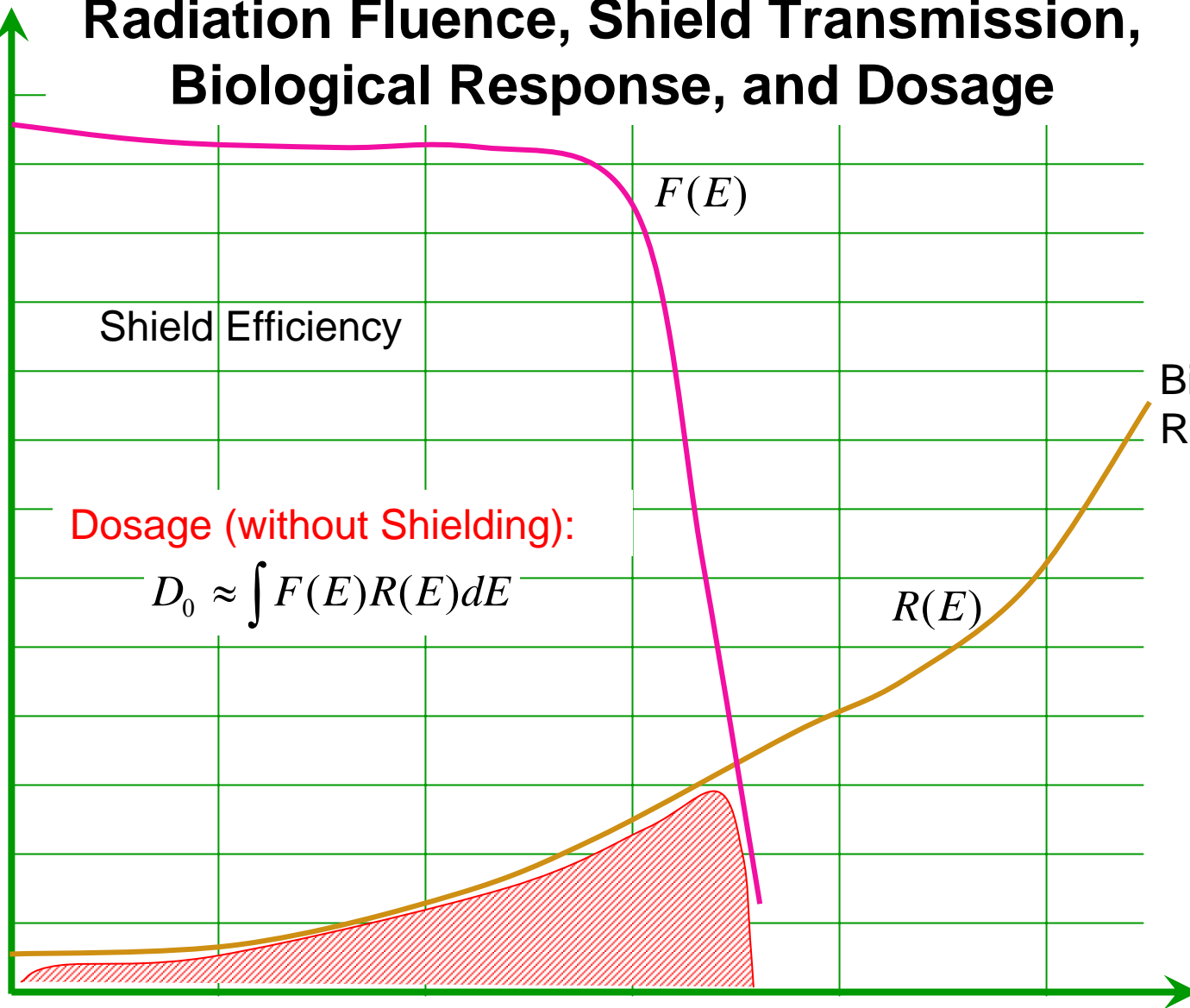
10

100

1000

10000

Particle Energy [MV]



Radiation Fluence, Shield Transmission, Biological Response, and Dosage

SPE

Radiation Intensity

Dosage with Shielding:

$$D \approx \int (1 - \xi(E)) F(E) R(E) dE$$

$F(E)$

$1 - \xi(E)$

Biological Response

$R(E)$

0.1

1

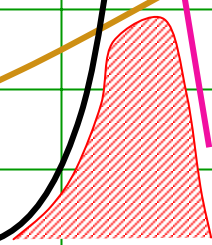
10

100

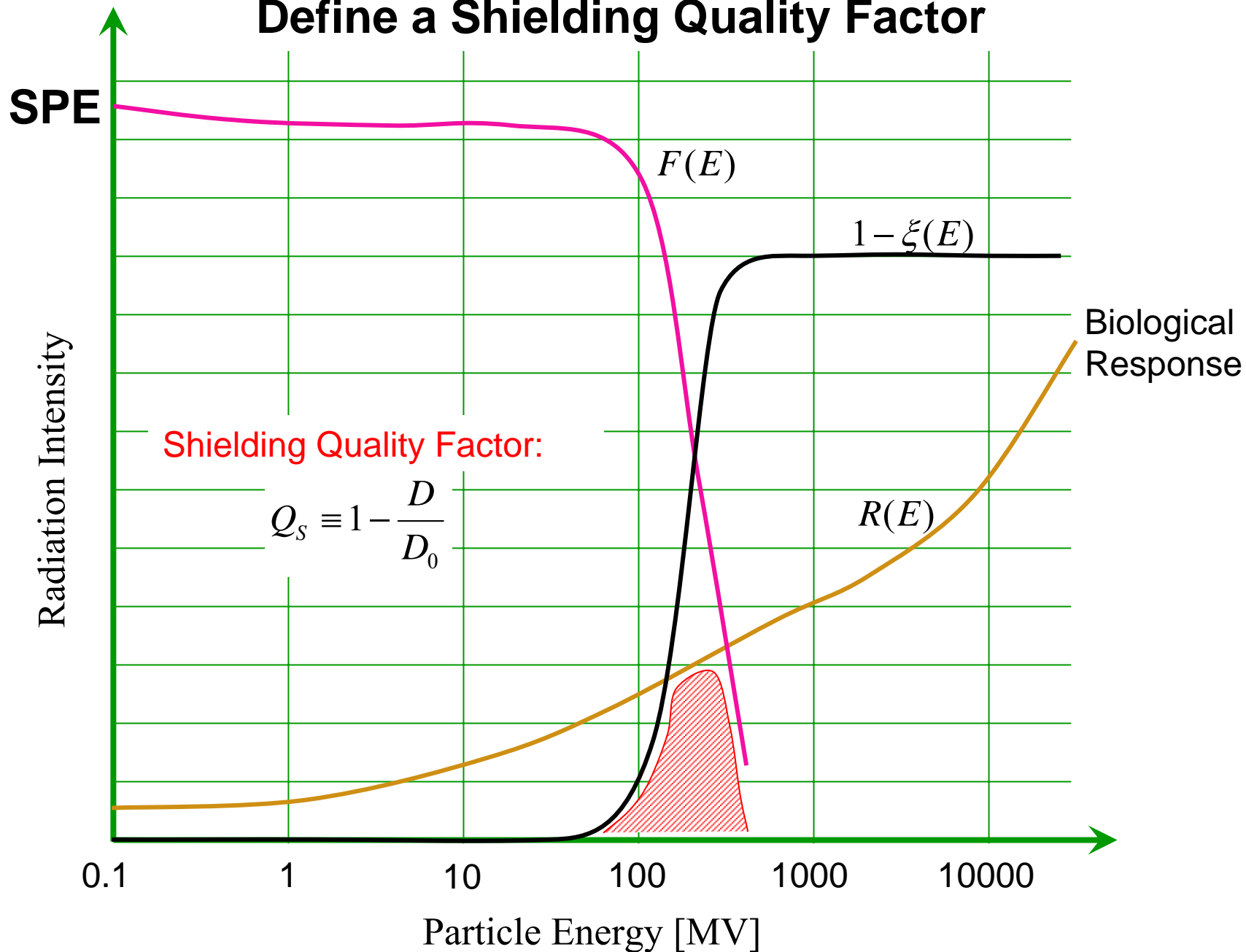
1000

10000

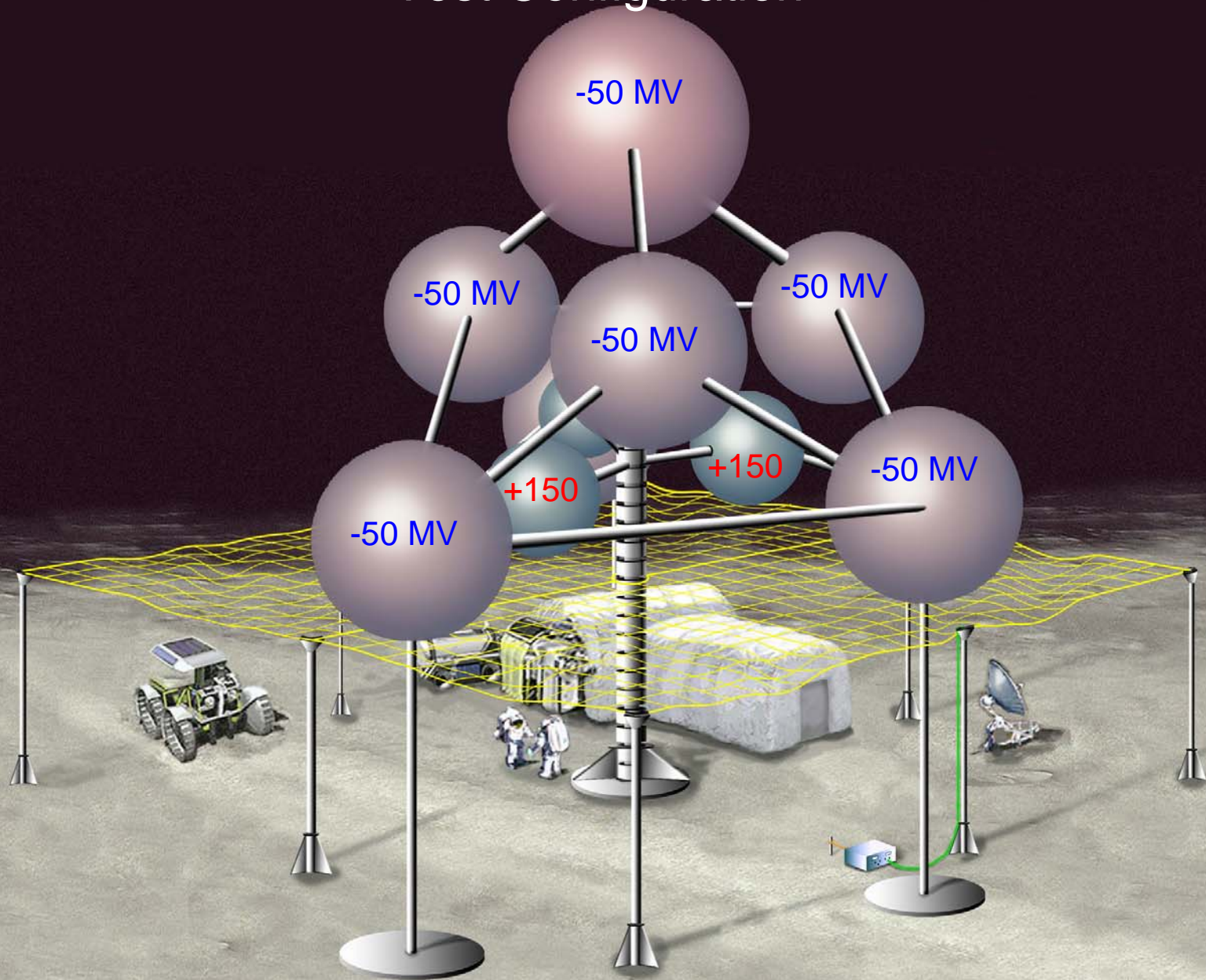
Particle Energy [MV]



Define a Shielding Quality Factor



Test Configuration



Simulation Run of Lunar Electrostatic Shield Model (LESM v1.2)

– User Interface and Sphere Configuration File

Lunar Electrostatic Shield Model (LESM) v1.2

Model Length Parameters

Ro = 20 [m] Ro' = 30 [m]

rho0 = 4 [m] rhoP = 30 [m]

Charged Particle Energy

Particle#1: Eo = 50.0 [MeV] sigE = 10.0 [MeV]

Particle#2: Eo = 5.0 [MeV] sigE = 1.0 [MeV]

Particle Composition

Particle#1: Ne = 0 Np = 1 Nn = 0

Particle#2: Ne = 1 Np = 0 Nn = 0

Monte Carlo Parameters

of particles = 10000 seed = 425001

☒ Semi-Sphere Isotropic Radiation (Lunar Version)

Plot Parameters

☒ Disengage Shield ☒ x-y

☐ Suppress Trails ☒ x-z

☐ Sliced Plots ON ☒ yzPlot.bmp ☒ y-z

☒ Wide Points

rho-Start Radius of Slices = 0.0 [m]

Separation Between Slices = 1.0 [m]

Base Filename = Rho

☐ Sphere Potential Plot Shading ON

Recursion Parameters

delta_t = 1.0 [ns] max iterations = 10000

delta_x = 0.1 [m]

Classical / Relativistic Mechanics

☒ Relativistic Acceleration ON

Program Output Log

v0(1) = 0.31582 [c] (initial particle velocity, fraction of c, corresponding to Eo(1))

v0(2) = 0.99578 [c] (initial particle velocity, fraction of c, corresponding to Eo(2))

fc = 41.96000 [%] (total particle / sphere collisions)

ft = 3.23000 [%] (percentage of particles arriving at rho0 protected radius)

vmin = 0.16051 [c] (minimum particle v of all particles, as a fraction of c)

vmax = 0.99854 [c] (maximum particle v of all particles, as a fraction of c)

vavg = 0.66046 [c] (average particle v of all particles, as a fraction of c)

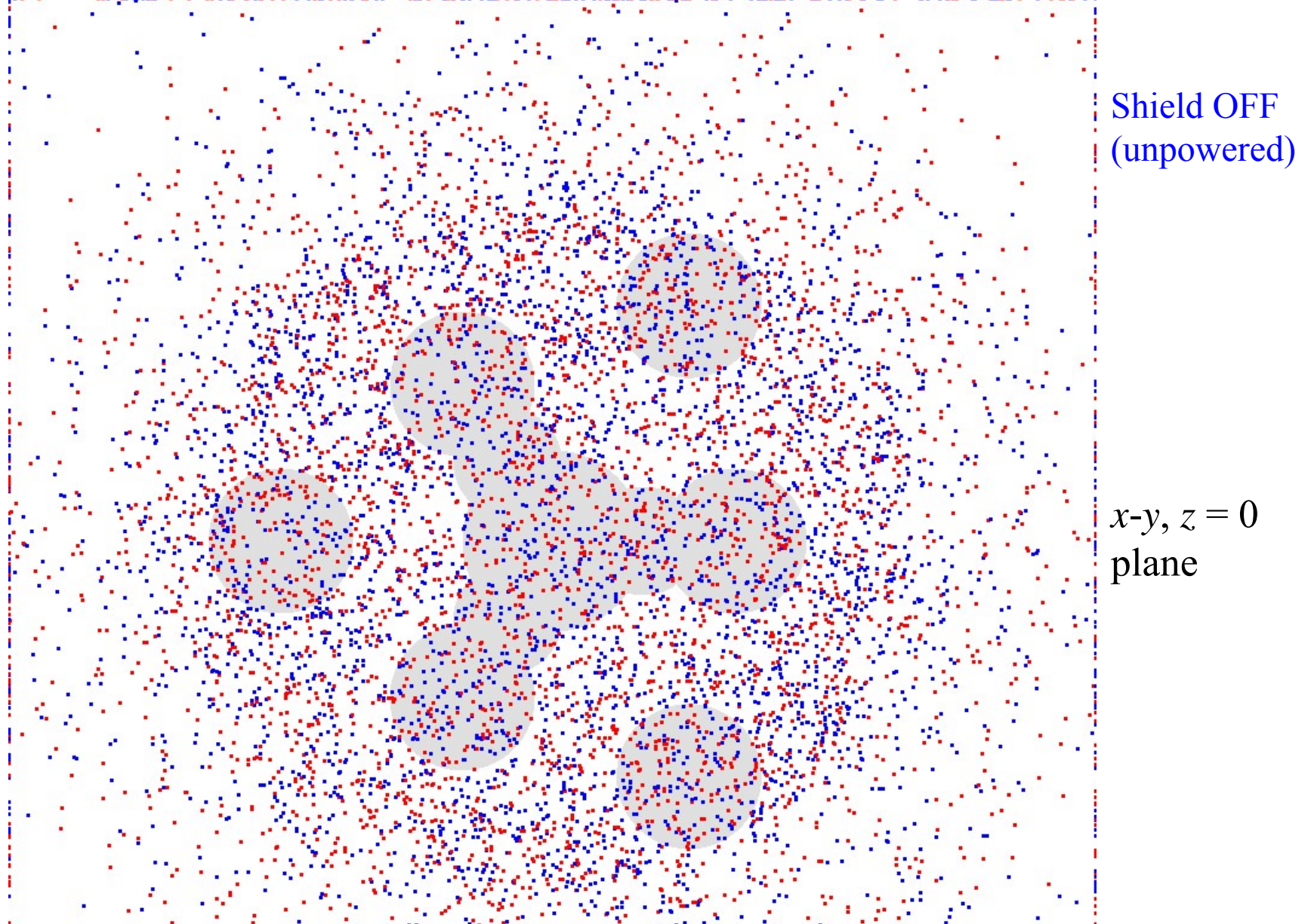
Buttons: Apply Run Status: Done Processing Exit

Shield OFF
(unpowered)

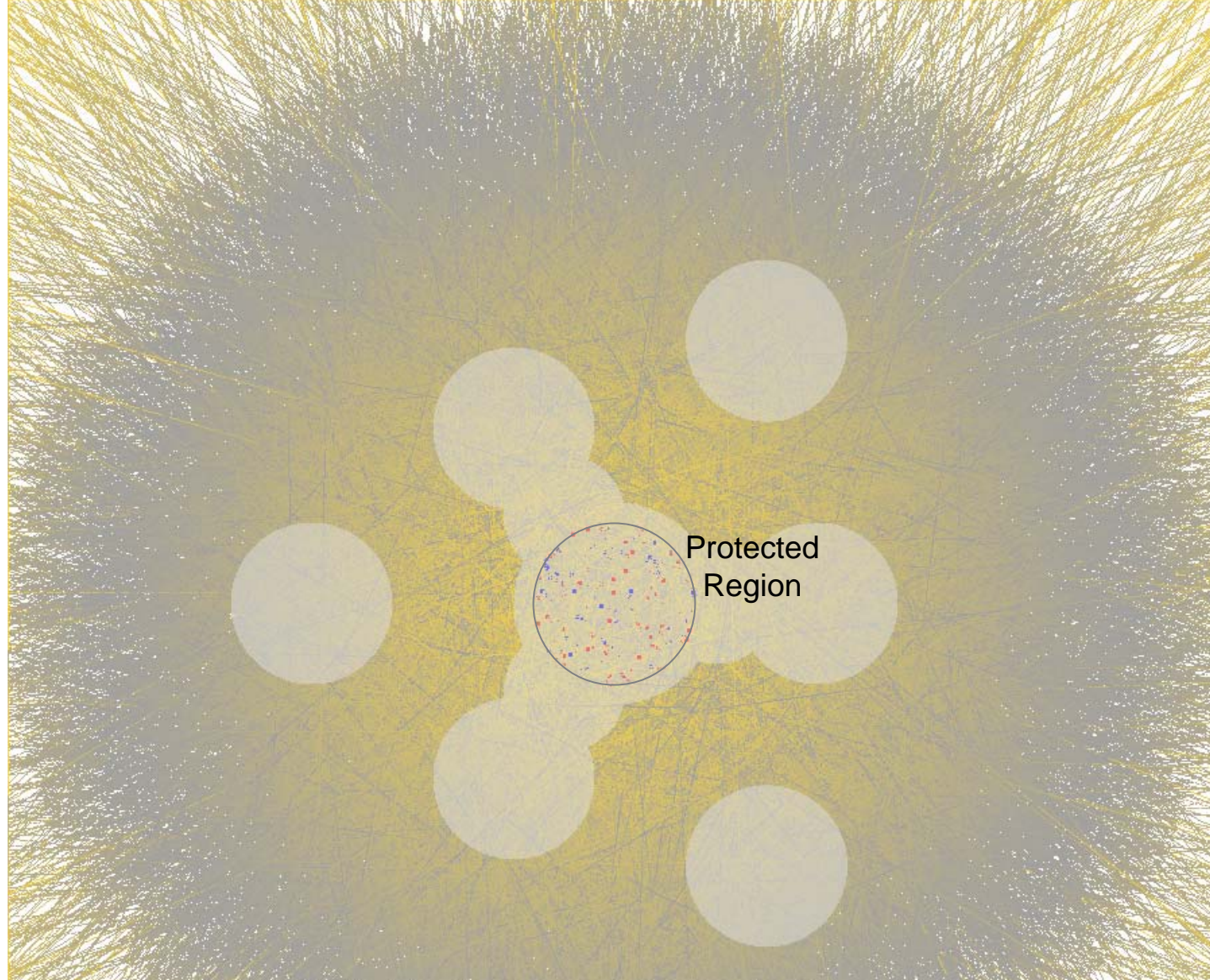
Sphere Configuration File
Shaded Entries Correspond To Image Charges Below Lunar Surface

20 Number of Spheres

V [MV]	R [m]	x [m]	y [m]	z [m]
150.0	3.0	5.0	0.0	8.0
150.0	3.0	-2.5	4.33	8.0
150.0	3.0	-2.5	-4.33	8.0
-50.0	4.0	10.0	0.0	12.0
-50.0	4.0	-5.0	8.66	12.0
-50.0	4.0	-5.0	-8.66	12.0
-50.0	5.0	0.0	0.0	16.0
-50.0	4.0	-15.0	0.0	8.0
-50.0	4.0	7.5	12.99	8.0
-50.0	4.0	7.5	-12.99	8.0
-150.0	3.0	5.0	0.0	-8.0
-150.0	3.0	-2.5	4.33	-8.0
-150.0	3.0	-2.5	-4.33	-8.0
50.0	4.0	10.0	0.0	-12.0
50.0	4.0	-5.0	8.66	-12.0
50.0	4.0	-5.0	-8.66	-12.0
50.0	5.0	0.0	0.0	-16.0
50.0	4.0	-15.0	0.0	-8.0
50.0	4.0	7.5	12.99	-8.0
50.0	4.0	7.5	-12.99	-8.0



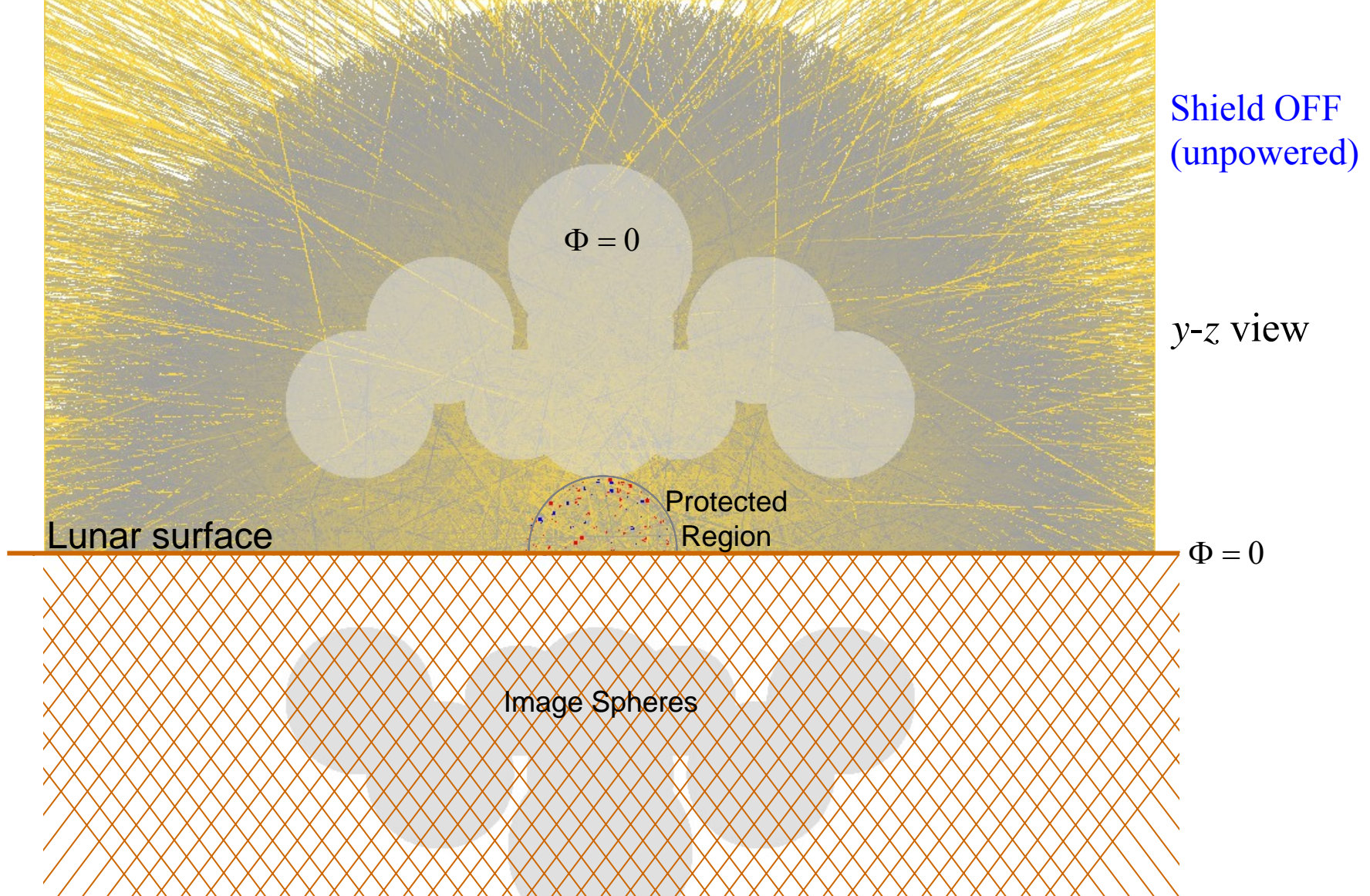
Simulation Run of Lunar Electrostatic Shield Model (LESM v1.2) – Red dots are intersection of electrons and blue dots are intersection of protons with lunar surface ($z = 0$). Gray circles are x-y projections of unpowered electrostatic spheres.



Shield OFF
(unpowered)

x-y view

Simulation Run of Lunar Electrostatic Shield Model (LESM v1.2) – Red dots are intersection of electrons and blue dots are intersection of protons with a 4 [m] radius sphere (protected area) centered at $x = 0$, $y = 0$, $z = 0$. Gray circles are x-y projections of unpowered electrostatic spheres. Yellow-gray trails are particle trajectory paths.



Simulation Run of Lunar Electrostatic Shield Model (LESM v1.2) – Red dots are intersection of electrons and blue dots are intersection of protons with a 4 [m] radius sphere (protected area) centered at $x = 0$, $y = 0$, $z = 0$. Gray circles are y-z projections of unpowered electrostatic spheres. Yellow-gray trails are particle trajectory paths.

Simulation Run of Lunar Electrostatic Shield Model (LESM v1.2) – User Interface and Sphere Configuration File

Lunar Electrostatic Shield Model (LESM) v1.2

Model Length Parameters

Ro = [m] Ro' = [m]

rho0 = [m] rhoP = [m]

Charged Particle Energy

Particle#1: Eo = [MeV] sigE = [MeV]

Particle#2: Eo = [MeV] sigE = [MeV]

Particle Composition

Particle#1: Ne = Np = Nn =

Particle#2: Ne = Np = Nn =

Monte Carlo Parameters

of particles = seed =

☒ Semi-Sphere Isotropic Radiation (Lunar Version)

Plot Parameters

☐ Disengage Shield ☒ Wide Points

☐ Suppress Trails

Output Filenames

 ☒ x-y

 ☒ x-z

 ☒ y-z

☐ Sliced Plots ON

rho-Start Radius of Slices = [m]

Separation Between Slices = [m]

Base Filename =

☐ Sphere Potential Plot Shading ON

Recursion Parameters

delta_t = [ns] max iterations =

delta_x = [m]

Classical / Relativistic Mechanics

☒ Relativistic Acceleration ON

Program Output Log

v0(1) = 0.31582 [c] (initial particle velocity, fraction of c, corresponding to Eo(1))

v0(2) = 0.99578 [c] (initial particle velocity, fraction of c, corresponding to Eo(2))

fc = 25.48999 [%] (total particle / sphere collisions)

ft = 0.20000 [%] (percentage of particles arriving at rho0 protected radius)

vmin = 0.00268 [c] (minimum particle v of all particles, as a fraction of c)

vmax = 0.99999 [c] (maximum particle v of all particles, as a fraction of c)

vavg = 0.57919 [c] (average particle v of all particles, as a fraction of c)

 Run Status: Done Processing

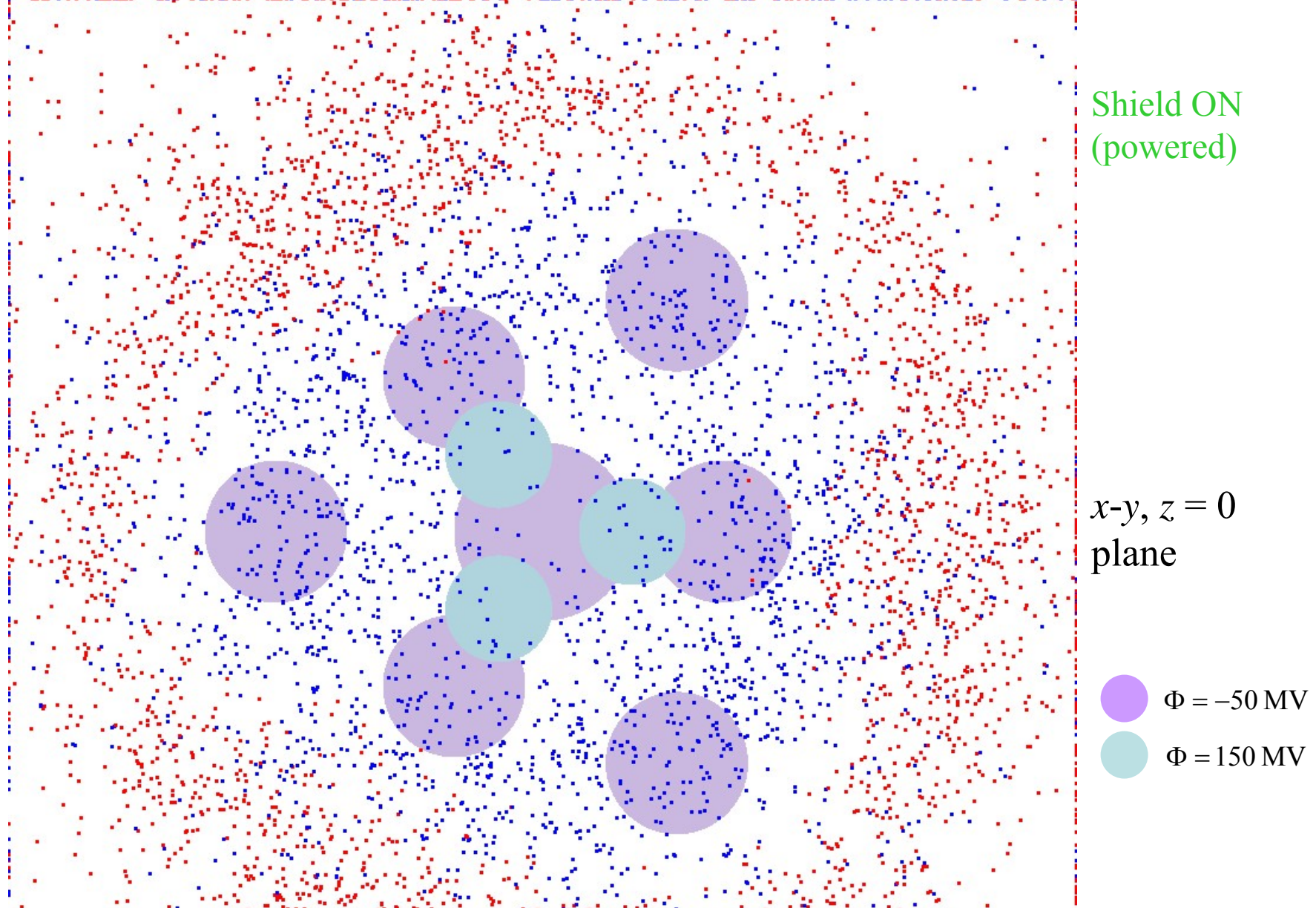
Shield ON
(powered)

Sphere Configuration File

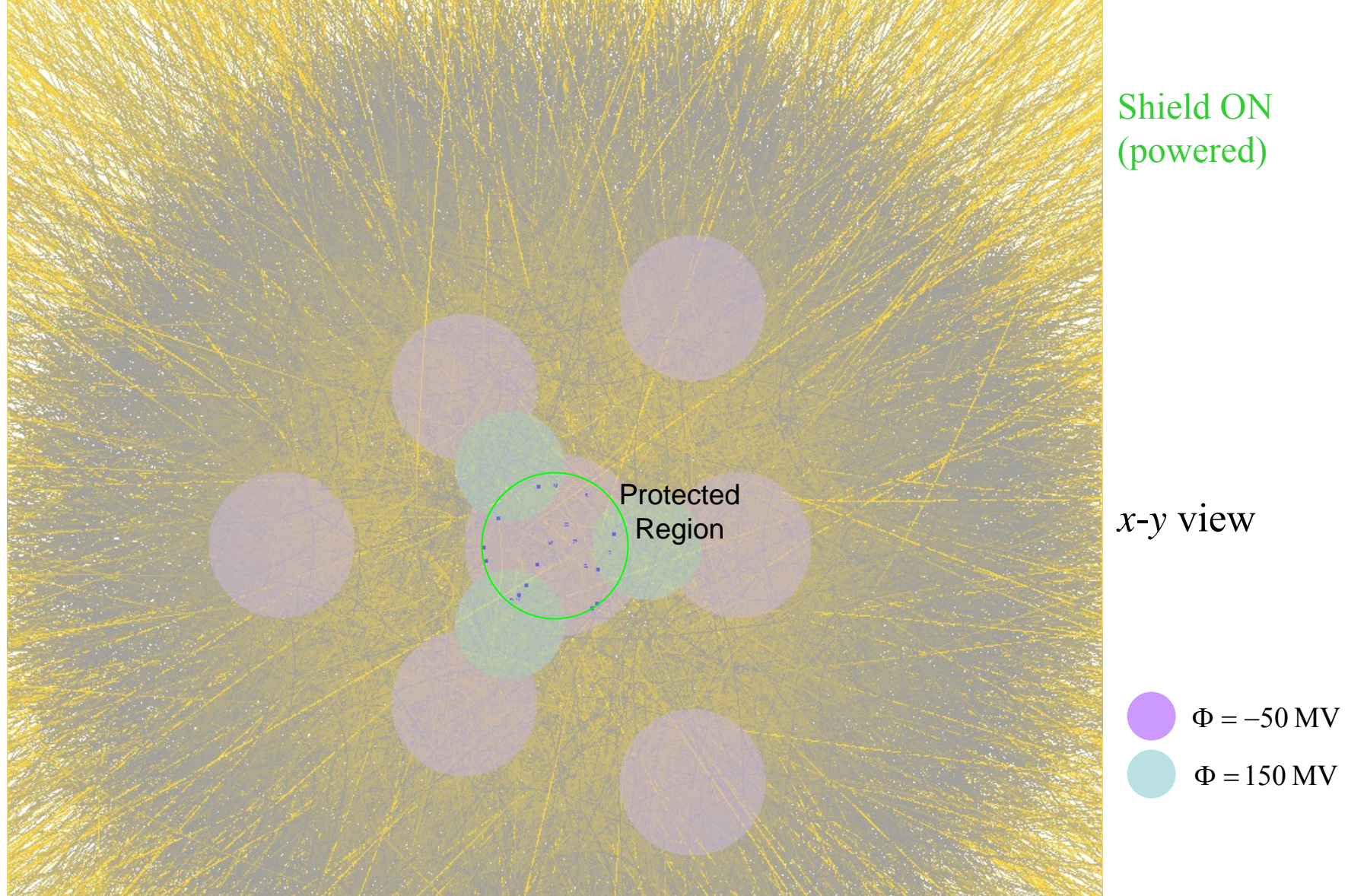
Shaded Entries Correspond To Image Charges Below Lunar Surface

20 Number of Spheres

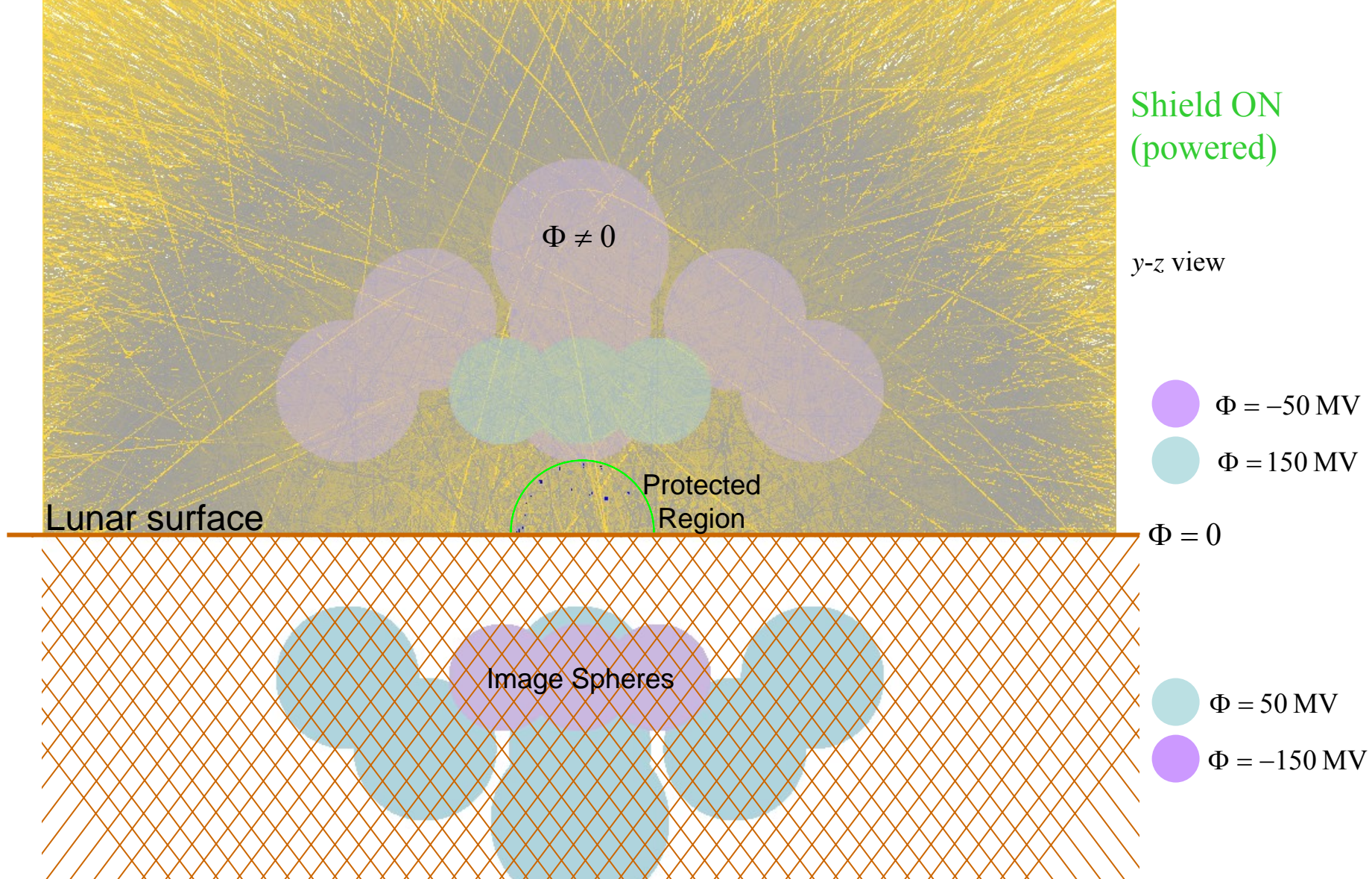
V [MV]	R [m]	x [m]	y [m]	z [m]
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150.0	3.0	-2.5	4.33	8.0
150.0	3.0	-2.5	-4.33	8.0
-50.0	4.0	10.0	0.0	12.0
-50.0	4.0	-5.0	8.66	12.0
-50.0	4.0	-5.0	-8.66	12.0
-50.0	5.0	0.0	0.0	16.0
-50.0	4.0	-15.0	0.0	8.0
-50.0	4.0	7.5	12.99	8.0
-50.0	4.0	7.5	-12.99	8.0
-150.0	3.0	5.0	0.0	-8.0
-150.0	3.0	-2.5	4.33	-8.0
-150.0	3.0	-2.5	-4.33	-8.0
50.0	4.0	10.0	0.0	-12.0
50.0	4.0	-5.0	8.66	-12.0
50.0	4.0	-5.0	-8.66	-12.0
50.0	5.0	0.0	0.0	-16.0
50.0	4.0	-15.0	0.0	-8.0
50.0	4.0	7.5	12.99	-8.0
50.0	4.0	7.5	-12.99	-8.0



Simulation Run of Lunar Electrostatic Shield Model (LESM v1.2) – Red dots are intersection of electrons and blue dots are intersection of protons with lunar surface ($z = 0$). Gray circles are x-y projections of powered electrostatic spheres.



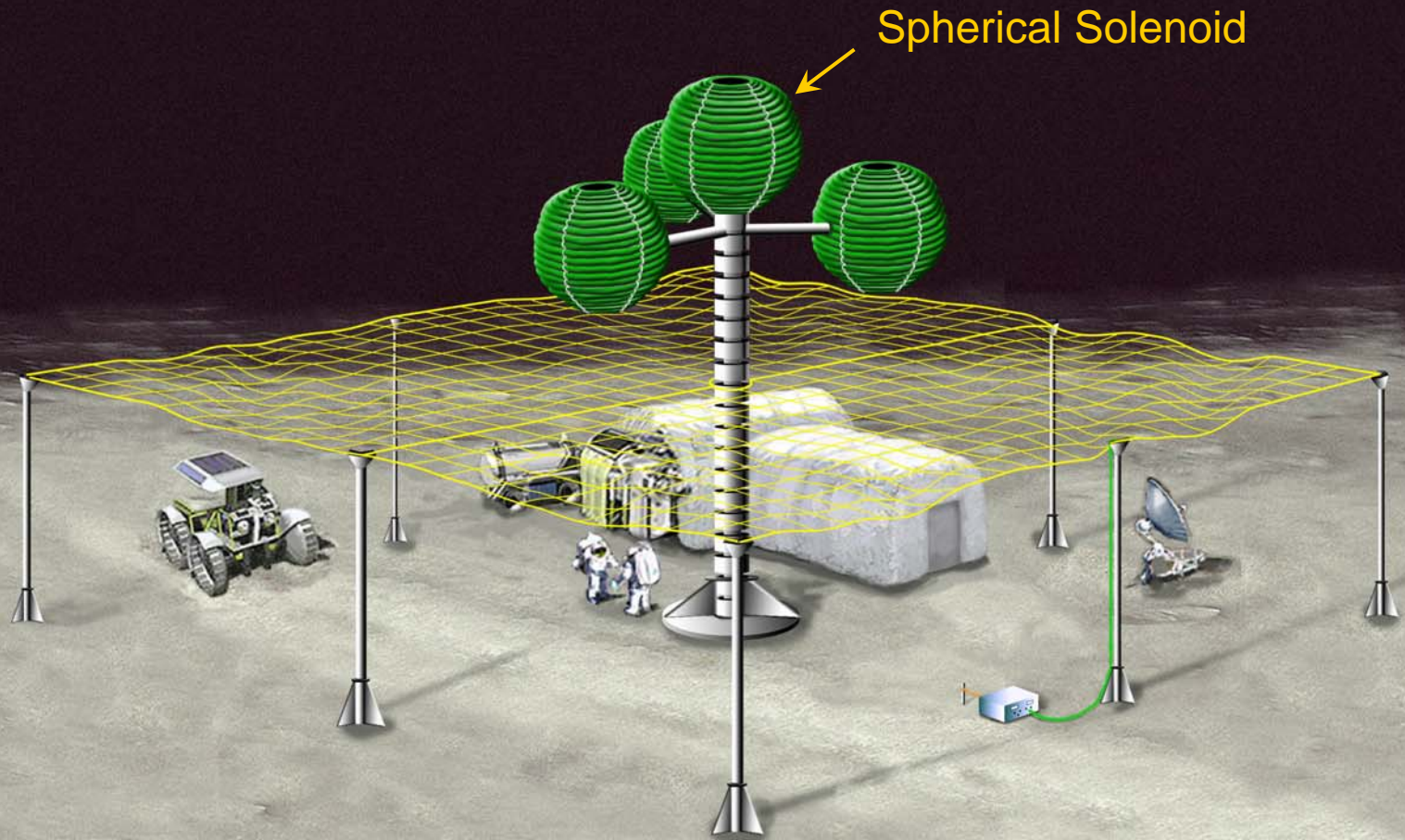
Simulation Run of Lunar Electrostatic Shield Model (LESM v1.2) – Red dots are intersection of electrons and blue dots are intersection of protons with a 4 [m] radius sphere (protected area) centered at $x = 0$, $y = 0$, $z = 0$. Gray circles are x-y projections of powered electrostatic spheres. Yellow-gray trails are particle trajectory paths.



Simulation Run of Lunar Electrostatic Shield Model (LESM v1.2) – Red dots are intersection of electrons and blue dots are intersection of protons with a 4 [m] radius sphere (protected area) centered at $x = 0$, $y = 0$, $z = 0$. Gray circles are y-z projections of powered electrostatic spheres. Yellow-gray trails are particle trajectory paths.

Model Simulation Results

Four Electrostatic Spheres and One Magnetic Field Coil



Simulation Run of Lunar Electrostatic Shield Model (LESM v2.3) – User Interface and Sphere Configuration File

Lunar Electrostatic Shield Model (LESM_Animin)

Model Length Parameters (all units in [m])

Ro = 30.000 Ro' = 40.000 rho0x = 0.000
 rhoP = 60.000 rho0 = 4.100 rho0y = 0.000
 rho0z = 25.000

Charged Particle Energy

Particle#1: Eo = 0.100 [MeV] sigE = 0.010 [MeV]
 Particle#2: Eo = 30.000 [MeV] sigE = 3.000 [MeV]

Particle Composition

Particle#1: Ne = 1 Np = 0 Nn = 0
 Particle#2: Ne = 0 Np = 1 Nn = 0

Monte Carlo Parameters

of particles = 4 seed = 425001
☒ Semi-Sphere Isotropic Radiation (Lunar Version)

Plot Parameters

☐ Disengage Shield ☐ Suppress Trails
☐ Sliced Plots ON ☒ Wide Points
 rho-Start Radius of Slices = 0.000 [m]
 Separation Between Slices = 3.000 [m]
 Base Filename = Rho
☐ Sphere Potential Plot Shading ON

Recursion Parameters

delta_t = 1.000 [ns] max iterations = 10000
 delta_x = 0.500 [m]

Classical / Relativistic Mechanics

☒ Relativistic Acceleration ON

Program Output Log

v0(1) = 0.55065 [c] (initial particle velocity, fraction of c, corresponding to Eo(1))
 v0(2) = 0.24843 [c] (initial particle velocity, fraction of c, corresponding to Eo(2))
 fc = 25.00000 [%] (total particle / sphere collisions)
 ft = 0.00000 [%] (percentage of particles arriving at rho0 protected radius)
 fo = 0.00000 [%] (trajectory timeout - incompleted)
 fs = 100.00000 [%] (percentage of particles intersecting boundary surface)
 vmin = 0.18487 [c] (minimum particle v of all particles, as a fraction of c)
 vmax = 0.99994 [c] (maximum particle v of all particles, as a fraction of c)
 vavg = 0.91785 [c] (average particle v of all particles, as a fraction of c)

Solenoid Parameters

a0 = 3.000
 dx = 0.010
 Mx = 5
 b = 0.005
 dz = 0.010
 Mz = 600
 i = 5000

Apply Run Status: Done Processing Exit

Shield ON
(powered)

Sphere Configuration File
Shaded Entries Correspond To Image
Charges Below Lunar Surface

8 Number of Spheres

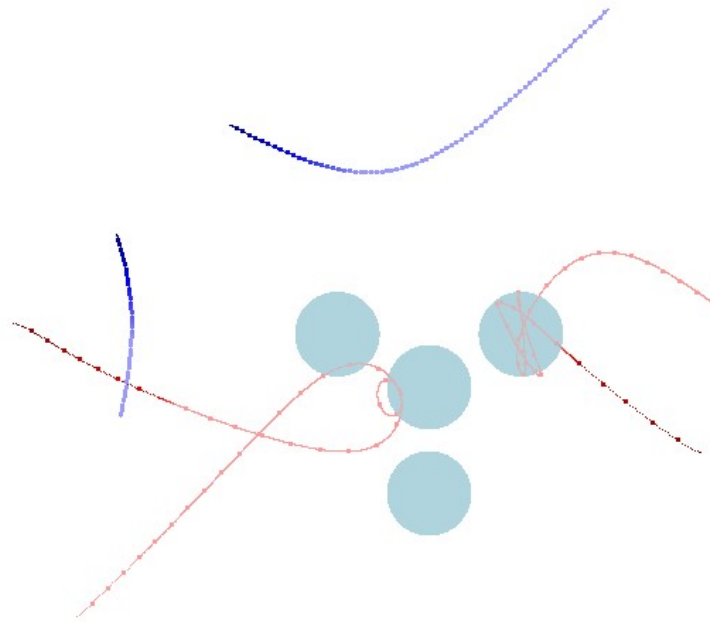
V [MV]	R [m]	x [m]	y [m]	z [m]
100	4.0	0.0	0.0	25.0
50	4.0	8.66	5.0	20.0
50	4.0	-8.66	5.0	20.0
50	4.0	0.00	-10.0	20.0
-100	4.0	0.0	0.0	-25.0
-50	4.0	8.66	5.0	-20.0
-50	4.0	-8.66	5.0	-20.0
-50	4.0	0.00	-10.0	-20.0

Model Simulation Results: x - y Plane

Two 30 MeV Protons and Two 1 MeV Electrons

$E \neq 0$

$B \neq 0$

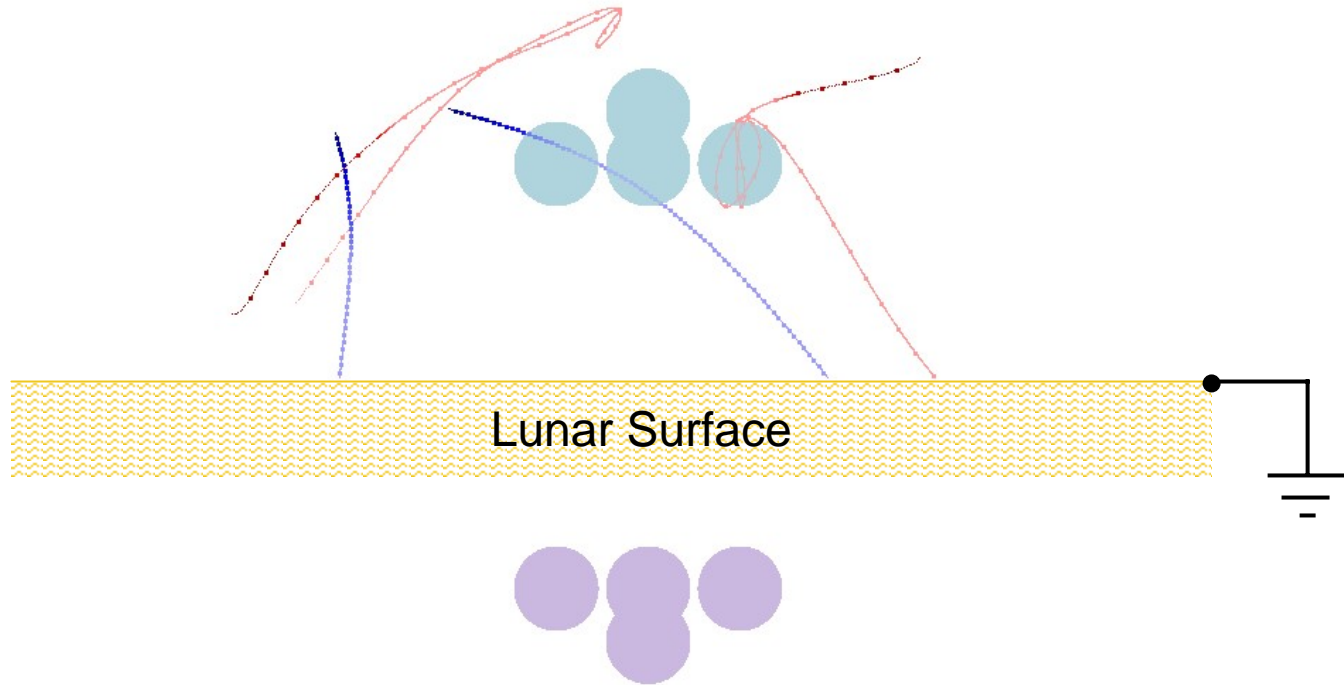


Model Simulation Results: x - z Plane

Two 30 MeV Protons and Two 1 MeV Electrons

$E \neq 0$

$B \neq 0$

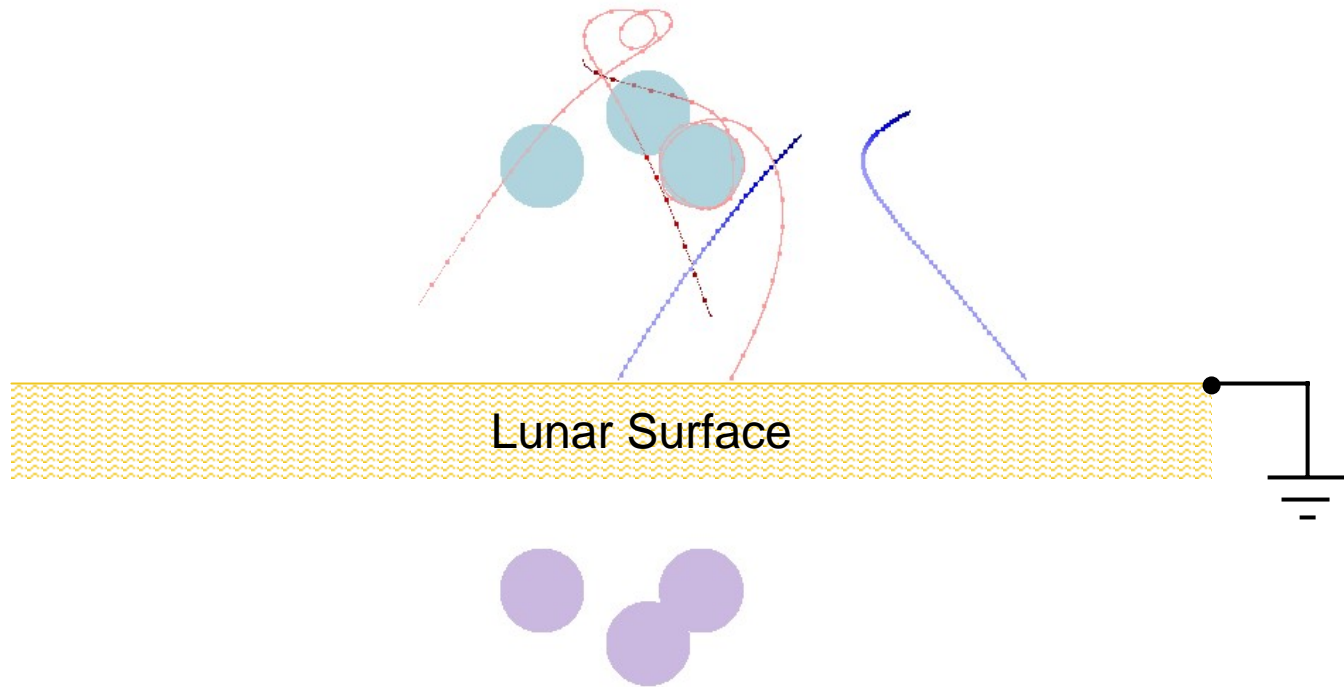


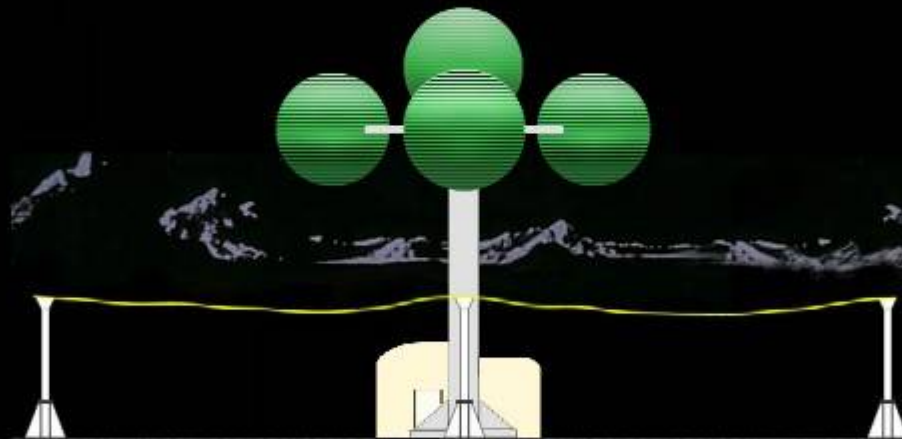
Model Simulation Results: y - z Plane

Two 30 MeV Protons and Two 1 MeV Electrons

$E \neq 0$

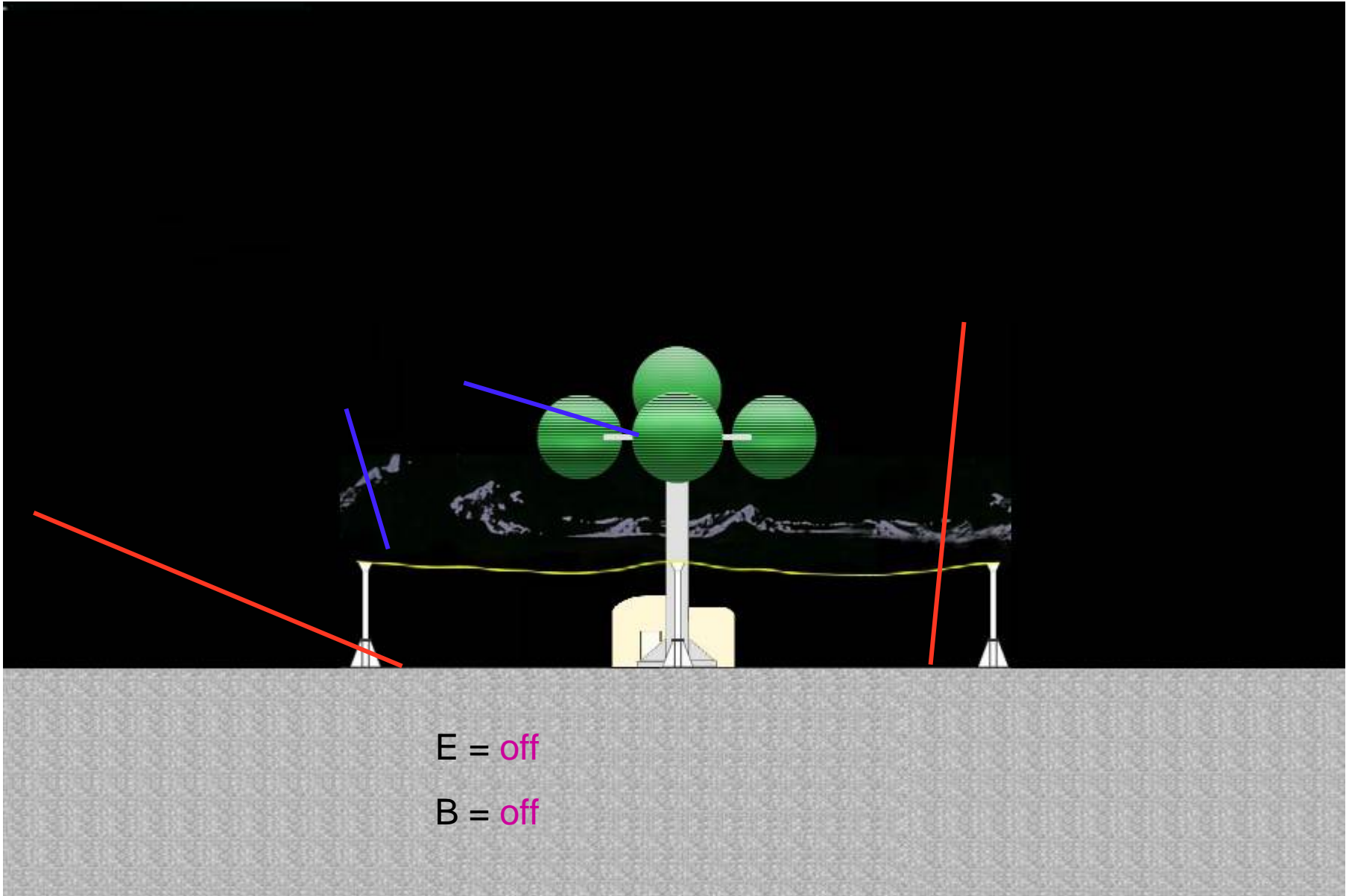
$B \neq 0$





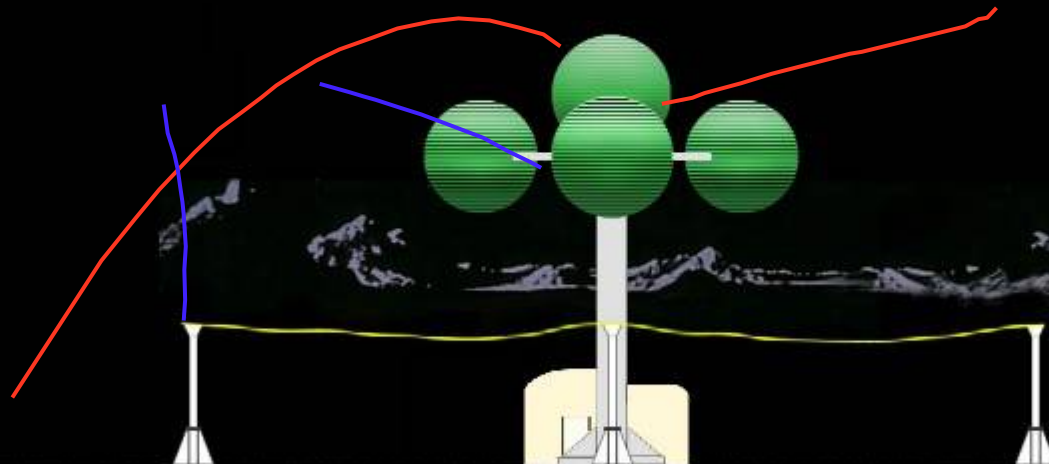
E = off

B = off



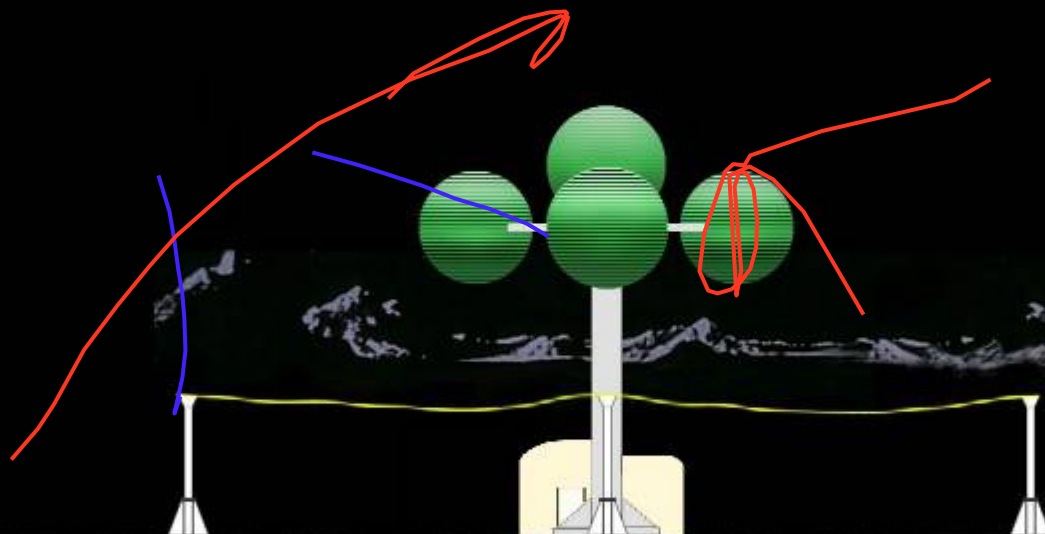
E = off

B = off



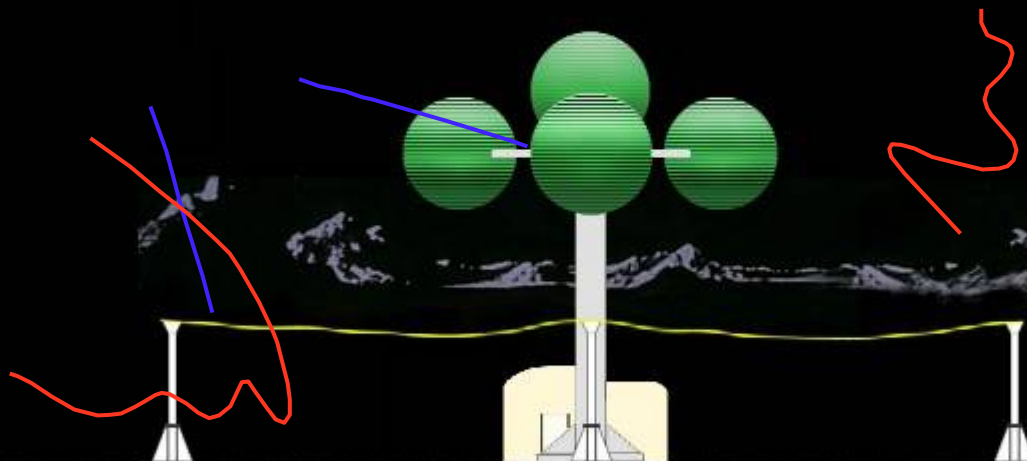
E = on

B = off



E = on

B = on



E = off

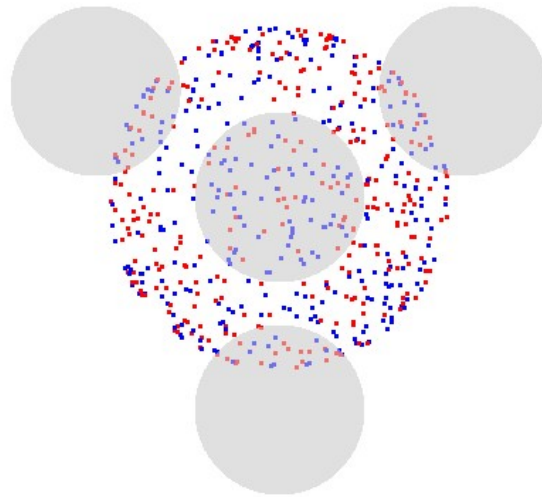
B = on

Model Simulation Results: x - y Plane

30 MeV Protons,
1 MeV Electrons

$$\mathbf{E} = \mathbf{0}$$

$$\mathbf{B} = \mathbf{0}$$



Model Simulation Results: x - z Plane

$$\mathbf{E} = \mathbf{0}$$

$$\mathbf{B} = \mathbf{0}$$

30 MeV Protons,
1 MeV Electrons

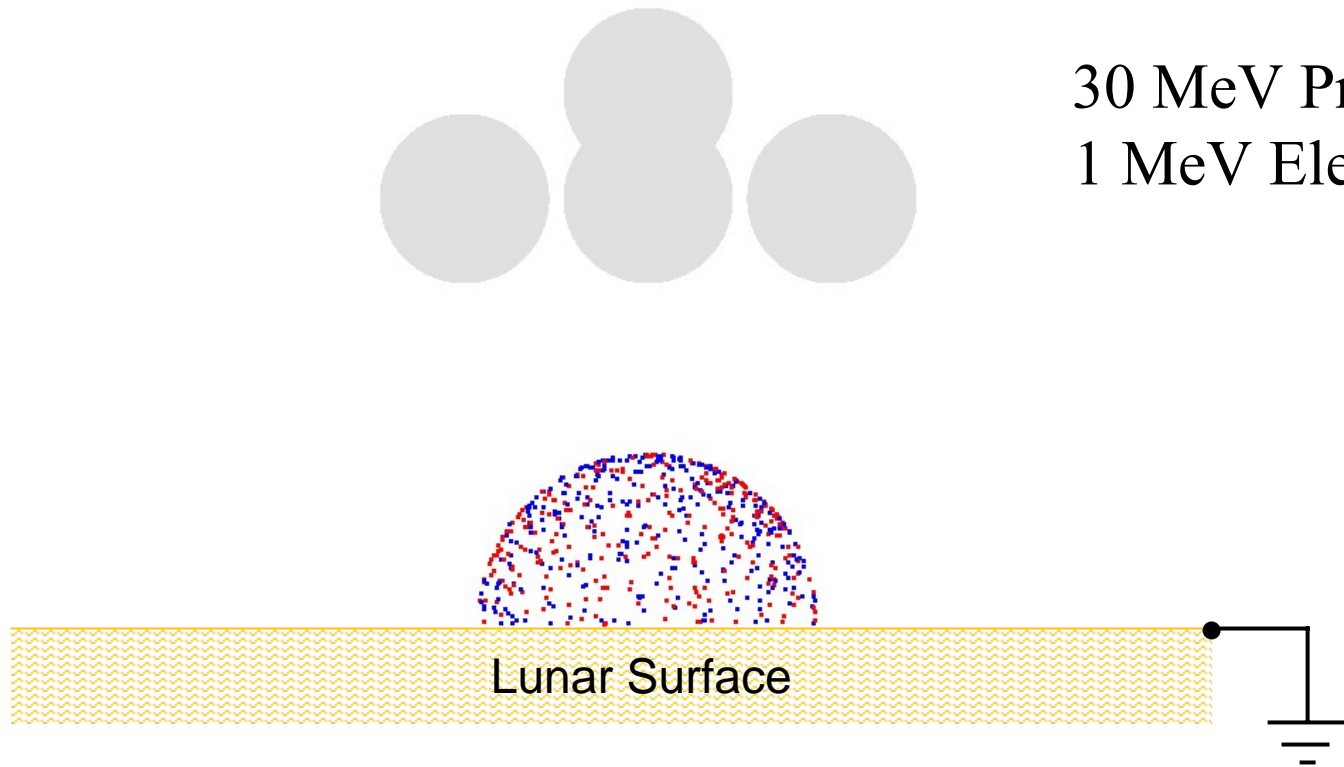


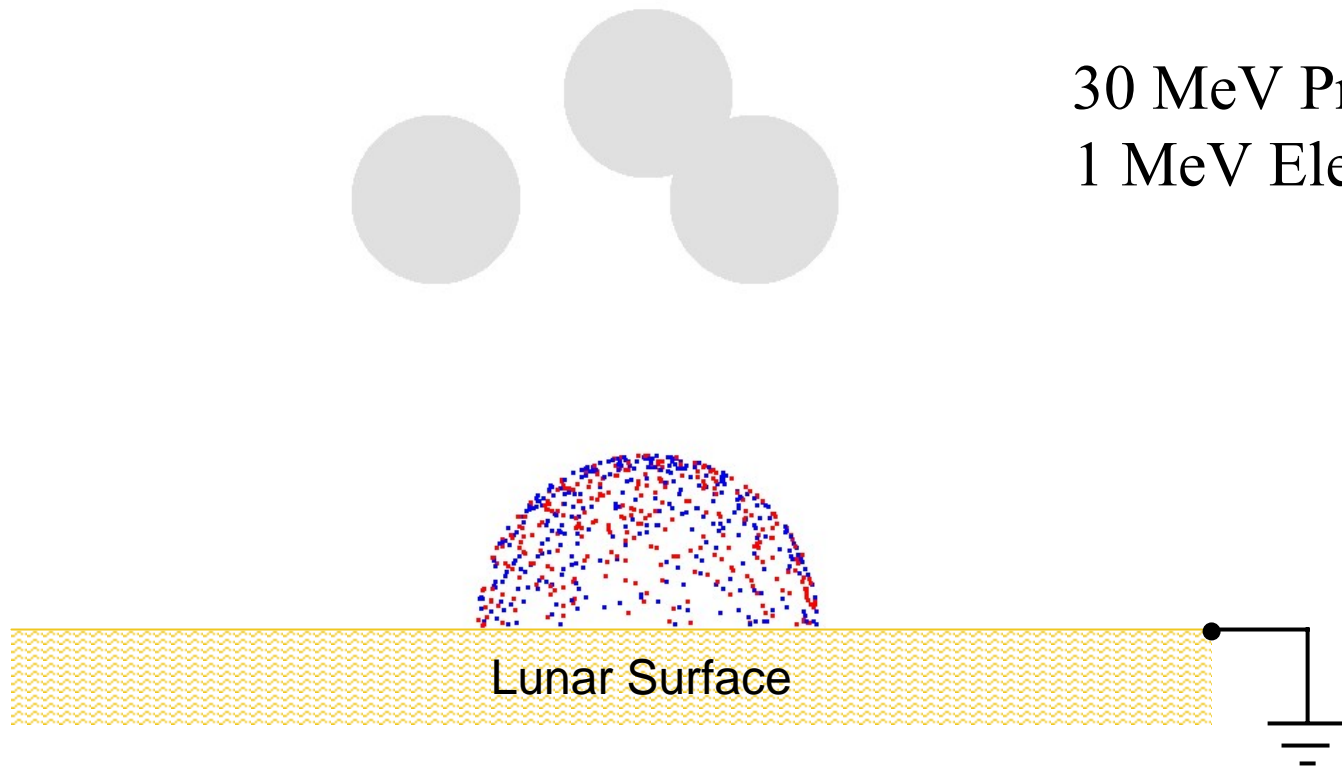
Image Spheres

Model Simulation Results: y - z Plane

30 MeV Protons,
1 MeV Electrons

$$\mathbf{E} = \mathbf{0}$$

$$\mathbf{B} = \mathbf{0}$$



Lunar Surface

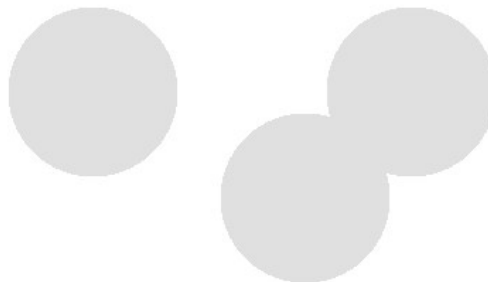
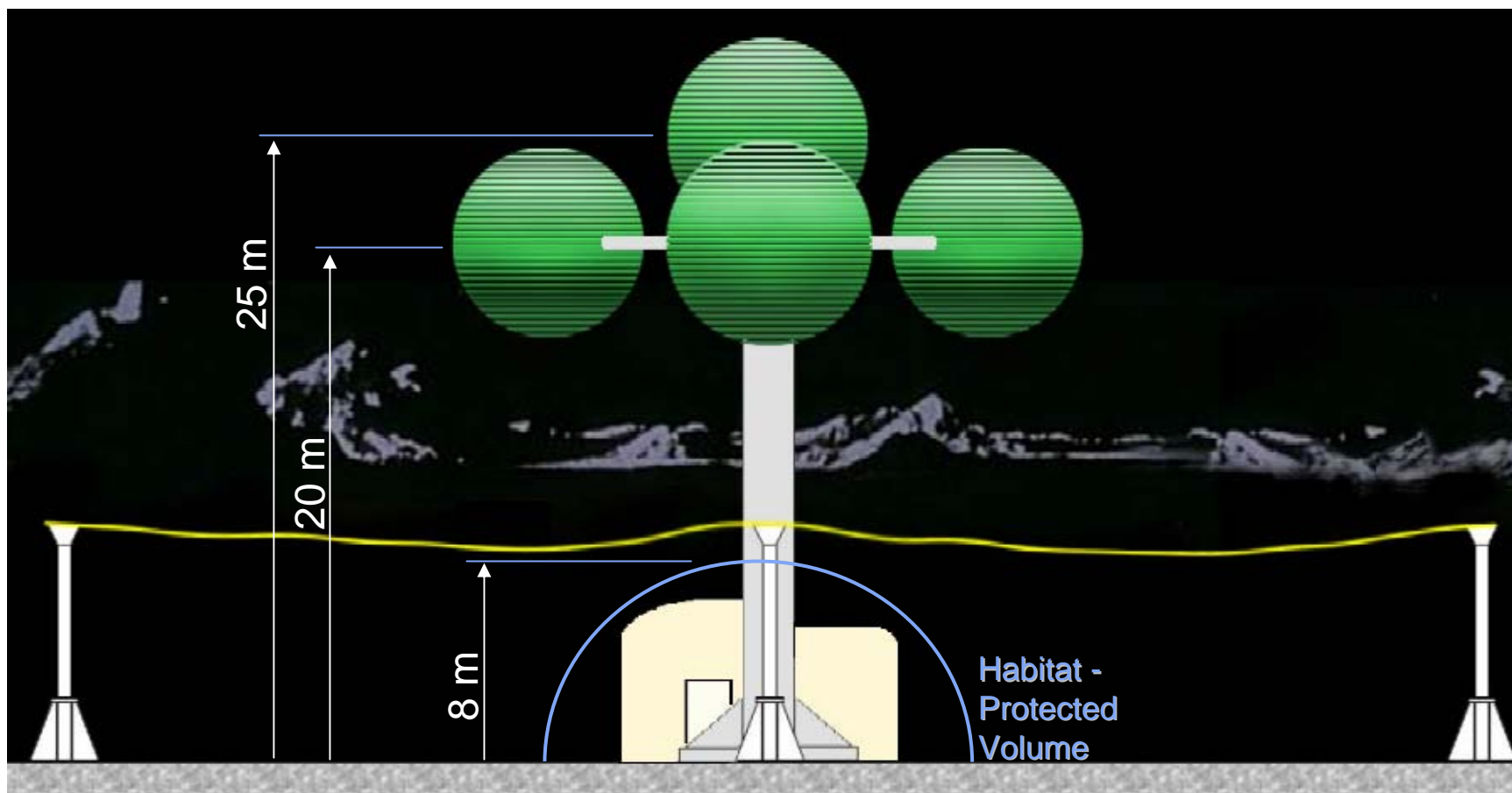


Image Spheres

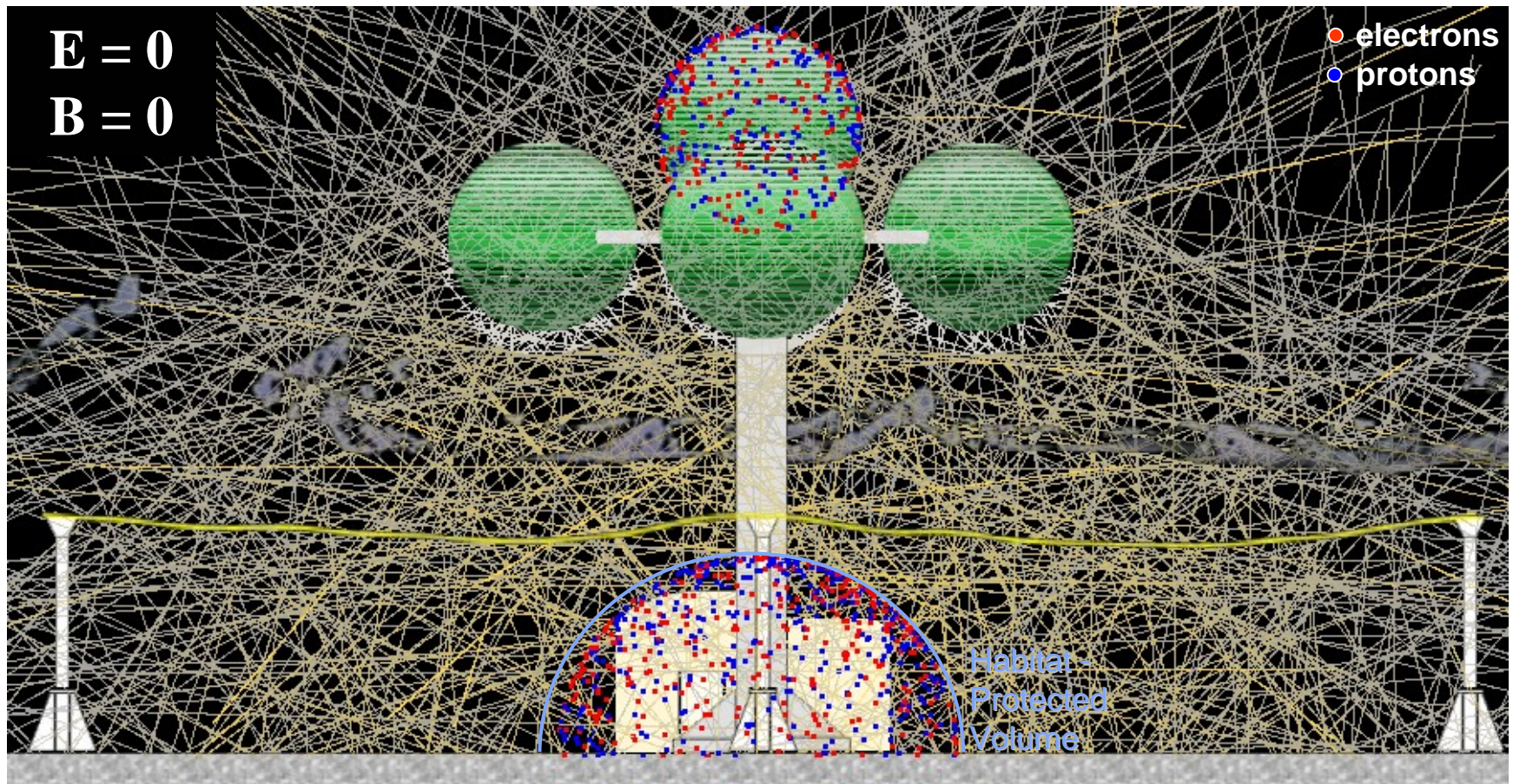
Model Simulation Results

**One +100 MV Sphere and
Three +50 MV Spheres**



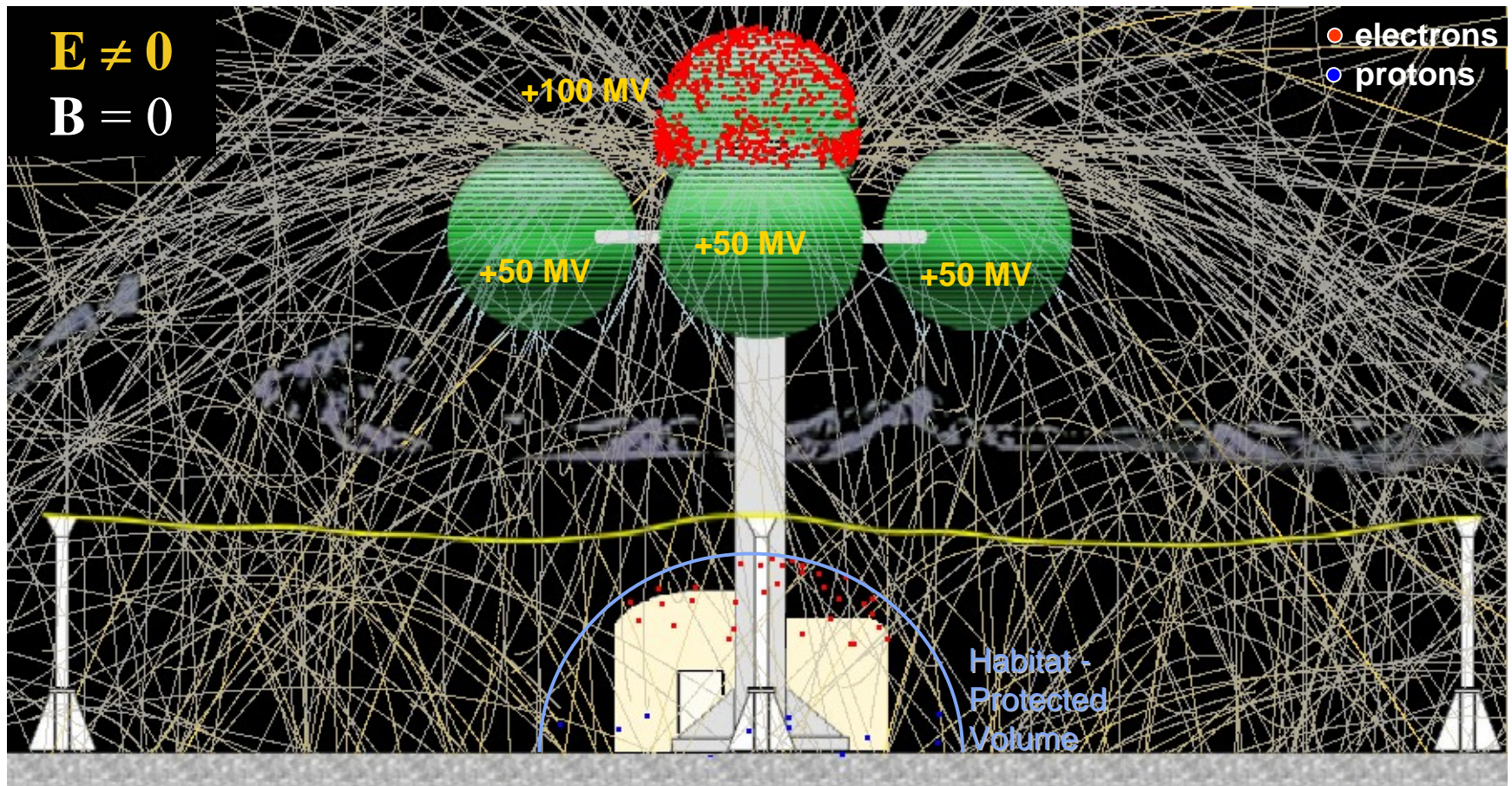
Model Simulation Results

30 MeV Protons, 1 MeV Electrons



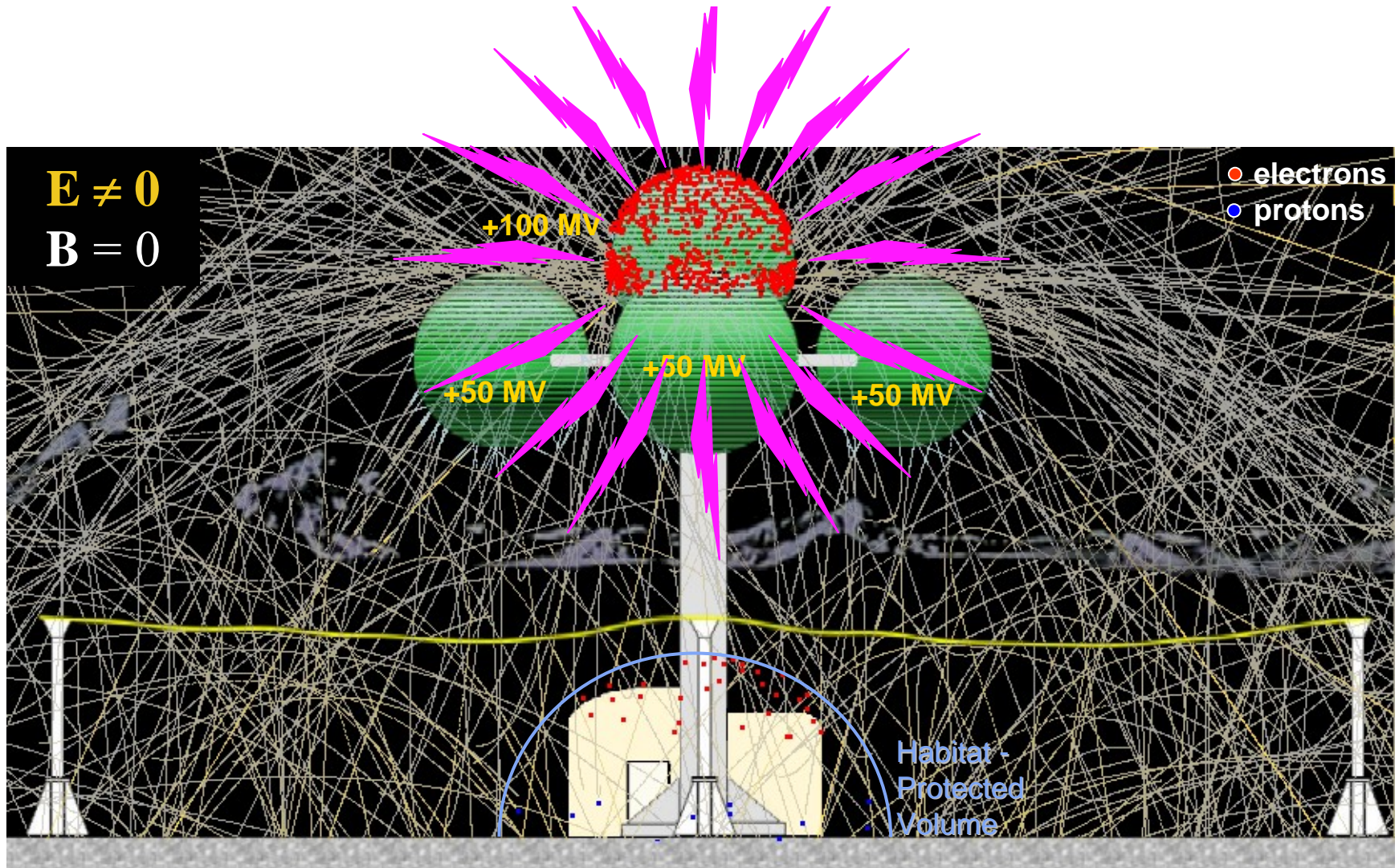
Model Simulation Results

30 MeV Protons, 1 MeV Electrons



Model Simulation Results

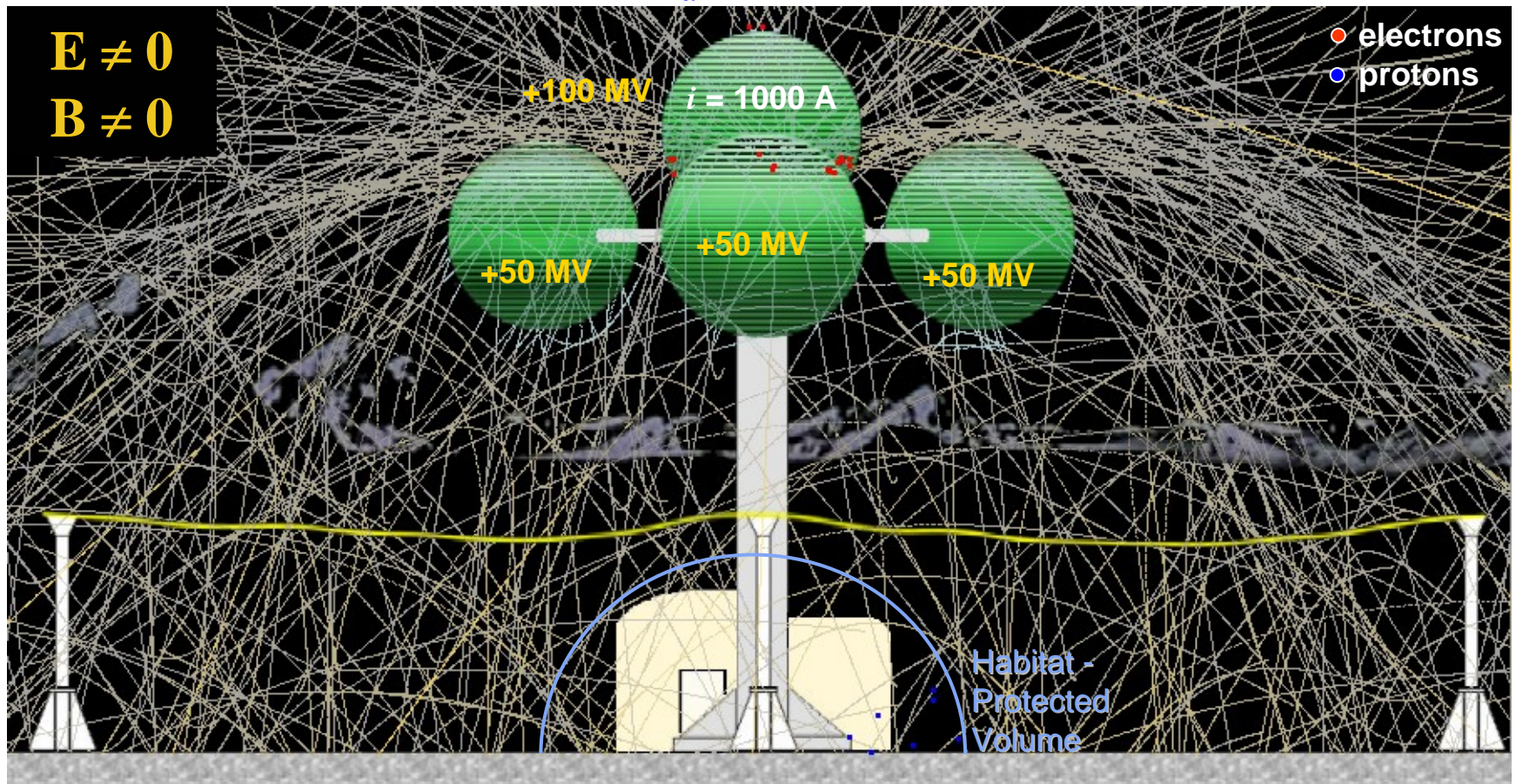
Bremsstrahlung X-ray Emission



Model Simulation Results

30 MeV Protons, 1 MeV Electrons

$$B_{\max} \approx 0.5 \text{ [T]}$$



Mathematical Model

Electric Field Due to a System of Conducting Spheres

The field due to a system of N point charges at a field point \mathbf{r} is:

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^N q_i \frac{\mathbf{r} - \mathbf{r}_i}{|\mathbf{r} - \mathbf{r}_i|^3} \quad (1)$$

where \mathbf{r}_i is the location of the i th point charge q_i

If the i th point charge q_i is implemented as a sphere with radius R_i and a *uniform charge distribution* at potential V_i , Equation (1) can be rewritten as:

$$\mathbf{E}(\mathbf{r}) = \sum_{i=1}^N V_i R_i \frac{\mathbf{r} - \mathbf{r}_i}{|\mathbf{r} - \mathbf{r}_i|^3} \quad (2)$$

Mathematical Model

Equation of Motion of a Charged Particle

A particle of charge Q , velocity \mathbf{v} and rest mass m_0 in combined static electric and magnetic fields, is:

$$\begin{aligned} Q\mathbf{E}(\mathbf{r}) + Q\mathbf{v} \times \mathbf{B}(\mathbf{r}) &= \mathbf{\dot{p}} \\ &= \frac{d}{dt}(\gamma m_0 \mathbf{v}) \\ &= \gamma m_0 \left(\mathbf{\dot{v}} + \gamma^2 \frac{\mathbf{v} \cdot \mathbf{\dot{v}}}{c^2} \mathbf{v} \right) \end{aligned} \tag{3}$$

where, $\gamma \equiv (1 - v^2 / c^2)^{-1/2}$

Mathematical Model

Solution to Particle Equation of Motion

The acceleration of the particle, $\mathbf{a} \equiv \frac{d\mathbf{v}}{dt}$, of the particle is calculated by re-writing Equation (3):

$$\mathbf{C} \cdot \mathbf{a} = \frac{Q}{\gamma m_0} (\mathbf{E}(\mathbf{r}) + \mathbf{v} \times \mathbf{B}(\mathbf{r})) \quad (4)$$

where,

$$\mathbf{C} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{pmatrix} = \begin{pmatrix} 1 + \gamma^2 \frac{v_x^2}{c^2} & \gamma^2 \frac{v_x v_y}{c^2} & \gamma^2 \frac{v_x v_z}{c^2} \\ \gamma^2 \frac{v_y v_x}{c^2} & 1 + \gamma^2 \frac{v_y^2}{c^2} & \gamma^2 \frac{v_y v_z}{c^2} \\ \gamma^2 \frac{v_z v_x}{c^2} & \gamma^2 \frac{v_z v_y}{c^2} & 1 + \gamma^2 \frac{v_z^2}{c^2} \end{pmatrix} \quad (5)$$

Mathematical Model

Solution to Particle Equation of Motion

Solving for \mathbf{a} in Equation (4),

$$\begin{aligned}\mathbf{a} &= \frac{Q}{\gamma m_0} \mathbf{C}^{-1} \cdot (\mathbf{E}(\mathbf{r}) + \mathbf{v} \times \mathbf{B}(\mathbf{r})) \\ &= \frac{1}{A_0} \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix}\end{aligned}\tag{6}$$

where,

$$A_0 = (c_{13}c_{22}c_{31} - c_{12}c_{23}c_{31} - c_{13}c_{21}c_{32} + c_{11}c_{23}c_{32} + c_{12}c_{21}c_{33} - c_{11}c_{22}c_{33})m_0\gamma\tag{7}$$

$$A_x = (c_{23}c_{32}E_x - c_{22}c_{33}F_x - c_{13}c_{32}F_y + c_{12}c_{33}F_y + c_{13}c_{22}F_z - c_{12}c_{23}F_z)Q\tag{8a}$$

$$A_y = (-c_{23}c_{31}F_x + c_{21}c_{33}F_x + c_{13}c_{31}F_y - c_{11}c_{33}F_y - c_{13}c_{21}F_z + c_{11}c_{23}F_z)Q\tag{8b}$$

$$A_z = (c_{22}c_{31}F_x - c_{21}c_{32}F_x - c_{12}c_{31}F_y + c_{11}c_{32}F_y + c_{12}c_{21}F_z - c_{11}c_{22}F_z)Q\tag{8c}$$

$$F_x \equiv E_x + v_y B_z - v_z B_y\tag{8d}$$

$$F_y \equiv E_y + v_z B_x - v_x B_z\tag{8e}$$

$$F_z \equiv E_z + v_x B_y - v_y B_x\tag{8f}$$

Mathematical Model

Trajectory Difference Equations of Particle Motion

Based on a Taylor series expansion about time point k , a set of difference equations for position and velocity can be expressed as:

$$\begin{aligned}\mathbf{v}_{k+1} &\approx \mathbf{v}_k + \cancel{\mathbf{v}_k} \Delta t \\ &\approx \mathbf{v}_k + \mathbf{a}_k \Delta t\end{aligned}\tag{9a}$$

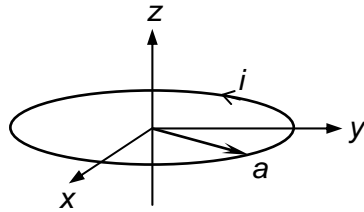
$$\begin{aligned}\mathbf{r}_{k+1} &\approx \mathbf{r}_k + \cancel{\mathbf{r}_k} \Delta t + \frac{1}{2} \cancel{\mathbf{r}_k} \Delta t^2 \\ &\approx \mathbf{r}_k + \mathbf{v}_k \Delta t + \frac{1}{2} \mathbf{a}_k \Delta t^2\end{aligned}\tag{9b}$$

where Δt a constant time step.

Mathematical Model

Magnetic Field due to a Current Loop

The magnetic field from a single current loop is:



Circular Current Loop.

$$B_x(x, y, z) = \frac{C x z}{2\alpha^2 \beta \rho^2} \left((a^2 + R^2) E(k^2) - \alpha^2 K(k^2) \right) \quad (10a)$$

$$B_y(x, y, z) = \frac{C y z}{2\alpha^2 \beta \rho^2} \left((a^2 + R^2) E(k^2) - \alpha^2 K(k^2) \right) \quad (10b)$$

$$B_z(x, y, z) = \frac{C}{2\alpha^2 \beta} \left((a^2 - R^2) E(k^2) + \alpha^2 K(k^2) \right) \quad (10c)$$

where $E(k^2)$ and $K(k^2)$ are the complete elliptic integrals of the first and second kind, respectively, and:

$$\rho^2 \equiv x^2 + y^2$$

$$\alpha^2 \equiv a^2 + R^2 - 2a\rho$$

$$k^2 \equiv 1 - \alpha^2 / \beta^2$$

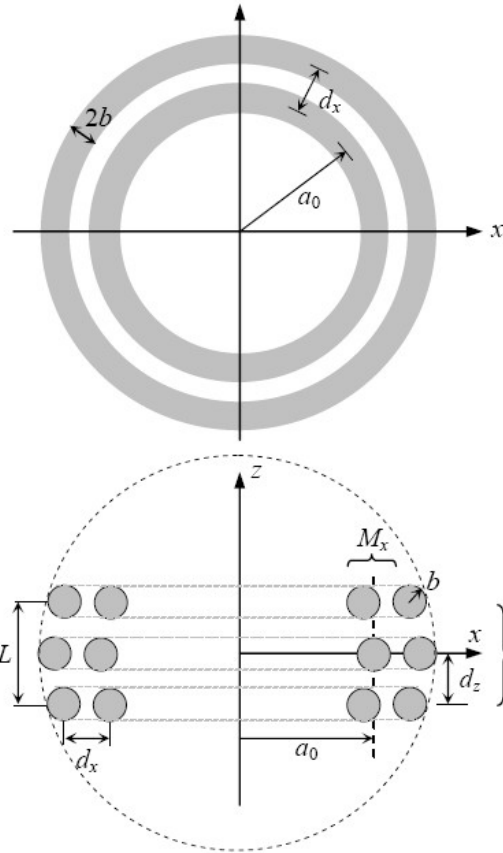
$$R^2 \equiv \rho^2 + z^2$$

$$\beta^2 \equiv a^2 + R^2 + 2a\rho$$

$$C \equiv \mu_0 i / \pi$$

Mathematical Model

Magnetic Field due to a Spherical Solenoid



Stacked multiple layer spherical coil.

$$B_x(x, y, z) = \frac{C x z}{2\rho^2} \sum_{m=-\frac{1}{2}(M_z-1)}^{\frac{1}{2}(M_z-1)} \sum_{n=0}^{M_x-1} \frac{((a_{mn}^2 + R_m^2) E(k_{mn}^2) - \alpha^2 K(k_{mn}^2))}{\alpha_{mn}^2 \beta_{mn}} \quad (11a)$$

$$B_y(x, y, z) = \frac{C y z}{2\rho^2} \sum_{m=-\frac{1}{2}(M_z-1)}^{\frac{1}{2}(M_z-1)} \sum_{n=0}^{M_x-1} \frac{((a_{mn}^2 + R_m^2) E(k_{mn}^2) - \alpha^2 K(k_{mn}^2))}{\alpha_{mn}^2 \beta_{mn}} \quad (11b)$$

$$B_z(x, y, z) = \frac{C}{2} \sum_{m=-\frac{1}{2}(M_z-1)}^{\frac{1}{2}(M_z-1)} \sum_{n=0}^{M_x-1} \frac{((a_{mn}^2 - R_m^2) E(k_{mn}^2) + \alpha^2 K(k_{mn}^2))}{\alpha_{mn}^2 \beta_{mn}} \quad (11c)$$

where $E(k^2)$ and $K(k^2)$ are the complete elliptic integrals of the first and second kind, respectively, and:

$$\begin{aligned} \rho^2 &\equiv x^2 + y^2 & \alpha_{mn}^2 &\equiv a_{mn}^2 + R_m^2 - 2a_{mn}\rho & k_{mn}^2 &\equiv 1 - \alpha_{mn}^2 / \beta_{mn}^2 \\ R_m^2 &\equiv \rho^2 + (z - md_z)^2 & \beta_{mn}^2 &\equiv a_{mn}^2 + R_m^2 + 2a_{mn}\rho & C &\equiv \mu_0 i / \pi \end{aligned}$$

The radius of each loop is: $a_{mn} \equiv nd_x + \sqrt{a_0^2 - (md_z)^2} \quad (12)$

where, $md_z < a_0$ and $\frac{1}{2}(M_z - 1)d_z < a_0$

Mathematical Model

Initial Particle Velocity calculated from Initial Particle Energy

Total energy of a particle of rest mass m_0 is:

$$E = mc^2 \quad (13a)$$

Total energy is the sum of rest mass energy and kinetic energy:

$$\begin{aligned} E &= E_0 + T \\ &= m_0 c^2 + T \end{aligned} \quad (13b)$$

Solve for T in term of v :

$$\begin{aligned} T &= mc^2 - E_0 \\ &= \gamma m_0 c^2 - m_0 c^2 = (\gamma - 1)m_0 c^2 \end{aligned} \quad (13c)$$

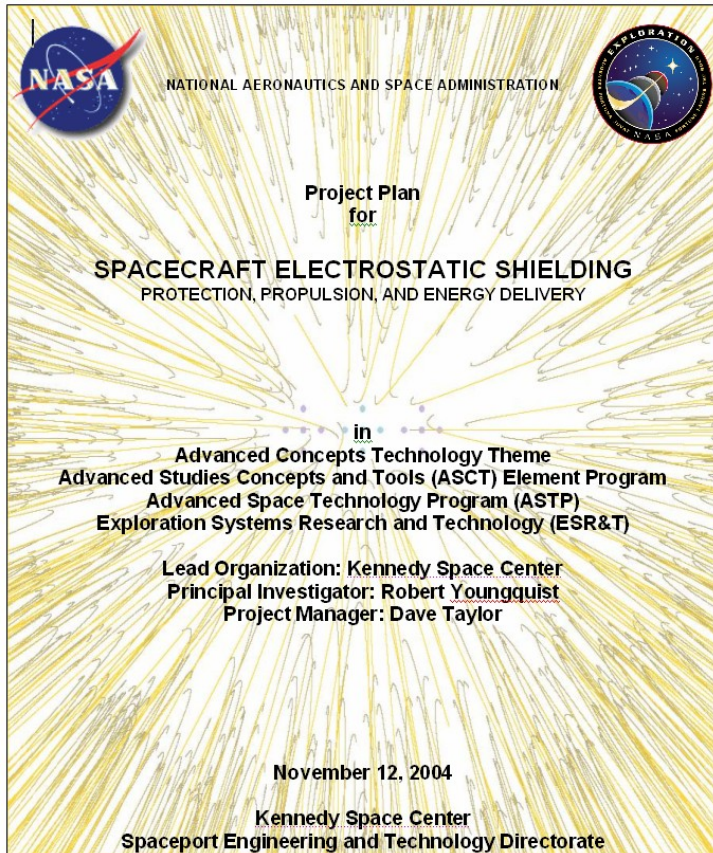
where, $\gamma \equiv (1 - v^2 / c^2)^{-1/2}$

Solve for v in term of T , with $\xi \equiv \frac{m_0 c^2}{qT}$:

$$v = \frac{c\sqrt{1 + 2\xi}}{1 + \xi} \quad (13d)$$

Present Radiation Shielding Studies at KSC

NASA (Spacecraft)



~ \$1.9 M / 4 yrs

ASRC

**Software-
Mathematical
Modeling**

**Software-
Mathematical
Modeling**

**Field Precision,
NM**

NIAC (Lunar)

**Analysis of a Lunar Base Electrostatic
Radiation Shield Concept
Phase I: NIAC CP 04-01**

Advanced Aeronautical/Space Concept Studies

Charles R. Buhler, Principal Investigator
(321) 867-4861

October 1, 2004

ASRC Aerospace Corporation
P.O. Box 21087
Kennedy Space Center, Florida 32815-0087



~ \$0.07 M / 6 mo

Problems already being addressed by NASA

Shield Configuration and Design for spacecraft

Shield Effectiveness

Material Tensile Strength/Dielectric Strength

Other Material Issues-Mechanical (Attachment, etc.)

Other Material Issues-Environmental (Temperature, UV resistance)

Other Material Issues-Misc. (Leakage current, Gamma resistance, Thickness/Weight, Crease resistance, Conductive coating (CNT or CVD Au), Aging)

Shield Forces

Net Shield Charge

Shield Discharge Calculations-Leakage, Corona, Plasma.

Charge Buildup on outside of Spheres.

Power Supply Feasibility-Voltage

Power Supply Feasibility-Current

Field Extent in a Plasma

Particle Entrapment

Safety-Total Stored Energy

Safety-Shield Stability

Safety-Electron Dosage Issue

Future Work required for a Lunar Solution

- Lunar Shield configuration
- Lunar gravity
- Lunar surface may or may not act as a sufficient electrical ground. [Power systems may have the benefit of free charges that a spacecraft will not have access to.]
- Lunar Shield will have to contend with Lunar dust.

Proposed Validation Experiment

