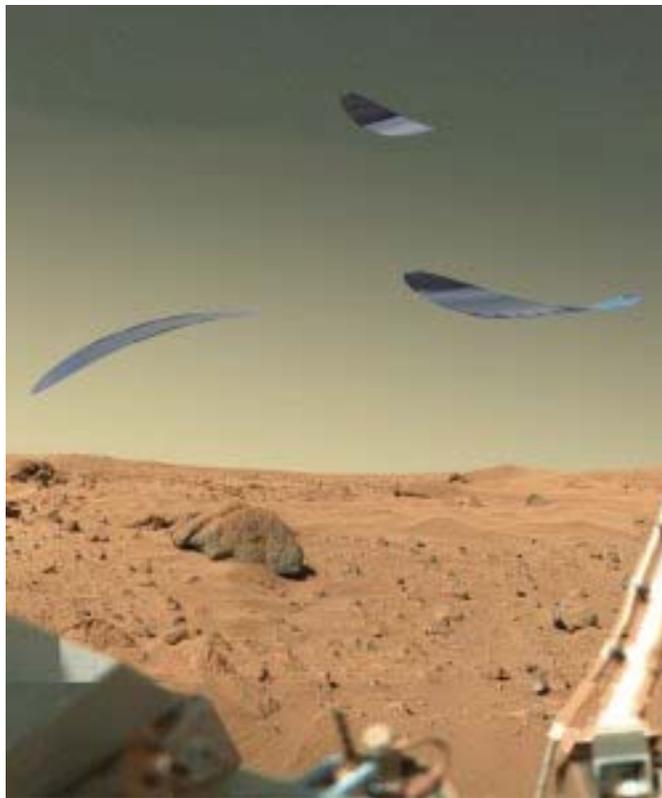


Artificial Neural Membrane Flapping Wing
NIAC Phase I Study
Final Report
May 3, 2006



Submitted by Pamela A. Menges, Ph.D.
Principal Investigator
Aerospace Research Systems, Inc.

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Aerospace Research Systems, Inc.
USRA Contract No. NAS5-03110

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Abstract

Artificial Neural Membrane Flapping Wing

Neural networks have evolved from a simple computer simulation to performance adaptive systems based on computational models derived from study of the ordering of neurons. Advances in materials, especially thin films, has provided a new environment for experimentation with the development of artificial neurons as device resident controllers for structures and vehicles. Advancement in the design of hypersonic and space launch systems has created the requirement for revolutionary technologies in the health-monitoring and control of highly complex autonomous flight systems. The concept of an artificial neural membrane flapping wing provides an integrated view of often non-related technologies and disciplines. The concept of such a device and potential vehicle may be used demonstrate the ability to advance the field of autonomous flight systems through biomimetics. The concept an ANM flapping wing is supported by the basic idea of integrating a neural network into a thin film or smart skin membrane for the purpose of demonstrating the ability of an artificial neural network or ANN to control simple locomotion for the purpose of flight. Simple contraction of the thin film has been demonstrated by running current through carbon fibers deposited on a polyamide sheet. It is necessary to evaluate mechanisms of locomotion and complete modeling of ANN controllers for different types of motion. It is the basis of this proposal to develop a virtual library investigating the potential types of motion that may be demonstrated by an ANM flapping wing.

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Preface

This project was undertaken as a feasibility study in the development of a highly complex functional structure. The choice of a flapping wing concept vehicle provided a familiar platform to investigate the potential of creating a functional adaptive neural network as both a distributed and hierarchical system. Distribute functions allowing inputs across a “membrane” structure from both external, remotely sensed data and internal vehicle system data as well as control-feedback functions and hierarchical as a structured learning, information processing and control system.

The “Artificial Neural Membrane Flapping Wing” Phase I study offered the opportunity to identify potential, developmental and emerging technologies that would make such a far reaching technology possible. Do to the nature of the study and the proprietary nature of work previously performed used to support the study, readers should be aware that proprietary material and certain related research data are not included. This report endeavors to reflect the complexity, potential and provide a beginning roadmap for the continued development of highly integrated intelligent structures. Also a primary component of this study was the development of a digital library of animations and simulations of various methods of flight, technology integration and modeling, and an evaluation of missions for exploration of planetary atmospheres and to support human operations on Mars and beyond. The digital library is on file with the NASA Institute for Advanced Concepts and many of the simulations may be requested through the Aerospace Research Systems, Inc. website www.arsispace.com.

This system concept creates a highly integrated synergistic, biomimetic structure that demonstrates the potential for a new class of functional intelligent structures and a revolutionary technology.

This system concept creates a highly integrated synergistic, biomimetic structure that demonstrates the potential for a new class of functional intelligent structures and a revolutionary technology. Applications extend to all forms of flight vehicles, propulsion, chemical and material processes, artificial organs and a growing list of adaptive structures including in the near term (2-5 years) aircraft armor, automated high-speed combustors, and

configurable solar sails and wind generators. In the near long term (6-10 years) configurable spacesuits and anthropomorphic exoskeletons for undersea and space exploration and paralysis patients. In the not so near long term (11-20 years) configurable flight systems for autonomous and human flight in a variety of planetary atmospheres, neural programmable pressure vessels for submarine and space environments, hypersonic and ion wave engines, adaptive board-level chemical and material processing (the portable pharmaceutical plant), micro- and nanoscale production of power, and implantable biotechnology enzyme production capsules.

The roadmap contained in this report is a partial roadmap that will be built upon in preparation for a NASA Institute for Advanced Concepts (NIAC) Phase II in the near future. Aerospace Research Systems, Inc. is investing in further research to prepare for the development of a demonstrator system under a Phase II. Continued work will be completed in proprietary technologies necessary to fabricate even the base system concept require further development. Materials processes and instrumentation methods are significant issues in this research. New methods in strain and force measurement at the micro- and nanoscale are required. Further, the fundamental issues surrounding the mathematical complexity of supporting advanced neural controllers within simulated networks also requires a highly effective set of computational models and supporting processing systems.

One comment as Principal Investigator I must make is overall level of surprise by me and my team in underestimating the level of complexity in materials, mathematics, processes and programming required to define this technology and evaluate its potential for eventual application. Many colleagues believe that we may be on the verge of a new discipline in the relationship between theoretical biology (biomathematics) and materials processing. If one visualizes the fabrication of an Artificial Neural Membrane (ANM) Flapping Wing (or any other ANM structure) one has to recognize the elements of automated programmable materials. The concept of automated programming of materials is not necessarily new, however the extension of a self-ordering, evolutionary neural network to materials programming creates evolutionarily and adaptive materials of a new class.

Pamela A. Menges, Ph.D.
April 30, 2006

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INTRODUCTION

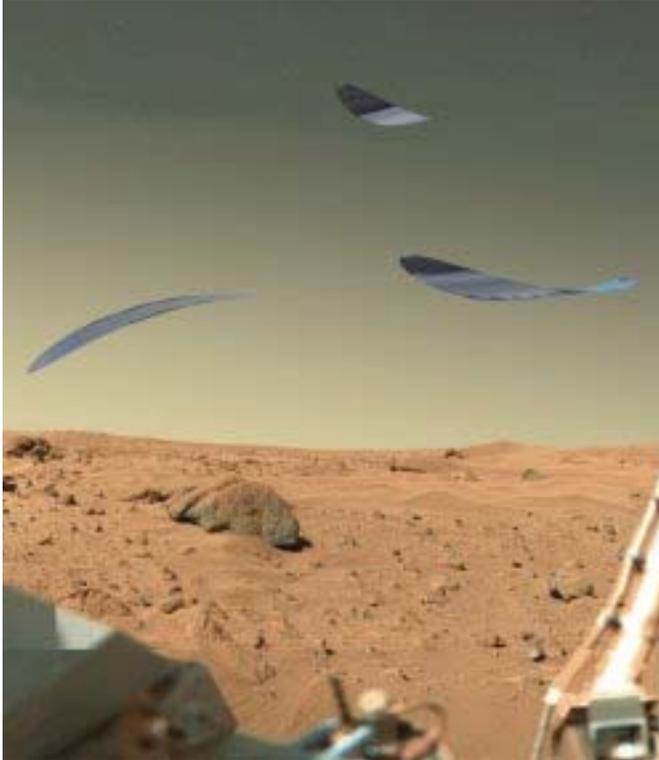


Figure 1-1: ANM Flapping Wing MAV concepts supporting exploration missions on Mars with atmospheric data and a large distributed delay tolerant network (DTN).

The Artificial Neural Membrane (ANM) Flapping Wing is a very complex concept that creates a synergistic biomimetic structure at the core of its development. Unlike other multifunctional materials the ANM structure is new class of functional structure that integrates distributed, multidimensional neural functions as the primary control and information processing system. It further integrates at the molecular level both neural and structural components of the ANM wing.

This Phase I NIAC Study evaluated the feasibility of developing a completely new concept of a “functionally intelligent material.” The diversity and complexity of thrusts that have been identified as being central to the continued development of the

ANM structural technology offer significant scientific and technological challenges that can be met over time.

Dr. Bob Cassanova describes the ANM flapping wing system on October 23, 2005 Cincinnati Edition NPR X-Star Radio Network, “This particular concept is using a potentially earth shaking concept for a material that would be a new material and structural development that would incorporate materials into a wing and would allow it to flap.”

This Phase I NIAC Study evaluated the feasibility of developing a completely new concept of a “functionally intelligent material.” The diversity and complexity of thrusts that have been identified as being central to the continued development of the ANM structural technology offer significant scientific and technological challenges that can be met over time. At the Los Alamos National Laboratory Center for Integrated Nanotechnologies (CINT) complex functional materials are defined as complex and collective interactions between individual components in materials to yield emergent properties

We proposed to investigate the development of an artificial neural membrane (ANM) flapping wing as a demonstration of the potential of neural engineering and advanced materials to create a revolutionary technology based in biomimetics and nanoscience we were aware that this was an area of investigation that had not been pursued to any great degree. The beginning of the study found hundreds of related applications yet nothing that directly created what amounted to be artificially intelligent structures with the capability to fly independently.

As principal investigator I spent many hours identifying possible emerging technologies to later find that they are less “emergent” than described. So starting from the fundamentals I began to identify how my organization could leverage our experience in artificial neuron or A-neuron design and materials engineering to increase the probability of developing a realistic developmental platform in the next ten (10) years.

Thrust areas are specifically directed at developing complex functional nanostructures, a new class of intelligent structure are considered in terms of a simple development matrix.

ANM Flapping Wing Development Thrust Areas	
Materials & Structures	Mechanics, force distribution, elasticity, mechanical coupling, interaction of nanoscale structures, integration of nems
Computational & Simulation	Flapping wing governing equation, neural algorithms, topological networks, embedded processing, control systems
Aerodynamics	Airfoil, stability, flow distribution, Flutter and Vibration, performance in earth and Martian atmospheres

The functional elements of these thrust areas are identified to be:

ANM Flapping Wing Development Thrust Area Functional Elements	
Complex Functional Nanomaterials	Interactions between individual components, properties and functions
Nanomechanics	mechanical behavior of nanoscale materials and structures
Computational	Advanced applied topology. Simulation of neural networks on mechanical networks, information and energetic stability in shuffle networks
Aerodynamics & Control	Aeroelasticity, optimal control theory for active membranes

Phase I Outline:

Artificial Neural Membrane Flapping Wing

New class of intelligent functional structures

Experience in functional structures

Current resident experience and technology

Identification of primary thrust areas for development

Materials & Structures, Computational & Simulation, Aerodynamics

Identification resources supporting thrust areas

Establish current and emerging science including resources to for further study

Artificial Neural Membrane (ANM) Technology

Applications and potential (Developed the ANM Flapping Wing Roadmap)

This study uniquely allowed for the investigation of the primary thrust areas and how they relate to each other in the development of an ANM functional structure. The parameters of the mathematical models and functional computational complexities were evaluated. Potential requirements for embedding complex networks within networks established. Several possibilities were evaluated for the engineering and integration of nanoelectromechanical (nems) in the ANM flapping wing as power, propulsion and sensory sources. The study also supported building of a virtual library and advanced concept visualization process which provides opportunities for discussion with other researchers in developing the nano-structures necessary to implement a functional neural membrane structure or vehicle. This library of vehicle structures and exploration mission concepts is on line at www.arsispace.com.

Phase I: Outcomes

Developed structured flapping wing visualization models, defined aerodynamic structures and dynamics of a conceptual ANM flapping wing, created a reasonable approach to algebraic mapping of embedded networks and evaluate existing and developmental technologies supportive of the fabrication of a neural flying structure.

It is important to recognize in the reading of this report that it does not contain proprietary data or materials.

ARTIFICIAL NEURAL MEMBRANE FLAPPING WING

Advanced Concept Description

The efficiency and functionality of autonomous intelligent systems (AIS) in the design of new generation of aerospace vehicles is almost acknowledged universally. AIS technologies are ideally used to increase the safety and reliability of complex advanced aircraft and launch systems. The perspective of AIS as a building block in the development of new vehicles should be recognized. It should no longer be seen as a luxury item to demonstrate an organization's technical competence. AIS should be not only the primary philosophy in

vehicle architecture design, but a fundamental tool in developing adaptive, modular, and highly integrated vehicle systems. When we say highly integrated we have to consider the change in paradigm from the application of parasitic systems engineering with high interdependence and cost to fault tolerant, robust systems allowing for multifunctional architectures supporting diverse technologies (Menges, 2005/2006).

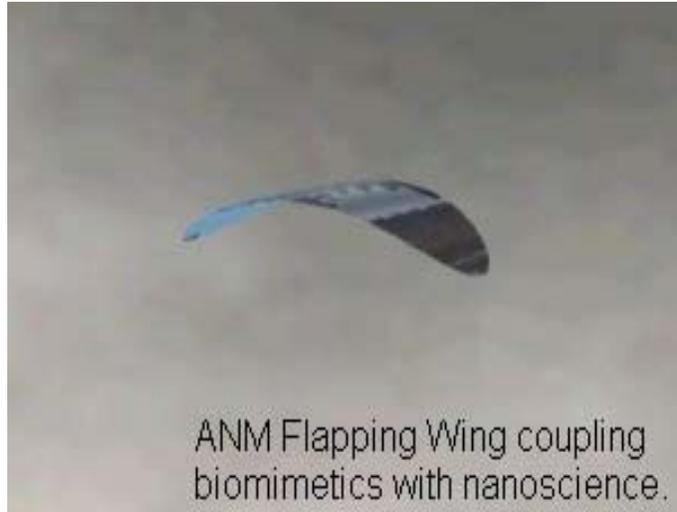


Figure 2-1

Over the past two decades, artificial neural networks or ANN have evolved from a simple computer simulation to performance adaptive systems based on computational models derived from study of the actual ordering of vertebrate neurons and signal analysis of neuronal operation. From these studies, artificial neurons or A-neurons have been modeled and fabricated as functioning electronic, electrooptic and optical devices (Menges, 2005/2006).

Since neural networks are perceived as simply an interconnected graph of nodes whose function is based upon the nerve cells of a living organism our concept of an ANM must be defined with somewhat new parameters for an artificial neural network (Mercadal, 1990). In more advanced ANN the nodes are processing elements that possess learning rules supporting a self-organizing or performance adaptive system. The development of this concept creates a set of models to support design of new ANN systems that will function to acquire and control multiple micro and nano devices integrated on thin film layers that comprise our artificial neural membrane.

In 1999 ARSI flight tested the ability of a software based ANN to control multifunctional smart structures panels for applications to detecting structural damage. The testing program demonstrated the use of an adaptive neural expert system to acquire distributed resistance

data and determine the “health” of the structures being monitored. The neural system is a conventional rule based expert system utilizing a neural network to acquire and control data from subsystems in this case test panels (Menges, 1999). This Neurogenesis algorithm is the evolutionary neural concept that supports the neural function on our ANM structure.

ANN software has been successfully demonstrated as controllers for a new generation of multifunctional smart structures. Now advancements in materials and thin film technologies have offered unique opportunities in the development of ANN device controlled smart skin sensors. ARSI simulated its first single neuron thin film (SNTF) sensor to control the response of a coherent light detector by differentiating between scanning and tracking functions of a Yag laser used in infrared scanning and tracking (IRST) systems.

The simulated A-neuron was a simple linear carbon sensor (Menges, 1999) and was used to demonstrate the ability to detect a signal and have a material react as part of a layered thin film device. More recently ARSI has integrated a multilayered A-neuron controller to initiate motion when a sensor detects coherent radiation. Both demonstrations indicate that materials will perform as sensors, circuits, and actuators. However, ordering an A-neuron to provide adaptive control requires the ability to support specific forms of effective flapping flight. The flapping flight must be further defined in a valid model or sets of model to establish a functional control network.



Figure 2- 2

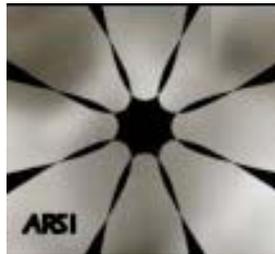


Figure 2-3

Figure 2-1: Aerospace Research Systems, Inc. simulated A-neuron sensor design required the precise deposition of metals for differentiation of signal thresholds depending on the resistance measured. Figure:2- 3 Concept for carbon tubule controlled membrane sensor where laser light excitation of the membrane creates current to cause the carbon tubules to realign causing the membrane to contract or flap.

Defining the function of a natural neuron we consider the spatial and the temporal aspects of its operation. This is also true for the A-neuron. Thus we are limited by lack of definition of the spatial and temporal mechanics of an ANM Flapping Wing. The development a set of virtual models of flapping motion also requires that we create an adequate ANM concept technologically and we attempted to provide a starting point to consider technologies and a functioning concept structure in this Phase I.

In conceptualizing an artificial neural membrane we have obviously taken some liberties in that natural neural membranes are highly complex and uniquely differentiated structures supporting neuron function and metabolism. Natural neural membranes generally are the body of the neuron that possess ion channels and are structured so that a thin bilayer of lipids isolates the extracellular side from the intracellular. The size of the neuron, especially the axon determines the speed at which it operates as does whether it is myelinated or not. Since myelination provides insulation, then myelinated axons provide faster conduction of signals

(Koch, 1999). This concept of insulating A-neurons may have its place. Since during the course of this study one of the significant issues that arose was the behavior of several energetic systems resident on a substrate so that issues of signal leakage, tunneling and other forms of transport must be addressed when considering the properties and function of the materials integrated. This will be discussed more in the ROADMAP section.

The similarities between our concept and that of the natural neural membrane only extend to the idea that we are conceptualizing an artificial thin film laminated membrane may be controlled by an artificial neuron or A-neuron. For most of our purposes will consider the A-neuron in fact an artificial neural network due to the requirement of configuring its circuits to control multiple functions including the flapping of the wing. However that does not detract from the fact that we can learn about the organization and function of our ANM through the study of the fundamental structures and functions of a natural neural membranes. This is the primary principal in biomimetics.

We are redefining the ANN to support our concept of an ANM by deriving some of the complex structures characteristic of a natural neural membrane and apply them to various aspects to miniaturize components of the ANM matrix. One such area is the ability to generate power and impart motion through a layered thick film matrix.

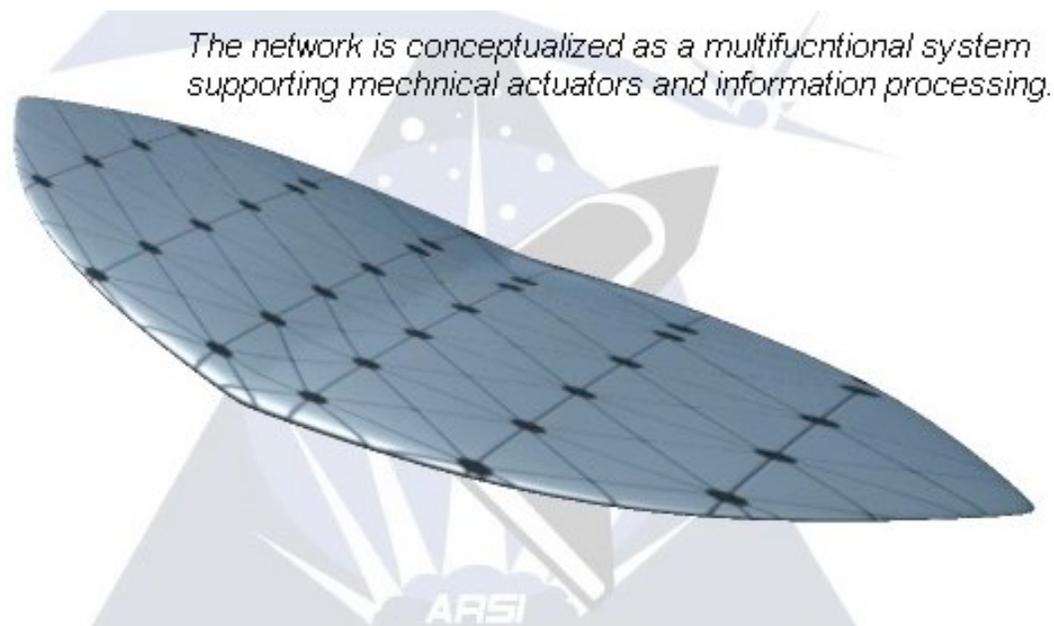


Figure 2-4: *Concept of the configuration of the carbon nanotubules or Nitinol Shaped Memory Alloy embedded in the outer polymer capsule of the membrane matrix.*

In Figure 2-4, the configuration theorizes either a herringbone or scissor structure to the nanotubules. The actual interface will rely on the direct contact of the metallic thin film layer by the carbon fibers or the NiTi actuators. The triangular shapes will contain the actual A-

neuron nodes which will be configured as part of the deposition process. The deposition of the nanotubules or NiTi will require the correct geometry for creating flapping motion consistent with the type of ANM flapping wing vehicle or structure. The two configurations under consideration and chosen from current experimental data are the dual chiral bundle nanotubule and the NiTi nanoparticles deposited thin film actuators (Menges, 1999, 2006).

We have chosen the concept of a matrix to describe the layered structures of our ANM. It is more closely related to the form of channels and organelles found in a natural neural membrane. The matrix also allows us to theorize about the potential of integrating ultra thin film technology with advanced enzyme films for both power and sensor functions.

The proposed ANM structure can be described as a micro-laminate supporting a matrix which is the case within certain parameters. The actual ANM is conceptualized as a thin encapsulated membrane fabricated through the layering of materials (metals and ceramics) through plasma deposition of thin film materials and nano-lithography components will be deposited on a high density polymer film as opposed to the conventional polyamide sheet. The conceptualized structure is integrated to create a multifunctional ANM vehicle providing remote sensing missions in several environments depending on the configuration of matrix and membrane level devices and materials.

Secondary power and thermal management will be supported by an organometallic structure that produces electric current through the motion of the ANM and supports communications between the two surfaces of the outer membrane by transporting data to the ANN. The organometallic gel will be surrounded by a permeable polymer membrane which will act in much the same way as the thin bilayer of lipids isolates the extracellular side from the intracellular in natural neural membranes. The semipermeable interface layer has been supported by this study a may be a means of isolating signal leakage and tunneling.

Current flapping wing motion is envisioned to be produced through a simple linear contraction. This study created simulated flapping motion of ANM that increased the potential motion to adapting to more sophisticated flight including wing twisting for stabilization (Menges 1995, 2005). Extension of leading edges through bending and rolling, turning and attitude adjustment through camber changes and bending of trailing edges mimicking ailerons and vertical and horizontal stabilizers.

The actual control of the flapping motion of the ANM will be determined in several ways. The ANN inputs will be from one or more sources depending on the complexity of integration. Natural neurons respond to inputs that can be defined as clustered and unclustered (Koch, 1999). If we considered our ANM to be composed of ANN interfaced actuators acting as artificial muscle to cause the flapping motion through excitation of the actuators, then we can determine the relative nature of possible inputs. These would include primary feedback from our ANN controlling the flapping and attitude and signals produced from any sensors or detectors. The inputs singularly or in clusters would be managed through

threshold level as well as preprogrammed signal detection in the ANN circuitry. The COMPUTATIONAL STRUCTURES section covers this in more detail.

The application of directly deposited devices and amorphous materials to integrate the existing state of the art in thin film deposition and thick film organic and organometallic materials and processes offers fabrication potential in the 10 to 15 year range. It is reasonable to predict that within a few short years methods in nano lithography will evolve allowing for the fabrication of the more complex NEMS on thin and ultra thin films. The convergence of ANN with NEMS will offer substantially more options in integrating multifunctional ANM vehicles and structures. Limitations in visualization of deposited thin film devices (i.e. using atomic force microscopy), prevents adequate real time visual control of thin film deposited devices.

Embedding nanotubules with coatings also reduces the efficiency of current and other signal exchange between the membrane matrix and the ANN necessitating the mechanical addition of interface channels. Improvement in deposition through lithographic processes will eliminate the need for coatings that may age differently than other materials used in ANM fabrication.

FUNCTIONAL MATERIALS

The two possibilities for creating a mechanical-data network, a network that is both functionally an actuator system with controlled mechanical linkages and a network capable of processing distributed data, are shaped memory alloy (SMA) and carbon nanotubes (CNT). In our introduction we discussed the importance of being able to sustain complex computational processes across a dynamic structure. Several possibilities from simulating networks on networks to creating shuffle algorithms to handle data morphing have been investigated in this Phase I. As discussed in the introduction the Phase I study has to a large degree been a feasibility study to the end two possibilities for materials and integration have been identified as being reasonable for our purposes and within reach in the next 10 years.

The first carbon nanotubes were synthesized by a vapor process was over a century ago. Nanotube hollow structures were not observed until 1958. Most carbon nanotubes are produced by a catalytic process over a gaseous species formed through the thermal decomposition of hydrocarbons.

Contemporary materials science has provided many more tools for both studying and producing carbon nanotubes. The single walled nanotube (SWNT) is nearly a perfect dimensionless structure. In recent years the use of SWNTs in developing nanoscale studies in electromagnetic systems has created an entirely new area nanoelectromechanical systems or nems. Nems have proven to offer a variety of geometries and capabilities including the potential for resident information processing.

Our interest in nems is supported by the potential offered by new methods of nanotube construction, embedding of metallic nano particles and doping of even more interesting structures created from Carbon-60, Buckyballs.



Figure 3 – 1

Dual chiral bundle concept potentially providing a very efficient 4 nm diameter actuator with and improved energetics over SWNT in simple helical configurations.

Computationally it has been shown that flattening the SWNT into a ribbon is energetically more robust than the tubule structure for any size greater than 2.5 nm. However, nanotubules have been created with diameters of 0.4 nm with the more reasonable energetic diameter of 1.4 nm. Nanotubes can be synthesized at any length including micrometer and millimeter scales. This characteristic provides single molecules structures with very large aspect ratios and potentially a dimensionally perfect structure for a variety of applications.

Another method for producing SWNTs is by rolling a sheet of graphene. This allows for a number of possible geometries based on the tube formation; straight, zig-zag and helical or chiral. The helical or chiral nanotube possibly offers the greatest potential for use as a nems components or as ANM wing actuators. The mathematical model derived for fabrication of nanotubules from graphene sheets is defined by the vector of helicity C_h and the angle of helicity θ (Bhushan, 2004), n and m are integers of the vector OA and a_1 and a_2 are the unit vectors. So that

$$OA = C_h = na_1 + ma_2 \quad \text{with}$$

$$a_1 = (a\sqrt{3}/2)x + (a/2)y,$$

$$a_2 = (a\sqrt{3}/2)x - (a/2)y \quad \text{where } a = 2.46 \text{ \AA},$$

$$\cos \theta = (2n + m)/2(n^2 + m^2 + nm)^{-1/2}$$

“The vector helicity $C_h = OA$ is perpendicular to the tube axis, while the angle of helicity θ is taken with respect to the so called zig-zag axis” so it is the helicity vector that creates the “zig-zag” of the tube (Bhushan, 2004). Finding the diameter of the nanotube is simplified by

$$D = \|C_h\|/\pi = [a_{CC}(2(n^2 + m^2 + nm))^{-1/2}]/\pi \quad \text{where } 1.41 \text{ \AA}_{\text{graphite}} \leq a_{C=C} \leq 1.44 \text{ \AA}_{\text{C60, Buckyballs}}.*$$

The effective diameter for structural purposes in less than 2nm but bundling SWNT may increase their strength as well as maintain structural effectiveness. This is the basis of several new processes that may provide dynamic response across SWNT assemblies supporting actuator system development.

Rolling two graphene sheets together and using a laser to cut the outer roll in equal rings from the inner section produces an interesting structure reminiscent of a bellows (See Figure 3-2). The concept creates a dimensionally controllable actuator with a greater potential for inducing displacements over longer distances than a chiral or helical nanotubule structure. This concept is under development at ARSI and offers a potential to change the paradigm in engineering and fabricating SWNTs and bundled nanotubules.

*Adapted from (Bhushan, 2004) page 42.

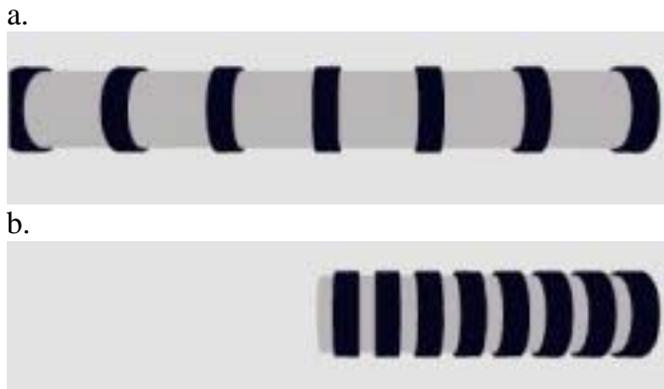


Figure 3-2: a. extended bellows. b. contracted bellows.

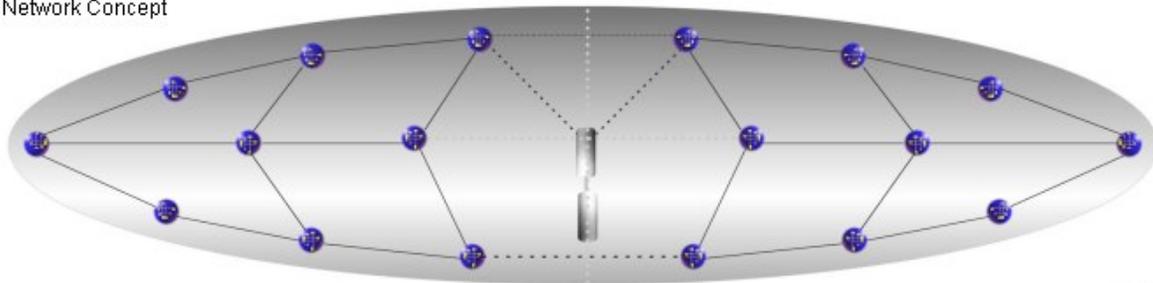
The bellow or coaxial actuator functions as a linear actuator producing stable motion in a specific direction with a controlled distance of travel. The significance of controlling the travel distance of such a device allows for increased precision in not just placement but force. The more controllable the force induced on the membrane is the greater the overall dynamic stability of the system, in our case a flapping wing vehicle.

Other potential opportunities lie with nanolithographic techniques that may include the use and arrangement of SWNTs or a variety of other materials including conductive and nonconductive metals.

Nitinol SMA

Nitinol (NiTi) is a Shaped Memory Alloys (SMA). The composition of NiTi alloys are nearly equivalent amounts of nickel and titanium. However the slightest change in the alloy ratios creates significant differences in the transition temperature of the alloy. Since NiTi contracts as it is heated it is easily controlled through resistance circuits. Further its unique physical properties make it a candidate for a heat engine. If the ANM flapping wing were integrated with an IR active photovoltaic film for power, it may also include a solar thermal film that could absorb solar energy as well as work energy and redirect it to the NiTi heat engine actuators (Redinoitis and Lagoudas, 2001).

ANM Flapping Wing NiTi Multifunctional Actuator
Network Concept



ARSI

Figure 3-3

The wing is conceptualized to be several polyamide or membrane type layers providing different levels of functional substrates. More than one set of type actuators may be embedded or deposited within the membrane structure. Power, thermal management, navigation, communications and sensor systems would also reside in the substrates. (See The ANM Concept Section).

In any case the relatively low power requirements, 8 to 20 V, for NiTi allow a number of possibilities in their application to a flapping wing powered vehicle. Actuator dimensions ultimately determine the voltage and related current required for contraction. The mechanical response exhibited by the NiTi wire is dependent on ohmic heating. The force the NiTi actuator would exert on the wing membrane depends on the counterforce provided by the membrane. The push-pull effect also affects the life span of the NiTi actuator, the lower the percentage of contraction of the wire the higher the number of cycles before failure.

Processes under development by ARSI for application of nanoparticles of NiTi to a thin film substrate theorize the integration of ceramics for improved insulation and activation of various components of the individual actuators. This allows for varying degrees of actuation and is intended to improve reactive function of the actuators while reducing fatigue related issues. Since the energy densities of NiTi are much higher than that of a nanotubule chiral structure there increased efficiency in inducing controlled displacements makes them ideal for application to membrane structures.

The physical and mechanical properties of NiTi are dependent on its crystalline structure. The NiTi crystal structure is a very dynamic and highly heat sensitive and when it is deformed in the martensite phase, the crystalline structure is not damaged. Instead the crystal structure transforms moving in a singular crystalline direction. When heated the material returns to the memory or austenite phase, to a state of less stress. The austenite phase is the phase above transition temperature. The transition temperature will vary according to the NiTi alloy composition. Most NiTi alloys have transition temperatures between 70-130°C with tensile strength 200,000 psi, melting point of 1,250°C, and resistance 1.25 Ω per inch/0.006 inch wire.

In non-memory metals, deformations cause dislocations of the molecular structure into new crystal positions. There is no "memory" in the crystal of where the atoms were before they moved. The mechanical responses of NiTi, is caused by internal molecular restructuring making it a very strong metal.

The current commercial candidate for our actuator development is an alloy produced by Dynalloy with the trademark name Flexinol. Flexinol wire has been successfully used as a NiTi actuator and has the potential to function at more than a million cycles before failure unlike other NiTi alloys. The 15 mil Flexinol wire produces a contraction force of 1.814 kg at up to 9% of its length.

ANM Power Sources and Systems

Several new technologies have been developed commercially over the past ten years that have allowed the application of thin film and amorphous photovoltaic materials. All until recently have been ultraviolet (UV) activated. Other potential power sources are piezoelectric and produce power through the motion of the membrane in flight. Air flow activated nems

may also be an emerging field where power is generated by gas and fluid flows over a nanofilms or membrane structure embedded with molecular rotors.

Membrane structures being extremely lightweight and easily deformed can also provide finer and more diversified types of control. The mass of the ANM flapping wing is also ideal for

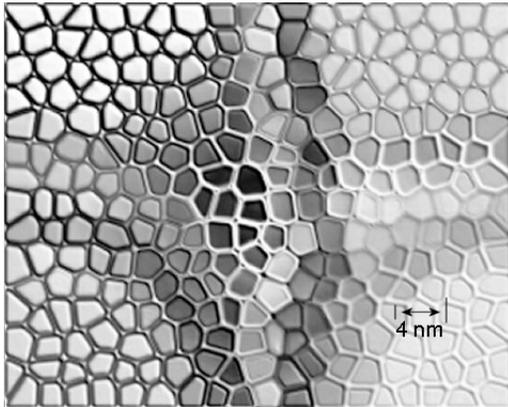


Figure 3-4: After T. Sargent, IR photo voltaic nanofilms, individual functional photo active “cells” seen at 4 nm.

very thin atmospheres like the Martian atmosphere. The Martian atmosphere is earth equivalent to approximately 132,000 feet. Making is significantly less dense than that of earth, but also allowing for increased solar thermal, UV and IR radiation. This is also to our advantage due to advances in photovoltaic nanofilms. Ted Sargent of the University of Toronto has developed a thin film deposited nanotechnology that produces deposit cells of approximately 4 nm and is photoactive in the infrared range. This increases the efficiency of the film by producing power with less solar radiation required than with the standard ultraviolet (UV) photovoltaic system.

As part of our roadmap of technologies required for the development and fabrication of a ANM flapping wing vehicle we have identified a number of methods both available and emerging.

Mechanical & Electrical measurements CNT

The measurement of mechanical and electrical properties of CNT is fundamental to developing force analysis of CNT actuators, further, the actual mechanisms by which helical or chiral CNTs function is not well understood particularly in the presence of other energetic systems. One of the primary areas of interest in creating an effective roadmap for the development of an ANM Flapping Wing vehicle will be the development and improvement of current methods of analyzing strain, force, and stresses in nems and mems. The next step in this process would be understanding the coupling forces and stresses induced in the membrane substrates. Current methods are very limited and include a few instruments including atomic force microscopy (AFM), high-precision capacitance bridges and laser stylus.

Electron transport properties of carbon nanotubes have been measured using scanning tunneling microscopes (STM) in conjunction with a probe and AFM may also be used with a conducting tip. The electrical properties of carbon nanotubes (SNT), both single and multi-walled, have been measured with a great deal of precision due to their length. STM offers molecular resolution, however new methods using high precision capacitance bridges can measure strain at the atomic level (Menges, 2006). Nanometer scale networks have been

measured providing current-voltage and current-distance curves important to us understanding the structure and capability of nanotubes to process information. Lithographic methods also offer both mechanical data and fabrication processes.

Emerging methods currently under development at the Center for Integrated Nanotechnology will improve SWNT and other nanoscale manipulation and increase the level of knowledge necessary to evaluate forces on nanoscale systems. Currently the best level of force induction on a ANM membrane would be available from the NiTi SMA data. The NiTi force coefficients and surface coupling with the membrane substrate or substrates can be approximated with a high certainty. The same cannot be said for a SWNT actuator system.

DEFINING COMPUTATIONAL STRUCTURES FOR ANM SYSTEMS

Introduction

During the is phase I study we investigated topological methods for implementing networks inside of networks and developed models for application to the functional geometries that will create the components of the neural structure. The neural structures and their function have been to a large degree defined by the ARIS Neurogenesis algorithms and software. The Neurogenesis software was developed from prior research in the differentiation of the central nervous system (CNS) of vertebrates particularly the differentiation from the notochord, or the CNS precursor, to higher functioning central and peripheral systems. The algorithm also integrates signal values and support on-board signal processing.

The underlying thrust areas of this Phase I created an opportunity to evaluate not just computational methods to produce networks on networks and model embedded processing but to experiment with animation methods and simulation programming. This section is supported by a series of animations posted on the World Wide Web and can be accessed through the Aerospace Research Systems, Inc. site at www.arsispace.com. These animations assist in the visualization of complex systems and functions as well as aid in conceptualizing integration. As with all the data made public in this report there is no proprietary data includes, however the simulations provide interesting insights into applications of nanotechnology and possible exploration missions.

The evaluation of a variety of computational methods identified significant requirements for the “ANM Flapping Wing Development Roadmap.” Due to the substantial complexity of the ANM functional structure, advances in algebraic topology along with network modeling and simulations applications drives the need for an independent effort to develop the necessary mathematical methods and algorithms to ultimately be successful.

COMPLEXITIES OF SUBMICRON AND MOLECULAR SYSTEMS

As discussed in the introduction much of the science required for the development of an ANM structure is in its infancy. The primary areas of thrust were defined at a top down level as Materials & Structures, Computational & Simulation, and Aerodynamics. From a more differentiated level we see that even the disparity between the thrust areas becomes less definite and an increasing interdependence occurs.

Returning to the current definition of the ANM flapping wing as a highly integrated, intelligent complex functional structure we need to discuss the potential for supporting the subsystem elements. If we recall that the functional nature of the ANM flapping wing is



Figure4-1: *Microscale sensors conceptualized as artificial super-neurons for multifunctional applications under development at ARSI.*

created by a new concept in computing and materials. The new intelligent functional material is organized to support information, signal and control processing simultaneously. It is the method whereby we simulate a network on a network so that the operational neural network is not just embedded but a functional subsystem of the actuator and information networks.

This description requires further consideration of the known science of these molecular and submolecular systems. In this section we consider electron transport theory and evaluate how E. G. Emberly and G. Kirczenow investigated electron transport using a molecular device whereby a molecular wire is attached to two leads. They chose 1,4 benzene-dithiol (BDT) attached to two Au leads. Due to the benzene ring structure of BDT it offers one of the better opportunities to form π -bonds in its delocalized state (Emberly & Kirczenow, 1998). This gives a good set of values for the consideration of the basic energetics observed in molecular switching.

We also made a leap from the energetics perspective to consider the application of a quantum algorithm in the application of quantum dot doped substructures to support co-processing of data using binary data extrapolated to qubits in order to sustain long chains of distributed code within virtually embedded networks. This possibility of maintaining binary information in a quantum environment and co-processing multiple or simultaneously simulated networks may be the concept that best provides the opportunity to create an ANM structure capable of complex autonomous operation. It is important to have the ability to support cross-functionality using binary for both the electrical and quantum processing. It resolves the difficulty in pre- and post-processing of data going from bit to quantum bit or qubit and back.

As a quick review we consider the function and type of natural neurons that exist in organic systems. Then we look at the issue of supporting circuit integrity and electron transport (data transport) in both simple molecular and quantum systems.

Defining Neuron Function

The structure and operation of a natural neuron is based on its function. Sensory neurons also known as afferent or receptor neurons, receive the initial or primary stimulus. Efferent or motor neurons stimulate effectors that produce some kind of response for instance a muscle contraction. The interneurons or connector neurons are generally part of the central nervous system whereas the sensory and motor are generally part of the peripheral nervous system. The interneurons would control the function of the other neurons based on an “interrogation” of the signals making them a primary signal processing node.

For our purposes we are conceptualizing an A-Neuron that can configure to any of the functions of a natural neuron based on the information input by the distributed network. This creates a dimensional processing capability within the network and allows for multiple functions across several levels.

Transmission of a nerve impulse across a natural neuron from one end to the other is a result of a series of chemical reactions. In the A-neuron it is a function of signal strength against noise from the network. The signal is further transmitted according to the function of nearby A-neurons that in turn define the function of each neuron they transmit to or interrogate. This form of communication allows the network to configure to its control functions based on flight dynamics of the wing. This differs from a natural neuron whose signal conductance is based on polarization. This may be a similar mode of operation for the nanotubule chiral structures given information on distributed functions as actuators. This may however not be translated to information processing on the nanotubules that may be more a function of transport theory than polarization.



Excitatory function of a dendrite activates the transmission of a signal based on voltage and transmitted through a particular conductance based on the type of neuron. The inhibitory function inhibits the signal over part of the dendrite depending on the type, voltage and energy of the signal. This is the basis by which we will model our A-neuron where the excitatory and inhibitory functions control voltages through the neural circuit. This provides a preliminary digital model for our neural data to be processed. *Right: Figure 4-2 Simple neuron geometry with the axon and body in grey and dendrites in black. Synaptic junctions not shown would in our A-neuron concept actually be gates triggered by signal strength allowing voltages across the circuits or nodes.*

Artificial Neuron Circuits Molecular Energetics and Quantum States

Natural dendrites have both fast and slow excitatory and inhibitory dendrites. These differences are based on structural elements that allow for different rates of transmission of signals through the neuron. The electrochemical process is driven by cell membrane permeability allowing a controlled exchange of ions producing an energetic system for signal conduction and processing. If we consider information processing as a function of signals, in the case of binary logic values of 1's and 0's, we can extrapolate that concept of an A-neuron circuit processing data in digital formats with associated thresholds for gate functions.

The next issue to contend with is the rate of firing. The temporal function is essential to the control of any mechanical system it is equally important for the processing of information on a neuron. It has been observed experimentally that natural neurons control temporal aspects through local as well as non-local inputs. The transmission of a signal to the actuator would be relayed through the non-local adjacent node. The rate of transmission is based on the governing equation for control of the wing geometry in response to the flight profile. *Right: Figure 4-3 ARSI developed A-neurons for photonic detectors control response to coherent light hazards.*



The speed at which our network operates is a function of the individual nodes and actuators in response to the flight model inputs as well as the materials properties of the actuators. Natural neurons vary firing rates according the structure of branches and firing rates vary in terms of local and non-local inputs. “Dendrite spikes initiated in response to local synaptic input can implement submillisecond temporal discrimination (Koch, 1999; Softky, 1994 & 1995).” For our purposes and due to the scale of our two possible systems, the NiTi and the carbon nanotubule systems, processing speeds across the network will operate in the millisecond to nanosecond range. Since we are dealing with systems of scale, the nanotubule structures will respond in the nanosecond range that translates in a macroscale to submilliseconds.

Energetics and Electron Transport

A-neuron circuit structure allows for total current I_{stim} transferred to each dendrite is divided relative to their input conductance, so the current (I) transferred to each branch with a diameter d_1 is equal to $I_{stim}d_1^{3/2}/(d_1^{3/2} + d_2^{3/2})$.

This is derived from the concept of cable theory used to describe spike initiation in natural branched dendrites exhibiting weakly excitable behavior (Koch, 1999; Emberly & Kirczenow, 1998).

By extrapolating from molecular conduction, electron transport in A-neurons can be seen as a tunneling phenomenon that is a function of the source of energy E has a probability $T(E)$ of being transmitted from its source to the drain (Emberly & Kirczenow, 1998). We can mathematically determine the transmitted current by calculating $I(V)$ or current times voltage where μ_s and μ_d are the chemical potentials of the respective molecules and correlate to the source and drain respectively.

$$I(V) = 2e/h \int dE T (1/\exp[(E - \mu_s)/kT] + 1) - (1/\exp[(E - \mu_d)/kT] + 1) \quad (\text{After Emberly \& Kirczenow, 1998})$$

Where μ_s and μ_d are defined by the following and V is the source drain bias voltage (Emberly & Kirczenow, 1998):

$$\mu_s = \epsilon_F + eV/2$$

$$\mu_d = \epsilon_F + eV/2.$$

If the transmission probability $T(E)$ is calculated extrapolating from the Lippmann-Schwinger equation we obtain

$$|\psi\rangle = |\Phi_0\rangle + G_0W|\psi\rangle$$

This gives us a unique opportunity to consider the energetics of a quantum system and its relationship to a binary system. Colin Williams of Caltech developed a method for entering binary code into a quantum computer this coupled with understanding the energetics of the molecular circuit based on Lippmann-Schwinger equation we can derive the potential for defining eigenstates within a quantum dot matrix that would support a larger variety of signal transmissions within in our A-neurons.

The quantum algorithm developed by Williams supports encoding vary large numbers of classical data bits by transforming data to smaller number quantum bits or qubits through different energy levels. A $\log_2 N$ -qubit state quantum data processing quantum dot matrix could quickly process large amount of structural and environmental data relayed from A-neuron control nodes and adjust the flight controls. The qubit processing nodes could in tern re-program the primary control neurons through learning sub-routines. This would be useful when first encountering turbulence or when first deployed within the upper atmosphere of a planet to respond to rapidly changing gas densities and temperatures.

Collins' algorithm uses a set of simply implemented steps taking a binary number or for example a four (4) bit binary string and convert it to four corresponding 2-bit eigenstates. So for the four bit string 0111 the corresponding eigenstates could be $|00\rangle$, $|01\rangle$, $|10\rangle$, $|11\rangle$. Collin's then constructs a superposition $|\psi\rangle$ of equal quantum states that peaked at only the eigenstates that correspond to 1's in the binary bit string (NASA Tech Briefs, March 2005, p.24).

So that the 2-qubit state would is:

$|\psi\rangle = 3^{-1/2} (|01\rangle + |10\rangle + |11\rangle)$ thus creating an entangled state of n-qubits that encode a sequence of binary bits (NASA Tech Briefs, March 2005, p.24).

The next step computes the unitary transformation required to obtain the superposition where the unitary maps the chosen state into $|\psi\rangle$. According to Collins the binary bit string 0111 would map as the unitary matrix:

$\begin{matrix} 0 & -1/(3)^{-2} & -1/(3)^{-2} & -1/(3)^{-2} \\ 1/(3)^{-2} & 2/3 & -1/3 & -1/3 \\ 1/(3)^{-2} & -1/3 & 2/3 & -1/3 \\ 1/(3)^{-2} & -1/3 & -1/3 & 2/3 \end{matrix}$	Creating the matrix first compute $ \psi\rangle \langle\psi $ for column 1 and then generate orthonormal vectors for the remaining columns. Then compute the most likely quantum circuit equivalent (NASA Tech Briefs, March 2005, p.24).
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Figure 4-4

Conclusions: Molecular Circuits Binary and Quantum Computing

Where trends in functional quantum dot matrices continue to increase in competencies, the reasonable potential for integrating quantum circuits in the membrane structure of our wing

presupposes the ability to differentiate between different energetic systems and assume no quantum tunneling or commingling. Isolating quantum circuitry will be a primary challenge for nanoscale systems. However, if the issue of contaminated signals and noise can be effectively addressed the ability to process binary data within a quantum system will create the best possible bridge between the two technologies in the next 10-20 years.

Fusing Data and Materials

Since we have an understanding that standard programming languages including such things as Prolog and LISP are not functional for our purposes, we have to determine the most effective way to process our data and provide performance adaptive controls for our wing. Assuming the basic quasi steady state model by Sane and Dickenson is adequate for our purposes at this time then we have to consider that as a basic computational starting point. Next by defining the possibilities of our carbon nanotubes in terms of movement and mechanical function we have a concept of the potential geometries and structures offered. However, we have been visualizing a different type of carbon lattice, one that may offer a structural-control system as well as a multifunctional environment for processing and power. We know that the US Navy has helped to develop carbon matrix batteries, we also know that new methods of doping carbon materials produces a photovoltaic material, and that quantum dot technology can be imbedded in any stable thin film material. These “knowns” allow us to consider the possibility of a carbon-60 superlattice offering not just a multifunctional thin film but a superstructure to program operations for flight, processing, control, propulsion and power.

Rénee Descartés (Aczel) made an interesting observation; geometric objects offer unique geometric values. In reading his least know work about his lost notebook, we see a structure to geometric operations that has not to the writer’s knowledge been applied to distributed (computing) operations. The unique structure of certain shapes lends themselves to binary structures. This basic idea of controlling photonic processing by specific geometries is not new however if we extend the idea to the ability to create superstructures within our superlattice using quantum dots and chirals, we then have a new material structure capable of self organizing both structural character and control of resident operations. In theory this may be reasonably accessible given current developments in carbon engineering. It was this set of observations that provided us (ASRI) with the concept of doping polymer molecules and bonding them to form a type of Nano Rotor. The chemistry for the application exists and is used in labeling reactions and genetic markers. The same method may provide a molecular structure capable of a rotary motion. The Nano Rotor may function as an impeller for moving gases and liquids or as a propulsion mechanism.

The Functional Network

In conceptualizing possible materials and structures for engineered neural membranes we also have determine the operational characteristics of the ANM. The first operational environment of primary concern is the “operating system.” The ANM operating system or

OS is the first of its type. Understanding that the ANM will reside in what amounts to be a programmable material matrix, the OS is not a traditional binary based system. In fact the methods by which the signals and base data are processed resemble a photonic system more than a digital. However, new advances in integrating binary data into photonic systems may allow a more rapid integration of a number of technologies.

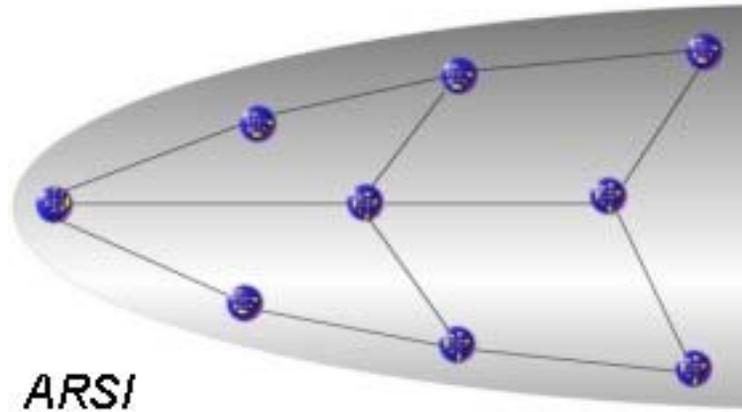


Figure 4-5

ANM wing section with multifunctional network supporting actuator, neural network simulation and information processing. The graphic is based on the NiTi actuator network with intelligent nodes. This concept is seen to be the most accessible technologically.

The matrix may be defined computationally as a superlattice, a topological structure that is embedded by other topologies. These resident topologies will comprise the components of the neural networks as well as the functional nems machinery. Assuming full scalability the embedded structures may include a number of variations not limited to single neurons, but cluster neurons and cortical structures. For our purposes an artificial cortical structure would simulate a supernode. Such a supernode would act not just as an artificial neuron or A-neuron, but a data resident structure or signal storage structure. Such a structure could provide the sensor and control nets with a history of activity over time and be used in more conventional computing devices as a random access memory where there is cross function compatibility.

Another attribute of engineered neural membranes not directly involved in common binary systems is the application of the fourth dimension as a specific coding element. Most computers have clocks integrated within some part of their hardware. Neural systems are highly dependent on temporal coding and thus reliant on 4-dimensions for function as opposed to two or three in conventional digital models. This reliance is due to the function of the natural neuron. Neurons in organic systems are often defined by their firing rates and signal thresholds, both temporal measurements. This element of natural neurons may permit development of a new concept of shared learning in A-neurons. The idea of shared learning or mimicking of neural function is not unusual in the natural world either and neurons that do this are referred to as mirror neurons.

Modeling the Functional Network

The concept of data processing networks folding into or onto other data processing networks is not necessarily new, however the dual character of our networks as processing nets and nems is unique. It requires us to investigate topological methods for implementing networks inside of networks and develop models for application to the functional geometries that will create the components of the neural structure (processing and memory components) and nanomachinery (actuators).

A method of evaluating network geometries and function may be seen in a basic exchange of data through simulation of two bounded networks. However, for our purposes of considering the structure of the “multifunctional” networks on our ANM flapping wing we take a few liberties to consider node function and evaluate data simulation, symmetry and correlative structures. So for application to our ANM the nodes for both networks are depicted as two types (Figure 4-6), the smooth or standard node by a black dot and the complex or multifunctional node by a spiky dot depicting multiple operations. In reality such a multifunctional node would exist in multiple data and set dimensions. Rendering such a complex node requires multiple sets. Such an array is possible through advanced differential methods and to a limited basis through assignment of topological vector spaces associated with neural topologies. At this point we are limiting ourselves to the possibilities of how to structure the network, mirror data sets through simulation and create equivalent function through two dissimilar networks. It may be possible through simulation to create an asymmetry between the complex and smooth nodes and permit the smooth nodes to take on the identity of the complex nodes with multifunctional structures and multidimensional processing capability.

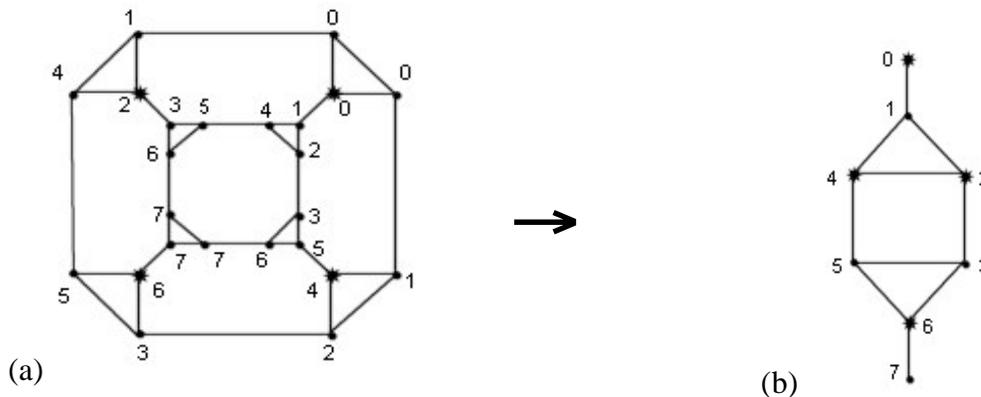


Figure 4-6 (a) Is a Cube-Connected Cycles (CCC) and (b) is a Shuffle-Exchange network the two may exchange data through algebraic mapping. The movement of data on each triangle in the CCC network is simulated by data movement on one of the two triangles in the shuffle exchange network. [Figures and networks after Rice (1994) p.215.]

The Cube-Connected Cycles (CCC) network though seemingly dissimilar from the Shuffle-Exchange network, geometric corollaries are created by the mapped triangles which directly

simulate the data from one to the other. This is an important distinction, that the networks themselves are simulated. Thus computationally embedding a simulated network on a physical or mechanical network is highly possible and may offer the solution to our embedded or multifunctional processing and nems structures.

Algebraic Mapping and Boundary Set as Discrete Analogs

In evaluating algebraic mapping and boundary set as discrete analogs we want to develop methods for describing formulations, according to Rice (1994), to satisfy two problems:

(1) simulating computation on one network by computation on another network

and

(2) network resident data driven computing.

Both problems are equivalent to our issues in developing algorithms for our programmable membrane. First the simulation of network computation on one network defining another is the best description for modeling the functional nature of our two- or three-dimensional structure of our ANM functional net. We are describing the ANM neural net as a functional net due to its nature as an active network driving complex computational functions. These complex computational functions can be further reduced as individual neural nets with computational nodes. Currently the individual or local neural nets are being conceptualized nominally as two-dimensional neural nets or a-neurons that “drive” or propagate specific signals through computational and routing nodes. The computational nodes may be other A-neurons or some type of solid-state device. The routing nodes may be traditional input-output devices or optical gate array capable of sorting far more data at several values per photon. In other words support multidimensional data transfer through a multidimensional network. This concept is based in advances in materials engineering allowing for the design and deposition of quantum dot superlattices replacing “traditional” electrooptical gradients applied to gate structures.

DEFINING A NEW CLASS OF FUNCTIONAL MATERIALS

Just as we begin to comprehend the potential of such a synergy of material structure and intelligent system we must also acknowledge the mathematical implications in defining the complexity of an “intelligent” structure computationally.

Initially we established several realistic models that mathematically would support networks in networks. The dimensional nature of such a complex networking system requires adaptive control of distributed data. The four dimensions attributed to our intelligent structure include standard metrics of three dimensions and a fourth temporal dimension. It is important to emphasize that the nature of even artificial neurons that are not only three dimensional, but

due to the function of neurons as rate based operators their temporal component is integral to their function.

In adjusting to a new exercise in mathematical modeling I referred to the fundamental criteria described by Kurt Gödel in his work to create a new “Calculus.” Even though ultimately he believed that any metamathematical exercise was limited only by our own limitations in logical process the reality is any processes of logic must have physical corollary in order to be applied to the control and analysis of a physical system. At one level it is comforting to see limitations in one’s own cognitive structures; however it is baffling how seemingly reasonable theorists can be averse to acknowledging the requirements of physical reality. To visualize the wing shape as a mathematical and topological structure allows us the opportunity to geometrically model not just the structure but the function of the intelligent structures.

The Wing and the Manifold

The intelligent structure in order to function as an integral machine requires mapping and mapping is a function of projective geometry. Our preliminary definition of network data exchange through simulation is supported by projective geometry. By reducing the requirements of distributed embedded networks to geometry we can visualize the concepts in order to extrapolate the functions to higher topological dimensions. Ultimately the ANM structure will be a highly structured, multiple surfaced manifold. The preliminary model will focus on analytical and differential manifolds and not include any duality. However, due to the finite structure of our manifold we must also look at our ANM as a minimal surface or set of minimal surfaces with boundaries that are not necessarily equivalent (Morgan, 1988 p. 69).

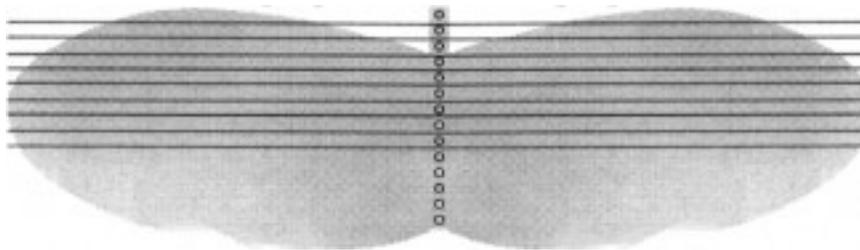


Figure 4-7: If we refer to our idealized quasi steady state model of a flapping wing we can see the surface of the wing as a manifold and as a minimal surface (Sane & Dickenson, 2002).

Mathematically a manifold is a complex topological structure that is defined by its surface(s). Simply and as a preliminary, an analytic manifold can be defined by the following:

M is a topological space and a *local chart* on M is the pair (U, ϕ) where U is an open set in M and ϕ is a homeomorphism of U onto an open set $\phi(U)$ in R^n for some n . (U, ϕ) is a *local chart* that may be defined as a system of local coordinates on M defined by the open set U . If we assume M is a smooth manifold, then M is a topological space together with a collection of local charts called an *atlas*. And simply an analytic manifold is a smooth manifold for which the mapping is analytic (Sattinger and Weaver, 1999). This definition creates physical boundary criteria as well as dimensional boundary criteria for our manifold system.

Visualization of Nanoelectromechanical Systems and Structures

Several mechanisms, materials and fabrication processes were considered for integration in the ANM flapping wing. One appears to be very promising and offers improvements in manufacture. If we visualize the membrane as a layered or laminated structure then we can create layers of nanoelectromechanical (nems) machines. One such nems unit is based on a molecular rotor where metal doped polymer molecules create a rotary motion through an electromechanical polarization mechanism. Such a rotor would provide power to propulsion systems as well as move fluids through nanotubules to create pressure gradients, exchange heat and pump gases into sensors for analysis.

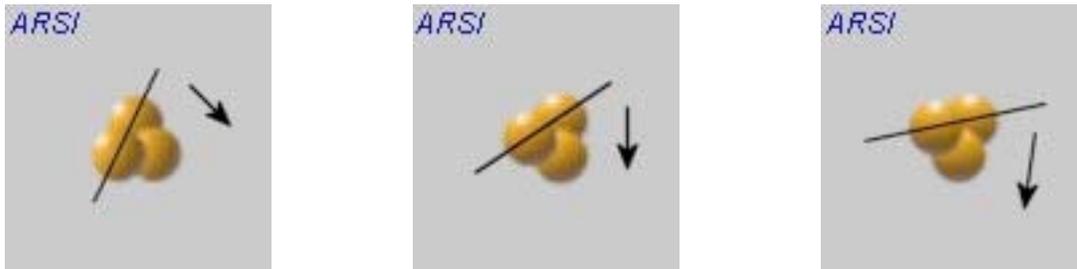


Figure 4-8 The NEMS Molecular Rotor visualized with line depicting rotation around a center axis. (ARSI/Aerospace Research Systems, Inc.)

Several actuator models were developed using current thin film deposition technology and ARSI's experience with adaptive apertures. The geometries investigated provide a positive response across a thin film polyamide structure with a known force. Response time is always primarily a function of the material or materials used and the mass of the membrane. Three actuator geometries modeled for appropriateness of motion, force distribution, and dynamic stability. The actuators named according to their relative motion include scissors, herring bone, and aperture concepts. These actuators were conceptualized as being a Nitinol (NiTi) shaped memory alloy (SMA) offering high cycle rates. The energy densities of NiTi are greater than chiral nanotubules so the NiTi actuators offer a higher efficiency in producing controlled displacements. However, the recovery time is greater for the NiTi actuators.



Figures 4-9: NiTi SMA conceptualized actuator configurations for thin film membrane structures; a-scissor induces expansion-contraction in a scissor motion force vectors resolving in two directions, b-herring bone providing expansion-contraction in a specific linear motion, c-aperture can be programmed to differentiate between two signals inducing either a rotational force or a liner-reducing motion where the actuator arms shorten. (ARSI Systems Engineering Laboratory, Cincinnati, Ohio)

In considering the possible geometries to implement actuator function, integrating carbon nanotubes we considered scissor and herringbone geometries as well as the chiral or helical structure unique to the single walled carbon nanotube (SWNT). This structure is discussed more in the FUNCTIONAL MATERIALS section. The multiwalled carbon nanotube coaxial nems was also conceptualized and investigated by Dr. Menges at the ARSI Systems Engineering Laboratory. The coaxial or bellows configuration may be possible with new CNT fabrication methods.



The unique properties of SWNT allows for bundling of chiral nanotubes. The chiral angle is determined by the vector of helicity. SWNT having the vector of helicity perpendicular to any carbon-carbon bond direction creates zig-zag type geometries offering excellent force induction for size and mass. SWNT actuators can theoretically be any length. (After Bhushan, Handbook of Nanotechnology2004).

Figure 4-10 Proposed dual chiral nanotube bundle actuator. Aerospace Research Systems, Inc.

Animation and Simulation Programming Visualizing NEMS

Animation of the actuator mechanisms and model of a bimodal wing were developed investigating possible actuator configurations, force and motion control. The exercise to model a bimodal wing with fidelity allowed the engineers to interface with the multimedia designers effectively and discuss how the quasi steady state flapping wing model chosen for the study may flap with the embedded data and actuator networks. The multifunctional networks are potentially the greatest challenge. Initially the neural processing capabilities combined with standard processing circuitry and nanotubes or molecular switches or rotors will create a functional neural structure, meaning a multifunctional smart structure capable of active motion control. The next step is to integrate a valid flight model with a limited ability to adapt to varying aerodynamic profiles structurally. The final goal is to integrate advanced sensory functions for flight control, remote sensing or communications applications. The potential for shared telemetry and delay tolerant network operations creates a new paradigm for distributed intelligent networks. The ability to fly the components of the network over significant distances and provide remotely sensed data, communications and a highly fault tolerant, reliable planetary exploration and surveillance tool.

COMMENTS ON AERODYNAMICS

In this section we will consider the basic structures of conventional aircraft and correlate their function to our ANM wing system. As discussed in the “Introduction” this Phase I study was primarily concerned with the feasibility of developing materials and structural systems that integrate evolutionary neural systems with advanced microscale and nanoscale components. Aerodynamic considerations were primarily concerned with the ability of the ARSI Neurogenesis Algorithms ability to recognize flapping wing flight.

The Neurogenesis software has been used to monitor and control multifunctional vehicle structures and combustors. However, governing equations beyond that of mechanical responses and sensory input detecting fatigue, cracking, ballistic or some other structural failure had not been integrated. In particular aeroelastic responses that will be observed with a membrane vehicle include a significant level of elastic response at a wider range of frequencies than in conventional wings. A single experiment using a well established algorithm for quasi-steady state flapping flight was used to provide a complex biomimetic model to evaluate the ability of the neural controller to recognize flapping flight. There was recognition of two degrees of motion and a brief description is included in this section.

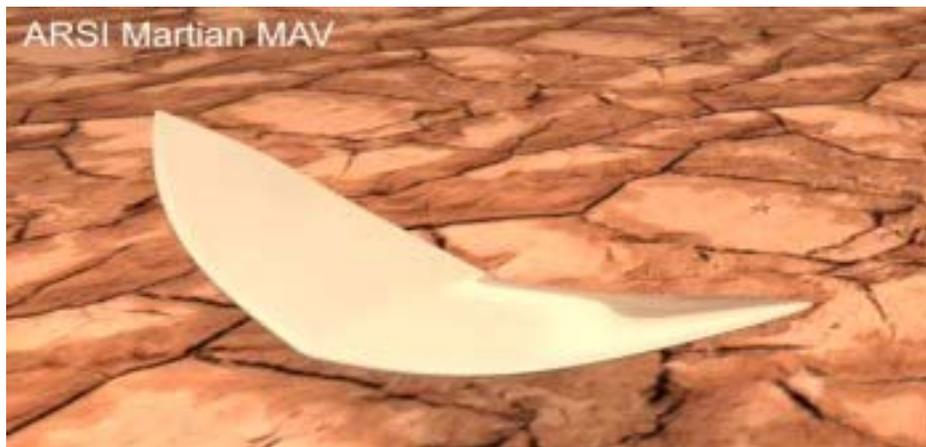


Figure 5-1

Aerodynamics: the basics

In equilibrium flight the fundamental equation of airfoils defines aerodynamic lift being generated by the wing as a force perpendicular to the wing and equal to the weight per unit of the vehicle which is expressed as:

$$mg = l = \frac{1}{2}\rho U_{\infty}^2 cC_l$$

where

m = mass per unit span, kg/m^{-1}

g = acceleration due to gravity, $g = 9.81 \text{ m/s}^{-2}$

l = lift generated by the wing, Nm^{-1}

ρ = fluid density or kg m^{-3}

$U_\infty =$ flight velocity, m/s^{-1}

$c =$ chord length or the maximum distance between the leading edge and the trailing edge

The following provides definition for a non-flapping wing in order to develop a basic understanding of the forces acting on a wing in flight and provide the basic aerodynamic vocabulary.

Aerodynamic lift is perpendicular to the wing. The mean aerodynamic chord (MAC) is the line that passes through the center of the plan area and is the primary axis for longitudinal stability of the wing. The MAC is not the average chord but is the length of the chord through the centroid area.

When the wing produces lift there is a pressure differential between the upper and lower surfaces and the static pressure on the upper surface is less than the lower. The spanwise flow components create vortices at the wing tips. “The rotational pressure flow combines with the local airstream flow to produce a resultant flow of the trailing vortex (Hurt, 1965).” This generates a downwash which we will see in our flapping wing model. However in flapping wing formation flight the downwash becomes negligible.

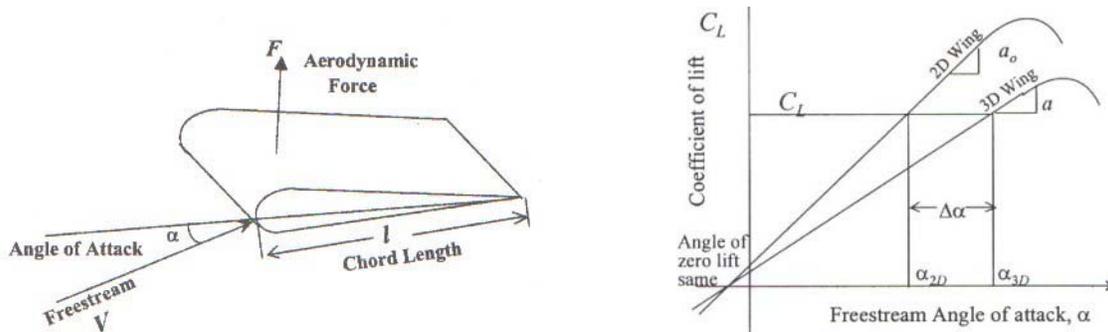


Figure 5-2: Diagram from the Society of Flight Test Engineers Reference Handbook 2001. Figure 5-3: Defined as $\alpha = dC_L/d\alpha = a_0/[1 + (57.3 a_0/\pi ARe)]$ where a is the lift curve slope and a_0 is the lift curve slope for an infinite wing. Diagram after Society of Flight Test Engineers Reference Handbook, 2001.

Drag is a significant factor in the performance of any aerodynamic surface. Drag is also a complex phenomenon where total drag is composed of induced drag, parasitic drag, wave drag, interference drag, profile drag, skin friction drag, and pressure drag. The drag effects of wing taper is directly associated with the distribution of lift along the span of the wing. Induced drag is caused when the lift is distributed in an elliptical fashion. However, the ANM flapping wing geometry simplifies the types of drag that are of consequence when not flapping.

Variations and Degrees of Flapping Flight

As was observed in the Phase I proposal and early in the actual study robust, controlled flapping wing flight as its traditional concept would not be easily accomplished. Other

options for a combination of a modified flapping wing model and adaptive surface structures could increase the stability of the ANM flapping wing and create a uniquely propelled vehicle using ionized surfaces to increase and control airflow over the wing.

Wing Theory and Function

It is necessary to define possible wing configurations through actuator control and how they correspond to traditional aircraft structures and control surfaces. This is also necessary to evaluate the actuator integration and operation. In keeping with the concept of a biomimetic vehicle visualizing the actuators as artificial muscle and the membrane wing as the fundamental structure of the aircraft we can begin to recognize the aerodynamic and mechanical components. This in turn provides the model for the neural systems function and integration.

In this section we will consider the basic structures of conventional aircraft and correlate their function to our ANM wing system. Starting at the basics maneuverability is the ability of an aircraft in flight to change direction. Aircraft utilize control surfaces to alter direction. These alterations affect climb, bank, descent, drag, and lift. In our ANM flapping wing MAV the control surfaces will be induced by wing twisting and warping.

If we consider the ANM wing or membrane surface as an ideally adaptable structure allowing twisting of the leading edge and trailing edge as well as camber changes we can visualize a structure capable of dynamic properties allowing flapping flight. Even though our ANM flapping wing does not have an aileron, it in fact can induce aileron motion through twisting of the trailing edges of the wing seen in Figure 5-4.



Figure 5-4

The ANM wing design is also complicated by the fact it is a tailless vehicle. This increases issues of stability particularly at speeds approaching stalls. Stabilization of a tailless aircraft can be improved by maintaining a downward apex or inducing an artificial dihedral when not in flapping mode. Since we have a geometrically adaptive wing structure it is possible to

induce surface twisting of the membrane structure to simulate a downward apex or an artificial dihedral.



Figure 5-5:

Dihedral induced through same mechanism as flapping motion except can vary to increase stability during steady state flapping flight and gliding flight.

During the course of the study a quasi steady state flapping wing model was used as part of an evaluation of a number of issues seen as being significant to the development and integration of flapping wing membrane vehicle. The model developed by Sane and Dickinson (2001) provided one of the few experimentally validated quasi steady state insect models. The insect model was chosen due to the preferred elliptical geometry of the ANM flapping wing. The quasi steady state model *is ultimately not* appropriate for an operational flapping wing vehicle but allowed a simple method of providing a computational model to compare to physical data generated by a NiTi actuated Mylar flapping wing. The wing flapped with two degrees of freedom one active and one passive. The experiment was placed in the ARSI 36 cm wind tunnel to isolate the wing from aberrant air flow.

The Neural Controller Flapping Wing Experiment allowed the correlation of mechanical data and a computational model to be used as a benchmark for learning flapping wing flight. The simple experiment provided a mechanical system to be recognized by the neural controller. However, the same experiment could be repeated with a basic two dimensional model offering no aerodynamic function for comparison. In order for an appropriate aerodynamic model to be used as a control function for the neural controller, the pressure distribution, air flow, aeroelastic response and flapping force would need to be measured and modeled. The experiment did more to evaluate the resistance model for controlling a NiTi actuator than demonstrate an aerodynamic system and it accurately assessed the degree of complexity required to integrate a flapping wing MAV and then instrument the structure. The instrumentation of a flapping wing membrane vehicle requires a degree and scale of systems integration that has never been attempted. The force and structural dynamic models will be simplified by the use of material actuators with a known force distributions, however the “avionics” of the vehicle will be a development effort in itself.



Figure 5-6: The experiment did more to define the eventual challenge in instrumenting very small vehicles. Flapping in 2-degrees of freedom one active and one passive.

The use of a shape metal alloy (SMA) like NiTi as a flapping wing actuator is complicated by thermal heating by a number of sources including ambient conditions. Heating of an SMA directly affects its function. However, if the heating of the NiTi actuators can be controlled, the NiTi actuators may be driven as heat engines and not require electrical power.

The experiment also showed the ability of the neural controller to provide adaptive control of the flapping wing model even as the ambient temperature rose. This is crucial in future study of SMA actuators. The adaptive function of the neural controller coupled with the ARSI proprietary SMA may provide a more accessible alternative to the carbon nanotubule chiral based actuators. The primary difference in the neural network simulation on the nanotubules and the SMA structures will be the size and type of circuitry used. Information processing on the nanotubules is influenced by thermal dislocations to a greater degree than the SMA based circuits which could be deposited separately on top of or below the SMA actuators linking the actuators with the control nodes.

This also has led to the acknowledgement that where the insect wing geometry providing a near perfect elliptical wing for analysis may not be as useful as a flapping wing concept extrapolated from a bird. The bird wing offers larger scales more easily defined and measured. Modeling is less complex and feathers and other attributes can be removed without detriment.

In flapping wing flight the aerodynamic characteristics of each section of the wing is determined by local angles of attack as influenced by wake effects. This is not the case with a fixed wing where the wake would be a downwash effect. Wake effects are a type of turbulence caused by the wing displacing air as it flies through a previously undisturbed air mass. Since the flapping wing has an unsteady shed wake the sectional aerodynamic forces

can be reduced computationally to provide valid governing equations for lift and thrust. Such simplified models have been effectively applied to large ornithopters (Mueller, 2001).

The aerodynamic geometry chosen for the flapping wing concept will also be a product of the mechanical function and the properties of the materials. Membrane vehicles whether flapping wings, gliders or rotors offer unique challenges, their weight, size and level of functional integration all induce varied levels of performance and aerodynamic behavior. The requirement for particularly robust actuators is also a function of low mass and the need to induce wing loading as a control function.

Natural flapping flight of birds is largely inefficient with the best efficiency at about 25% (J.M.V. Rayner, 2001). Mechanical actuators will provide a greater efficiency, however the increase in efficiency over that of a bird in flapping wing flight will depend on the type of actuator. Efficiency for an ANM flapping wing vehicle may not be as significant if the actuators can maintain wingbeat kinematics and mechanical efficiency simultaneously (J.M.V. Rayner, 2001, Rediniotis & Lagoudas, 2001).

The continuation of this study will focus on advances made through the study of ornithopters. The highly synergistic relationship between the flapping wing model used and the data control structures of the algorithms requires advanced on-board data processing. This however may be facilitated by the thin film substrates being identified for use in the ANM wing. Such thin films are being used as ultra thin substrates for a new class of circuitry support both more conventional electronic systems and photonic processing. The ANM Flapping Wing Roadmap discusses the possibilities in greater depth.

ROADMAP FOR THE DEVELOPMENT OF AN ANM FLAPPING WING

The ROADMAP evaluates concepts developed from the study, explores the thrust areas, and discusses the issues relevant to the data obtained in the Phase I. The fundamental materials issues still exist only in a much more refined context. Directions for future research have been developed after careful consideration of all the available information and in the context of the larger scientific community. New materials and methods of integrating them as functional components are being developed continuously. However, certain issues stand out as being significant in the continuation of the development of an ANM flapping wing structure.

Specifically advancement in understanding the structure and application of NiTi alloys to thin film or membrane structures has been a real success in this study. The potential for developing a new class of programmable materials and structures is a definite possibility and work will continue in this area independently.

Also the substrate material chosen as a high density polymer or polyamide in conjunction with the ability to deposit a variety of metal and ceramic materials allows for applications in high-temperature, marine and space environments that normally would be inaccessible for highly sensitive sensors and controllers.

The opportunity to visualize the function of the wing and the actuator concepts has created new opportunities in development, simulation and visualization. In the near future we will begin experimenting with a neural driven simulator to evaluate the potential of a neural controller to learn effectively and at a valid level within a virtual system. This also leads potentially to creating a “mirrored” learning system for artificial neural networks.

Issues and Directions

In evaluating the possible materials and methods by which the ANM may be engineered several issues must be addressed. These are listed below and are to varying degrees important to our selection of materials and design of the ANM flapping wing sub-structures and systems. Issues 1, 3 and 4 are likely to be the most prevalent; however Issue 2 may provide the beginnings of a second Phase I.

- (1). Complications of defining components (and even subsystems) in a systems matrix where components (and subsystems) may be no more than an atom.
- (2). Quantum computational stability in nonlinear matrices using superlattice structures.
- (3). Defining topological vector spaces in 4-dimensions simultaneously. Maybe not active, but reactive. Neurons are reactive before becoming active.

(4). Permeability in engineered membranes with active matrices; threshold leakage, chiral switch isolation (insulation/shielding), mechanical matrix stability and force variable vectors (flapping wing flight dynamics)

(5) Signal permeability which may be a function of overall electronic transport and/or photonic activity and not just material characteristics.

Primary Components Phase I Outcomes: Building the Roadmap

- Preliminary flapping wing model → Topological manifold derived from wing geometry to allow for algebraic mapping of structures and networks → Aerodynamic structures and function of the ANM wing structured defined.
- Materials, technologies and nems concepts investigated and conceptualized for integration.
- Neural models defined by function → Embedded networks derived from neural structures
- Systems integration matrix evolves and fabrication processes investigated.
- Partners for support and integration of ANM flapping wing vehicle investigated and identified.
- Flapping Wing Virtual Library Completed offering visualization of ANM flapping wing flight, wing structures, networks, system and sub-system functions, and mission concepts.
- Broader discussion of application of “ANM technology” to other structures and systems (spacesuits, artificial organs, programmable pharmaceuticals, membrane technology satellites and microspacecraft, rotating machinery, and space and civil structures.)

One area discussed extensively in the COMPUTATIONAL STRUCTURES section may be the most fundamental area of development, aside from the materials aspects, this is the ability to model and define the energetics of the system or the ANM flapping wing as a complete structure as well as modeling the energetics of subsystems to determine levels of interaction and interference. Tunneling and leakage effecting the transport functions of the network control and actuator components may reduce there performance or interfere completely with their operation. Since we are considering molecular scale systems it is a known that simple molecules like chiral switches can leak signals. It is likely that the transport models currently available are inadequate for understanding any cabernet phenomenon.

Any future research must include a significant portion of the effort directed at understanding the energetics of the system. It is suggested as part of a potential Phase II study that ARSI partner with one of the leading centers on nanoscale energetics. Three separate organizations have been approached and discussions concerning capabilities and interests were undertaken. These include California Institute of Technology, Los Alamos National Laboratory Center for Integrated Technologies and University of Notre Dame. All offer unique capabilities and

opportunities for advanced experimentation and fabrication. Los Alamos is particularly supportive of developing new investigative processes and instrumentation.

Another research center, the University of Toronto has exceptional experience with functional nanofilms and is the residence of James DeLaurier, the ornithopter designer of note. He could not only provide aerodynamic but dynamic stability support to the development of an ANM flapping wing and has voiced interest in doing so.

Since the components and technologies supporting the development of the ANM flapping wing are so diverse several limited thrust area components are of singular importance. They include force measurement and actuator-membrane coupling and thermal management of SMA actuators like NiTi and the definition of nanoscale energetics already discussed. Potentially the thermal management issue may render an NiTi actuators inoperable for a period of time as ambient temperature rise around it. The integration of a “Peltier device” like film may be a possibility. Such a film would also aid in the dispersion of dust from the membrane or wing surfaces.

The nanomechanics of the system must also be modeled with some degree of precision in order to produce a stable flight system. This also directly effects the issue of force distribution and coupling and may be considered as a single thrust area component.

Advanced Space Exploration Missions and Missions to the Home Planet

ANM MAVs operating in the earth’s atmosphere could be integrated with precision Global Positioning Satellite (GPS) systems. Martian ANM MAVs would require a much different set of references for navigation. Altitude, airspeed, attitude, and direction would be a function of vehicle resident systems.

It is possible as part of the Martian ANM MAVs system to develop a terrestrial and atmospheric navigation, communications and remote sensing system, as a spin off concept of this Phase I ARSI is developing the Sophia ANM Global Environmental Monitoring System or GEMS. The Sophia ANM GEMS vehicles are technologically within much nearer reach than the ANM vehicles. Utilizing atmospheric systems the Sophia GEMS vehicles can provide a variety of terrestrial, oceanic and atmospheric information to a variety of end users in government and industry. The Sophia Cloud Rider concept uses the physics of clouds and cloud systems to transport themselves over large distances in between independent flight stages or legs.

A similar concept not as reliant on cloud systems could function on Mars as a global positioning system or Martian Positioning Systems (MPS) to support a diverse and expanded set of exploration missions. Such global or planetary monitoring systems could provide direct data to a SpaceWEB or solar system internet that could support cost reduction and improved data for the study of planetary atmospheres. Such data could be used to correlate information on solar activity and the different interaction that occur in microscale and nanoscale systems.



Figure 6-1

The ARSI ANM Flapping Wing MAV could provide a robust multifunctional platform covering hundreds or thousand of square kilometers providing a Delay Tolerant Network (DTN) for planetary studies and support for communications, weather, radiation monitoring for future astronauts. The ANM flapping wing Adaptive A-Neural Systems provide advanced learning platforms for investigating unknown environments. Other mission objectives could include Martian weather and atmospheric sampling, radiation survey, near IR spectrometry essential to analyzing mineral deposits, and high spatial resolution magnetometry that may provides answers to the origin of high crustal magnetism seen from orbit.

Advantages of airborne platforms for planetary studies:

- Airborne platforms can achieve science objectives difficult to complete from orbit or from surface rovers.
- They can cover much larger distances in a single mission and are not limited by the terrain.
- Improved imaging and sensor missions using airborne platforms.
- Produce images of more than a magnitude higher resolution than state-of-the-art orbiting spacecraft.

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