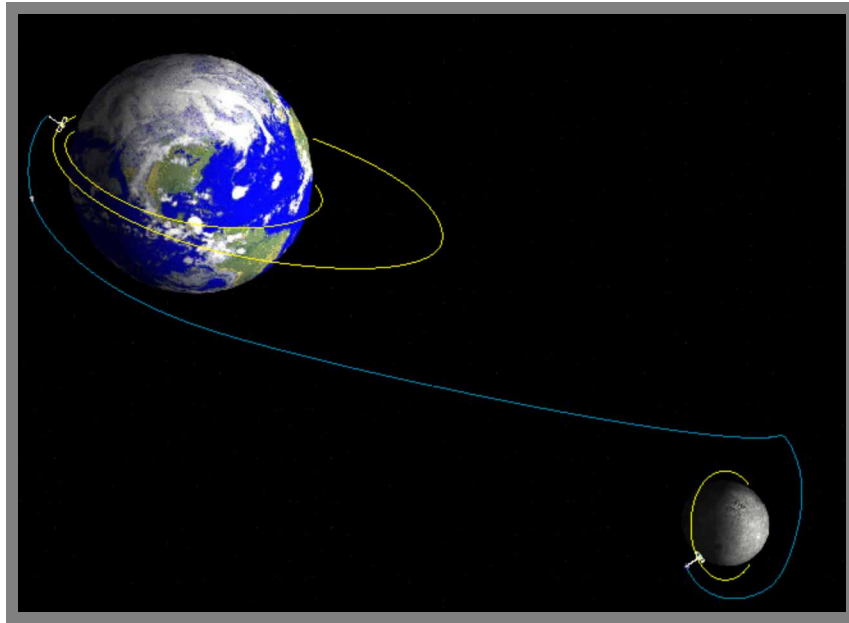


FINAL REPORT
on
NIAC Phase I Contract 07600-011
with
NASA Institute for Advanced Concepts, Universities Space Research Association

CISLUNAR TETHER TRANSPORT SYSTEM



Report submitted by:

TETHERS UNLIMITED, INC.
8114 Pebble Ct., Clinton WA 98236-9240
Phone: (206) 306-0400 Fax: -0537 email: TU@tethers.com
www.tethers.com

Report dated: May 30, 1999

Period of Performance: November 1, 1998 to April 30, 1999

PROJECT SUMMARY

PHASE I CONTRACT NUMBER **NIAC-07600-011**

TITLE OF PROJECT

CISLUNAR TETHER TRANSPORT SYSTEM

NAME AND ADDRESS OF PERFORMING ORGANIZATION (Firm Name, Mail Address, City/State/Zip)

Tethers Unlimited, Inc. 8114 Pebble Ct., Clinton WA 98236-9240 TU@tethers.com

PRINCIPAL INVESTIGATOR

Robert P. Hoyt, Ph.D.

ABSTRACT

The Phase I effort developed a design for a space systems architecture for repeatedly transporting payloads between low Earth orbit and the surface of the moon without significant use of propellant. This architecture consists of one rotating tether in elliptical, equatorial Earth orbit and a second rotating tether in a circular low lunar orbit. The Earth-orbit tether picks up a payload from a circular low Earth orbit and tosses it into a minimal-energy lunar transfer orbit. When the payload arrives at the Moon, the lunar tether catches it and deposits it on the surface of the Moon. Simultaneously, the lunar tether picks up a lunar payload to be sent down to the Earth orbit tether. By transporting equal masses to and from the Moon, the orbital energy and momentum of the system can be conserved, eliminating the need for transfer propellant. Using currently available high-strength tether materials, this system could be built with a total mass of less than 28 times the mass of the payloads it can transport. Using numerical simulations that incorporate the full three-dimensional orbital mechanics and tether dynamics, we have verified the feasibility of this system architecture and developed scenarios for transferring a payload from a low Earth orbit to the surface of the Moon that require less than 25 m/s of thrust for trajectory targeting corrections. In addition, the Phase I effort investigated the feasibility of using a similar tether system to provide rapid round-trip travel between low Earth orbit and low Mars orbit. A key technology required for both tether systems is hardware and techniques for rendezvous between the payloads and the rotating tethers. Automated rendezvous and capture systems currently under testing by NASA should, with further development, be capable of facilitating the tether-payload dockings. By providing a fully reusable infrastructure and by minimizing the need for propellant expenditure, tether transport systems can significantly reduce the cost of frequent travel to and from the Moon and Mars.

TABLE OF CONTENTS

I.	INTRODUCTION.....	1
II.	RESEARCH OBJECTIVES.....	2
III.	PHASE I RESULTS.....	2
III.A.	DESIGN OF THE CISLUNAR TETHER TRANSPORT SYSTEM	3
III.A.1.	Cislunar System Architecture	3
III.A.2.	LEO-to-LTO Tether Boost Facility Design	5
III.A.3.	Lunavator™ Design.....	6
III.A.4.	Cislunar System Dynamics Verification Through Simulation	7
III.A.5.	Analyses of Lunar Transfer Targeting.....	7
III.A.6.	Stability Analyses of Lunavator™ Orbits.....	8
III.A.7.	Maintenance of Rotating Tether Orbits by Tether Reeling	8
III.B.	LEO HEFT FACILITY ANALYSIS AND DESIGN	9
III.C.	TETHER SYSTEMS FOR EARTH ↔ MARS TRANSPORT.....	11
III.C.1.	Mars-Earth Rapid Interplanetary Tether Transport System	11
III.C.2.	Tether Boost Facility Design for the Human Mars Mission.....	12
III.D.	COMPARISON TO COMPETING TECHNOLOGIES.....	13
III.E.	HIGH-STRENGTH TETHER MATERIALS	14
III.F.	HIGH-SURVIVABILITY TETHER STRUCTURES	16
III.G.	KEY FEASIBILITY ISSUE: PAYLOAD RENDEZVOUS & CAPTURE WITH A ROTATING TETHER	16
III.H.	INCREMENTAL SYSTEM DESIGN	18
III.H.1.	Tether Transport System Technology Design Effort.....	19
III.H.2.	STOTS: <u>S</u> pinning <u>T</u> ether <u>O</u> rbital <u>T</u> ransfer <u>S</u> ystem Experiment.....	19
III.H.3.	TORQUE: <u>T</u> ether <u>O</u> rbital- <u>R</u> aising <u>Q</u> ualification <u>E</u> xperiment.....	19
III.H.4.	Earth-orbit Tether Boost Facility.....	20
III.H.5.	Lunavator™ Facility	20
III.H.6.	MarsWhip Tether Facility	20
IV.	SUMMARY.....	20

APPENDICES

- A. Design of Earth-Orbit Tether Facilities for Lunar Transfer Orbit Injection
- B. Lunavator™ Tether and Orbital Design for the Cislunar Transport System
- C. Cislunar System Dynamics Verification Through Simulation
- D. Analyses of Lunar Transfer Options
- E. Stability of Lunavator™ Orbits
- F. Maintenance of Rotating Tether Orbits by Tether Reeling
- G. LEO HEFT Facility Design
- H. Mars-Earth Rapid Interplanetary Tether Transport (MERITT) System, AIAA Paper 99-2151
- I. MarsHEFT
- J. The Hoytether™
- K. Momentum-Exchange Tether White Paper
- L. Cislunar Tether Transport System: AIAA Paper 99-2690

I. INTRODUCTION

Motivation

If mankind is to move beyond its current tenuous foothold in low Earth orbit and develop a sustained and prosperous presence on the Moon, Mars, and elsewhere in the solar system, the cost of transporting supplies, equipment, and personnel to these locations must be reduced by several orders of magnitude. The US space program is currently seeking to achieve such cost reductions for Earth-to-orbit transport by developing reusable launch vehicles. To achieve these cost reductions for the in-space propulsive needs of an interplanetary civilization, it will be necessary to develop a highly reusable transportation architecture that minimize the amount of mass that must be launched into orbit to provide in-space propulsion.

Background: Momentum-Exchange Tethers

Momentum-exchange tethers can provide a means for transporting many payloads without utilizing propellant, and thus can provide the infrastructure of a low-cost in-space transportation system. A momentum-exchange tether is essentially a long, high-strength cable rotating in orbit. This cable can provide a mechanical connection between two objects in orbit, enabling one object to transfer momentum and energy to the other object, much like a hunter can cast a stone with a sling.

A momentum-exchange tether facility will consist of a central station, a long, tapered, high-strength cable, and a grapple vehicle at the tether tip. The tether will be deployed from the station, and the system will be induced to spin using tether reeling maneuvers or electrodynamic forces. The direction of tether spin is chosen so that the tether tip is moving behind the tether facility's center-of-mass on its downswing, and moving ahead of it on its upswing, as illustrated in Figure 1. With proper choice of tether orbit and rotation, the tether tip can then rendezvous with a payload when the tether is at the bottom of its swing and later release the payload at the top of its swing, tossing the payload into a higher orbit. The orbital energy and momentum given to the payload comes out of the energy and momentum of the tether facility. The tether's orbit can be restored by reboosting with propellantless electrodynamic tether propulsion or with high specific impulse electric propulsion; alternatively, the tether's orbit can also be restored by using it to de-boost return traffic payloads.

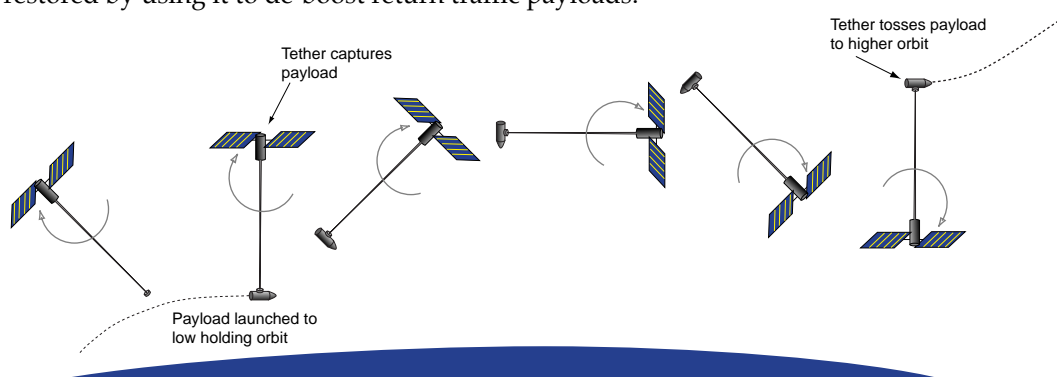


Figure 1. Schematic of a rotating momentum-exchange tether boosting a payload.

The Cislunar Tether Transport System

A system of several momentum-exchange tethers can provide a means for repeatedly exchanging payloads between low Earth orbit (LEO) and the surface of the Moon, *with little or no propellant expenditure required*. The basic concept of the “Cislunar Tether Transport System” is to use one rotating tether in Earth orbit to pick payloads up from LEO orbits and toss them to the Moon, where second rotating tether in lunar orbit, called a “Lunavator™”, would catch them and deliver them to the lunar surface. As the Lunavator™ delivers payloads to the Moon’s surface, it can also pick up return payloads, such as water or aluminum processed from lunar resources, and send them down to the Earth-orbit tether, which will deliver them LEO. If the flow of mass to and from the Moon is balanced, the orbital momentum and

energy of the system can be conserved, eliminating the need to expend large quantities of propellant to move the payloads back and forth. By providing a fully reusable transportation infrastructure and by greatly reducing the amount of mass that must be launched into orbit, the Cislunar Tether Transport System can reduce the costs of frequent travel to and from the Moon.

II. RESEARCH OBJECTIVES

The objective of the Phase I effort was to determine the technical and economic feasibility of constructing a system of several rotating tethers in Earth and Lunar orbit for the purpose of sustaining propellantless round-trip travel between Earth orbits and between LEO and the Lunar surface. As specified in the Phase I proposal, the Phase I effort addressed the following technical tasks:

II.A.1. Astrodynamic Design of the Cislunar Tether Transport System

This part of the Phase I project addressed the celestial mechanics issues in the design of a tether transport system for repeatedly transferring payloads from LEO orbits to the lunar surface.

II.A.2. Incremental System Design and Economic Analysis

The second task studied the possibility of constructing a Cislunar Tether Transport System incrementally, so that early stages can generate revenue to support the construction of later stages.

II.A.3. LEO HEFT™ Facility Analysis and Design

We also investigated the concept of using electrodynamic force tether propulsion in a rotating tether system to create a means of repeatedly boosting payloads from LEO to higher orbits without requiring propellant.

In addition to the tasks specified in the proposal, we also pursued the following task:

II.A.4. Mars-Earth Rapid Interplanetary Tether Transport (MERITT) System Feasibility Study

In this part of the Phase I project, we investigated the feasibility of using momentum-exchange tethers to create a system for exchanging payloads between Earth and Mars without requiring propellant expenditure.

III. PHASE I RESULTS

The Phase I effort successfully accomplished all three goals specified in the Phase I proposal, developing a baseline design for a tether transportation system capable of exchanging payloads between LEO and the surface of the moon, demonstrating the operation of this system in a numerical simulation, and developing techniques for using electrodynamic tether propulsion to reboost rotating tethers. In order to solve some of the challenges posed by the orbital mechanics in the Cislunar system, we have also developed methods for controlling the stability and orientation of rotating tether orbits using modest tether reeling maneuvers.

During this Phase I effort, we also invented a concept for using rotating tethers to provide frequent round-trip travel between Earth and Mars, and performed an initial system design and feasibility study of this concept.

We have sought to make this report readily accessible to the reader by presenting summaries of each of the tasks and their results on the following pages. The full details of the larger project tasks are then presented in separate papers given as Appendices A-K to this report. We have also presented a condensed form of the study results in a technical paper, given Appendix L to this report.

III.A. DESIGN OF THE CISLUNAR TETHER TRANSPORT SYSTEM

III.A.1. Cislunar System Architecture

The primary purpose of this Phase I study was to determine the feasibility of constructing a tether system capable of exchanging payloads between low-Earth-orbit and the surface of the moon. In 1991, Forward showed that such a system is theoretically possible from an energetics standpoint.¹ A later study by Hoyt and Forward developed a first-order design for such a system.² These previous studies, however, utilized a number of simplifying assumptions regarding orbital and tether mechanics in the Earth-Moon system, including assumptions of coplanar orbits, ideal gravitational potentials, and infinite facility ballast masses. In this Phase I effort, we have endeavored to remove these simplifying assumptions and develop a system architecture capable of accounting for the effects of the Earth's oblateness, the inclination of the Moon's orbit, and other complications.

The basic concept of a Cislunar Tether Transport System is to use one or more rotating tethers in Earth orbit to pick up payloads from LEO orbits and throw them to the Moon, where a rotating tether in lunar orbit, called a "Lunavator™", could catch them and deliver them to the lunar surface. As the Lunavator™ delivers payloads to the Moon's surface, it can also pick up payloads such as water or aluminum processed from lunar resources and send them down to LEO. By balancing the flow of mass to and from the Moon, the orbital momentum and energy of the system can be conserved, eliminating the need to expend large quantities of propellant to move the payloads back and forth. Such system is illustrated conceptually in Figure 2.

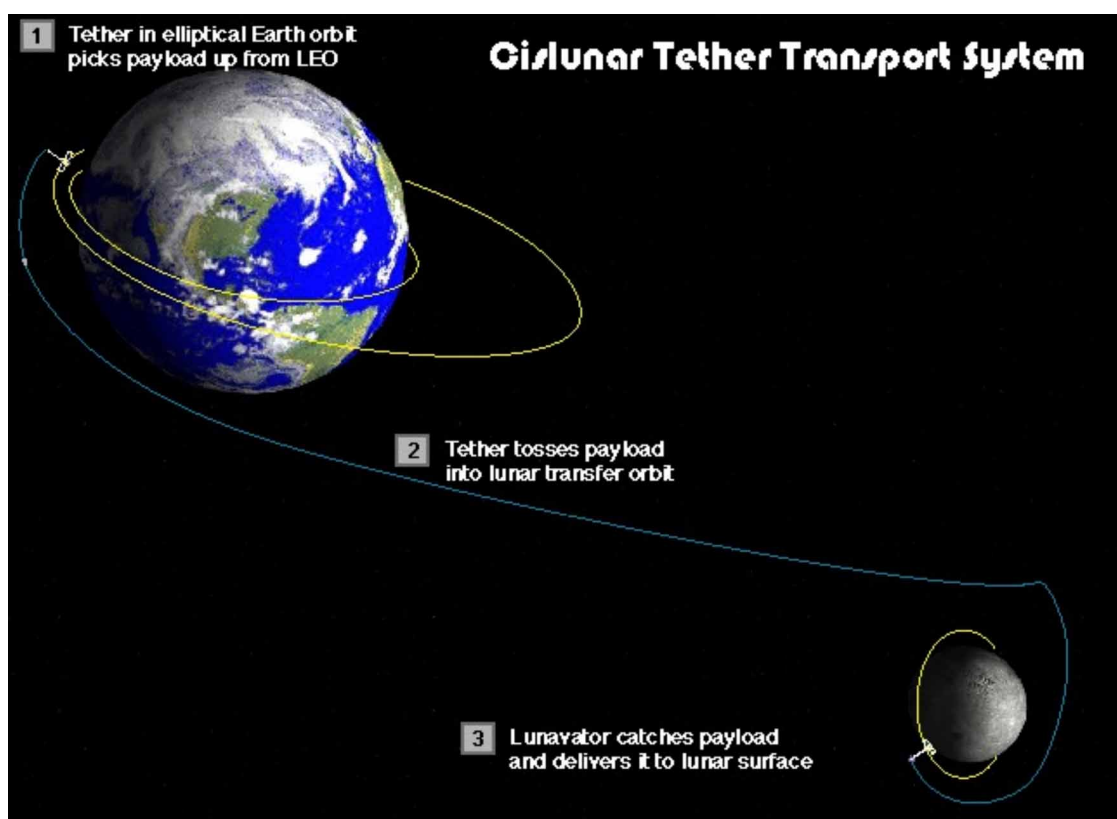


Figure 2. Conceptual illustration of the Cislunar Tether Transport System.

Orbital Mechanics of the Earth-Moon System

Orbital mechanics in cislunar space are made quite complex by the different and varying orientations of the ecliptic plane, the Earth's equatorial plane, the Moon's orbital plane, and the Moon's equatorial plane. Figure 3 illustrates these different planes. The inclination of the Earth's rotational axis to the pole of the Earth's orbit (the "obliquity of the ecliptic"), is approximately 23.45° , but varies due to tidal forces exerted by the sun and Moon, as well as other effects. It can be modeled over the short term as³

$$i_e = 23^\circ 27' 8'' - 0.4684''(Y - 1900), \quad (1)$$

where Y is the year. The inclination of the Moon's orbit relative to the ecliptic plane is constant, about $\lambda_m = 5^\circ 9'$. The angle i_m between the Moon's equatorial plane and a plane through the Moon's center that is parallel to the ecliptic plane is also constant, about $1^\circ 35'$.⁴ The line of nodes of the Moon's orbit regresses slowly, revolving once every 18.6 years. As a result, the inclination of the Moon's orbit relative to the Earth's equator varies between 18.3-28.6 degrees. The Moon's orbit also has a slight eccentricity, approximately $e_m = 0.0549$.

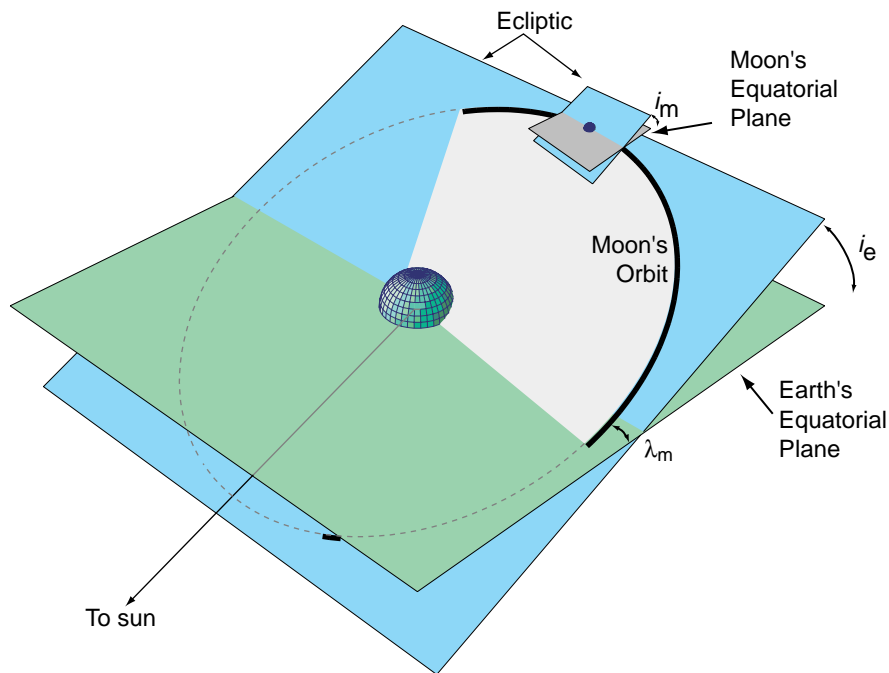


Figure 3. Schematic illustrating the geometry of the Earth-Moon system.

Tether Orbits

After considering many different options, we have determined that the optimum configuration for the Cislunar Tether system is to utilize one tether in elliptical, equatorial Earth orbit and one tether in a circular lunar orbit, as illustrated in Figure 2. This two-tether system will provide the lowest system mass, lowest system complexity, lowest ΔV requirements, and the most frequent transfer opportunities. The Earth-orbit tether will pick payloads up from equatorial low-LEO orbits and throw them towards one of the two points where the Moon crosses the Earth's equatorial plane. As the payload approaches the Moon, it will need to perform a small ΔV maneuver to set it up into the proper approach trajectory; the size of this maneuver will vary depending upon the angle between the Moon's orbit plane and the Earth's equatorial plane, but under most conditions it will only require about 25 m/s of ΔV .

The designs of these two tether facilities are summarized in the following two subsections, and more detailed descriptions are presented in Appendices A & B.

III.A.2. LEO-to-LTO Tether Boost Facility Design

(Appendix A)

In this task, we developed an architecture for a tether boost facility designed to exchange payloads between low-LEO orbits and Lunar Transfer Orbits (LTO). Our analyses of several different system architectures concluded that a tether system utilizing one tether facility in an elliptical orbit would provide the lowest system mass and complexity. The orbital design of this system is illustrated in Figure 4. Analysis of the system architecture indicates that a facility massing just 10.5 times the payload mass can inject payloads into minimum-energy lunar transfer trajectories. After boosting a payload, the facility can use propellantless electrodynamic tether propulsion near its perigee in LEO to rapidly reboost its orbit so that it can boost additional payloads. As a reference design, a tether facility massing 26 metric tons, with a power supply of 11 kW, can boost a 2.5 metric ton payload to the moon once every 95 days. We also found that apsidal precession of the tether's orbit can be handled either using tether reeling maneuvers or by selecting the tether's orbit so that the orbit's precession rate is resonant with the lunar orbital period. This tether boost facility will provide a means for repeatedly transferring payloads from LEO to the Moon without requiring propellant expenditure. If several of these facilities are deployed, the system could handle traffic to and from the moon as frequently as once every two weeks. This tether boost facility design is described in more detail in Appendix A.

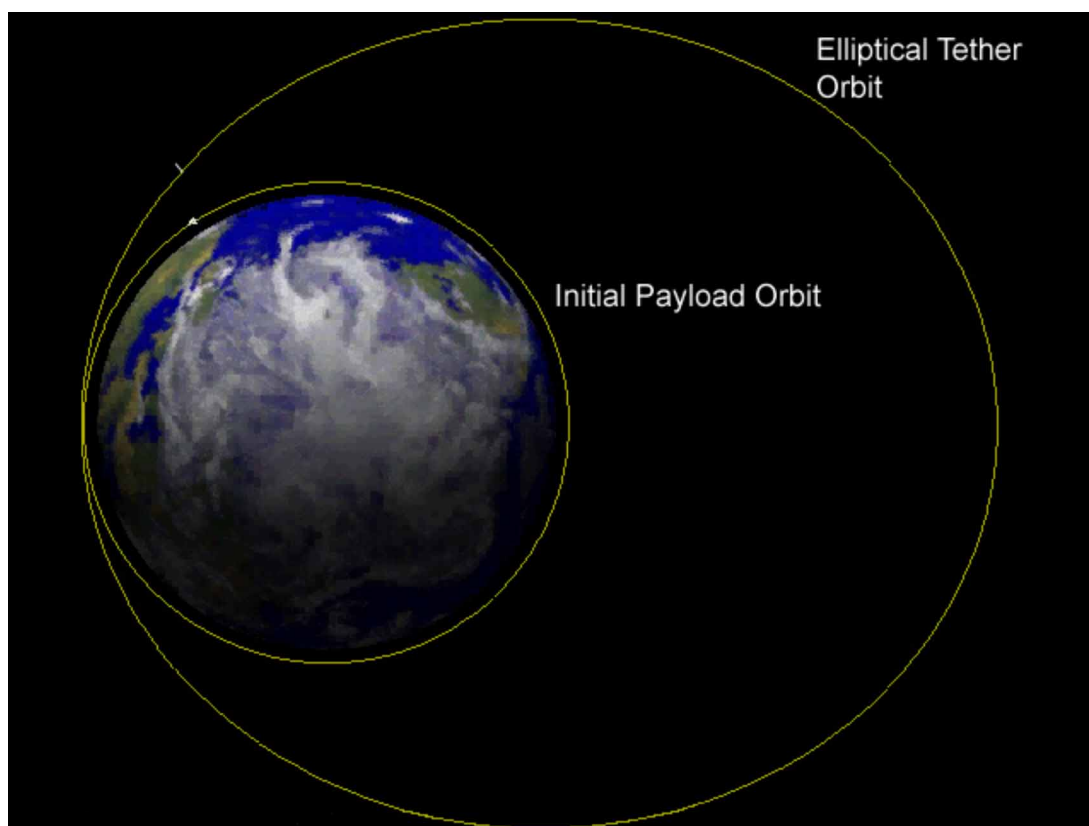


Figure 4. The initial payload orbit and the initial tether orbit for the LEO-to-LTO tether boost system, shown to scale.

III.A.3. Lunavator™ Design

(Appendix B)

In this task, we developed a design for a tether system capable of capturing payloads sent from the Earth to the Moon on minimal-energy trajectories and transferring them to the lunar surface. The challenge addressed was the need to enable a low-lunar-orbit tether facility that has an orbital velocity of 1.6 km/s to catch a payload from a hyperbolic lunar trajectory with a perigee velocity of 2.3 km/s (catch ΔV of ~ 0.7 km/s) and then deposit the payload on the moon with zero velocity relative to the surface (drop ΔV of 1.6 km/s). To enable this maneuver, we invented a tether system in which the tether ballast mass is divided between a counterbalance at one end of the tether and a central facility that can adjust its position along the tether. This reeling maneuver is illustrated in Figure 5, and Figure 6 shows the orbit of the Lunavator™ before and after capturing a payload sent from Earth. Using this method, we have designed a Lunavator system massing under 42 metric tons that can exchange 2.5 metric ton payloads between low-energy lunar transfer orbits and the lunar surface. This facility can be sent to the moon with a relatively low initial mass and build up its “ballast mass”, and thus its payload capacity, by picking up lunar materials. The Lunavator™ can be placed in either an equatorial or a polar lunar orbit. We have also developed a method of stabilizing perturbations of the Lunavator’s orbit using modest tether reeling operations. The Lunavator™ design is described in more detail in Appendix B.

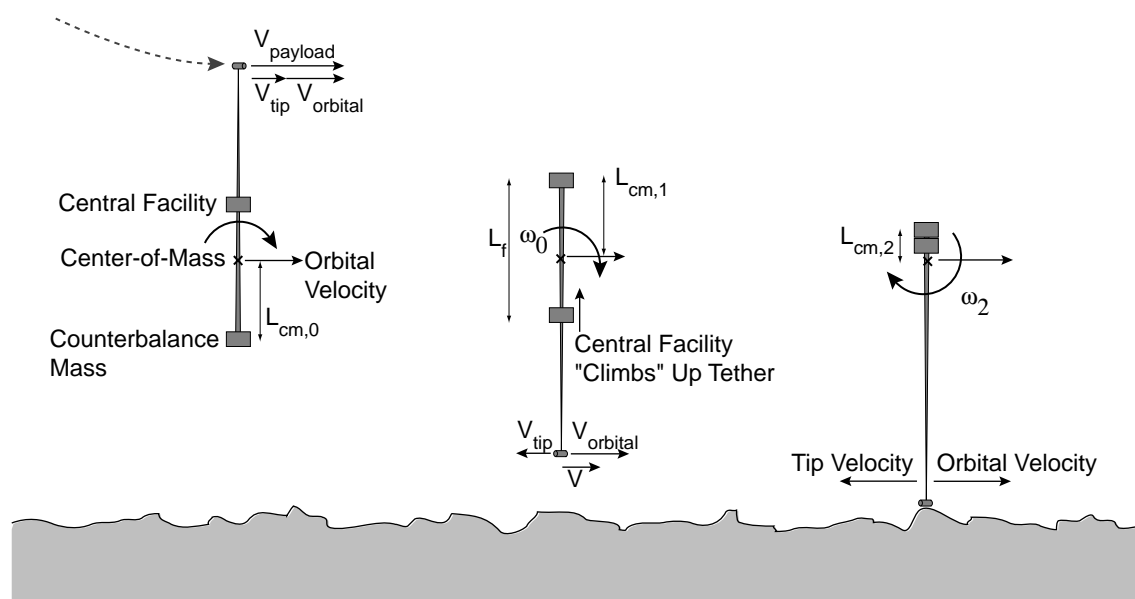


Figure 5. Method for a Lunavator™ to capture a payload from a minimal-energy LTO and deposit it on the lunar surface.



Figure 6. Lunavator™ orbit before and after catching a payload sent from Earth.

III.A.4. Cislunar System Dynamics Verification Through Simulation

(Appendix C)

In order to verify the design of the orbital dynamics of the Cislunar Tether Transport System, we have developed a numerical simulation called “TetherSim” that includes:

- The 3D orbital mechanics of the tethers and payloads in the Earth-Moon system, including the effects of Earth oblateness, using Runge-Kutta integration of Cowell’s method.
- Modeling of the dynamical behavior of the tethers, using a bead-and-spring model similar to that developed by Kim and Vadali.⁵
- Modeling of the electrodynamic interaction of the Earth-orbit tether with the ionosphere.

A screenshot of the TetherSim program is shown in Figure 7. Using this simulation tool, we have developed a scenario for transferring a payload from a circular low-LEO orbit to the surface of the Moon using the tether system designs outlined above. We have found that for an average transfer scenario, mid-course trajectory corrections of approximately 25 m/s are necessary to target the payload into the desired polar lunar trajectory to enable rendezvous with the Lunavator™. A simulation of a transfer from LEO to the surface of the moon can be viewed at www.tethers.com/Cislunar.mov.

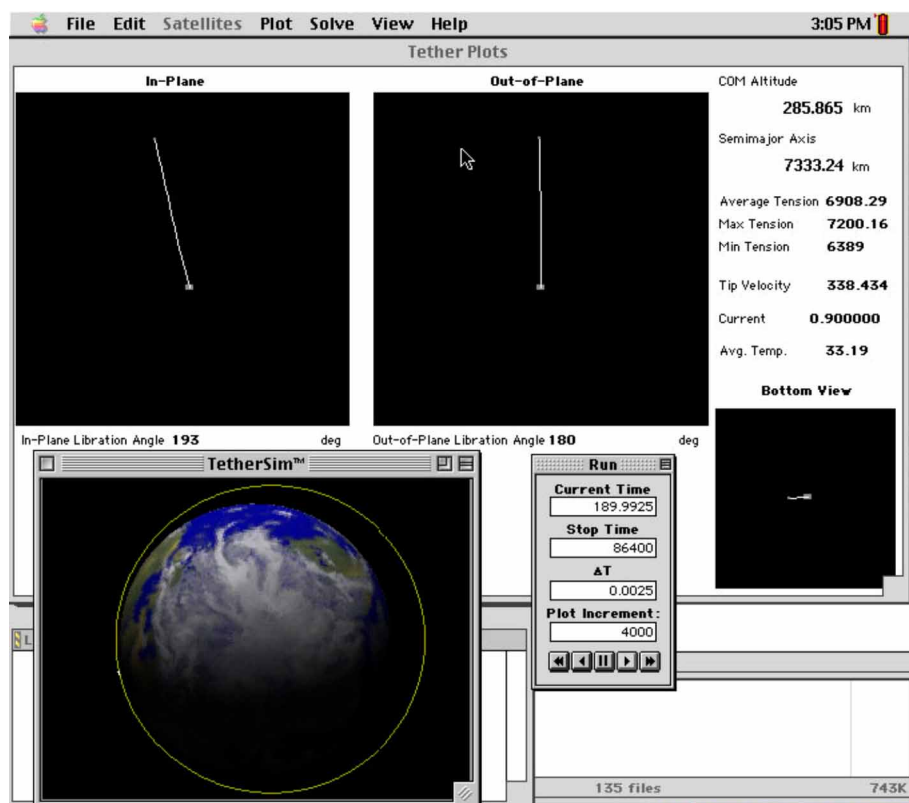


Figure 7. Screen shot of the TetherSim program simulating orbital reboosting of a 25 km HEFT Tether Facility.

III.A.5. Analyses of Lunar Transfer Targeting

(Appendix D)

In addition to the tether modeling conducted with TetherSim, we have also conducted a study of the Earth-Moon transfer to verify that the payload can be targeted to arrive at the Moon in the proper plane to rendezvous with the Lunavator™. This study was performed with the MAESTRO code,⁶ which includes the effects of lunisolar perturbations as well as the oblateness of the Earth. We studied transfers to both equatorial and polar lunar trajectories, and found that both options can be achieved with zero or minimal propellant expenditure.

Transfer to Equatorial Lunar Trajectories

Transfer of a payload from an equatorial Earth trajectory to an equatorial lunar trajectory can be achieved with zero propellant expenditure, but this requires use of a one-month “resonance hop” transfer in which the moon’s gravity is used to “slingshot” the payload into an Earth orbit that returns to the moon in the lunar equatorial plane. In further study, we found that it is possible to eliminate the one-month transfer time if we use a small ΔV maneuver to bend the payload’s trajectory into the lunar equatorial plane. Simple zero-point patched conic analysis predicts that this maneuver would require roughly 132 m/s of ΔV , but more detailed analysis with the MAESTRO code revealed that luni-solar perturbations can provide most of the bending needed, and the total ΔV required from the payload vehicle is only about 25 m/s.

Transfer to Polar Lunar Trajectories

We also studied transfer of payloads from the tether boost facility in equatorial Earth orbit to a polar lunar trajectory. We have found that by varying the energy of the translunar trajectory slightly and adjusting the argument of perigee, it is possible to target the payload to rendezvous with a polar orbit Lunavator™ with a wide range of ascending node positions of the Lunavator orbit. Our simulations indicate that the viable nodal positions ranges at least $\pm 10^\circ$ from the normal to the Earth-Moon line. This control will enable us to adjust the payload’s trajectory to account for slow variations in the Lunavator™’s orbit caused by the Moon’s non-ideal gravitational potential. Under some conditions, this transfer may be achieved with zero propellant expenditure; under average conditions, some propellant expenditure will be required, but the ΔV needed will again be on the order of only 25 m/s.

Thus our analyses of the lunar transfer scenario indicate that the celestial mechanics of the Earth-Moon system will permit a tether transport system to exchange payloads between LEO and the Moon with zero or minimal propellant expenditure. These analyses are described in more detail in Appendix D.

III.A.6. Stability Analyses of Lunavator™ Orbits

(Appendix E)

In order to provide the most consistent transfer scenarios, it is desirable to place the Lunavator™ into either a polar or equatorial lunar orbit. An equatorial lunar orbit has the advantage that it is relatively stable. An equatorial Lunavator™, however, would only be able to service traffic to equatorial lunar bases.

A polar orbit would be preferable for the Lunavator™ for several reasons. First, direct transfers to polar lunar trajectories are possible with little or no propellant expenditure required. Second, because a polar lunar orbit will remain oriented in the same direction while the moon rotates inside of it, a polar Lunavator™ could service traffic to any point on the surface of the moon, including the potentially ice-rich lunar poles. Polar lunar orbits, however, are unstable. The odd-harmonics of the Moon’s potential cause a circular, low polar orbit to become eccentric. Eventually, the eccentricity becomes large enough that the perilune is at or below the lunar surface. For the 178 km circular orbit, the rate of eccentricity growth is approximately 0.00088 per day.

III.A.7. Maintenance of Rotating Tether Orbits by Tether Reeling

(Appendix F)

In tether transportation systems such as the Cislunar Tether Transport System⁷ and the Mars-Earth Rapid Interplanetary Tether Transport (MERITT) System⁸, maintenance of the shape and orientation of the tether facility orbits will be critical to enabling frequent opportunities for these systems to exchange payloads between Earth, the Moon, and Mars. The orbits of tether facilities around the Earth, the Moon, and Mars will experience perturbations due to the oblateness of the planetary bodies, lunisolar or geosolar gravity fields, solar pressure, atmospheric drag, and other effects. Although high-specific impulse thruster propulsion might be considered for orbital maintenance of the tether facilities, thrusters require propellant expenditure. If tether systems are to achieve their full potential for reducing the cost of in-space transportation, they must be able to operate with a minimum of propellant expenditure. Propellantless electrodynamic tether propulsion may provide a very effective means of performing some

of the orbital maneuvers required for the low-Earth-orbit portions of the tether systems, but tether facilities around the Moon, Mars, and in high-Earth-orbit will not be able to avail themselves of electrodynamic tether propulsion due to the paucity of magnetic field and ambient plasma in those orbits.

Fortunately, tether reeling maneuvers can provide a means to modify or maintain the orbits of tether facilities without requiring propellant consumption. Previous work has studied tether reeling maneuvers in hanging tether systems, but did not study rotating tether systems in depth. In this subtask, we developed analytical methods for determining the effectiveness of tether reeling maneuvers in rotating tether systems. These analyses indicate that modest tether reeling maneuvers can provide an effective method of dissipating the eccentricity perturbations that would threaten the long-term orbital stability of a lunar tether, and for modifying the rate of apsidal precession of Earth-orbit tether facilities.

Stabilization of a Polar Lunar Orbit

One such application of tether reeling would be to stabilize the orbit of a polar-orbit Lunavator™ facility. As noted in Section III.A.6, polar lunar orbits are unstable. The odd elements of the tesseral harmonics of the lunar gravitational field cause the eccentricity of a polar lunar orbit to change. If no countermeasures are taken, an orbit that begins circular will eventually become eccentric enough that its perilune intersects the surface of the moon. Figure 8 illustrates a method for using tether reeling to counteract the growth of eccentricity of a polar lunar orbit. Essentially, by reeling the tether in when it is near apilune, where the lunar gravity is lower, and then reeling the tether out near perilune, where the gravity is higher, the tether can work against the non-linearity of the gravitational field to extract energy from the orbit, returning it to its circular shape.

Tether Reeling to Counteract Apsidal Precession of an Earth-Orbit Tether

In the Cislunar Tether Transport System, it is most advantageous to place the Earth-orbit tether facility in an equatorial, elliptical orbit. In order to permit payloads to be exchanged between the Earth and other planetary bodies, the tether system's orbit must be controlled so that the orbit's line of apsides points at or near one of the moon's nodes so that it can throw a payload to the moon when it crosses its node. If the Earth were perfectly spherical, this would not be an issue, because the orbit orientation would remain fixed. However, the Earth's oblateness causes the line of apsides of elliptical orbits to precess.

Again, tether reeling can provide a means of addressing this issue without requiring propellant expenditure. By reeling the tether in and out slightly once per orbit, it is possible to either counteract the apsidal precession to hold the line of apsides pointed at one lunar node, or to enhance it so that the apsides line up with one of the moon's nodes at the right time for a transfer to the moon.

A detailed analysis of tether reeling techniques for maintaining the orbits of rotating tethers is given in Appendix F.

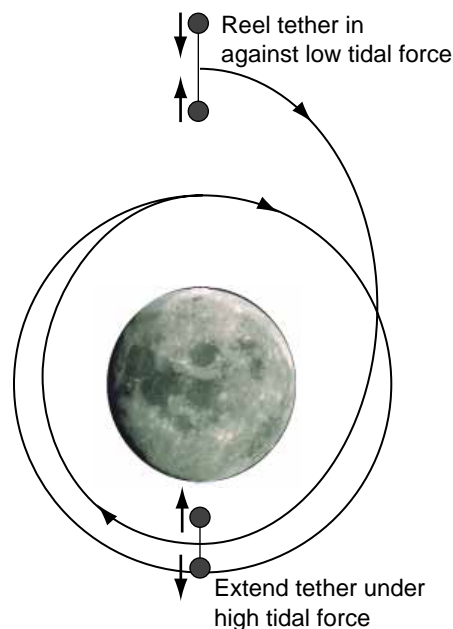


Figure 8. Schematic of tether reeling maneuver to reduce orbital eccentricity.

III.B. LEO HEFT FACILITY ANALYSIS AND DESIGN

(Appendix G)

When the Earth-orbit tether facility boosts a payload to the Moon, it does so by transferring some of its own orbital energy and momentum to the payload. Once a two-way tether system has been set up, the Earth-orbit tether facility can restore its orbital energy by catching and deboosting payloads sent back by the second tether. In the period before return traffic from the Moon has been established, however, the Earth-orbit facility will require some form of propulsion to reboost itself in order to prepare for its next payload boost operation.

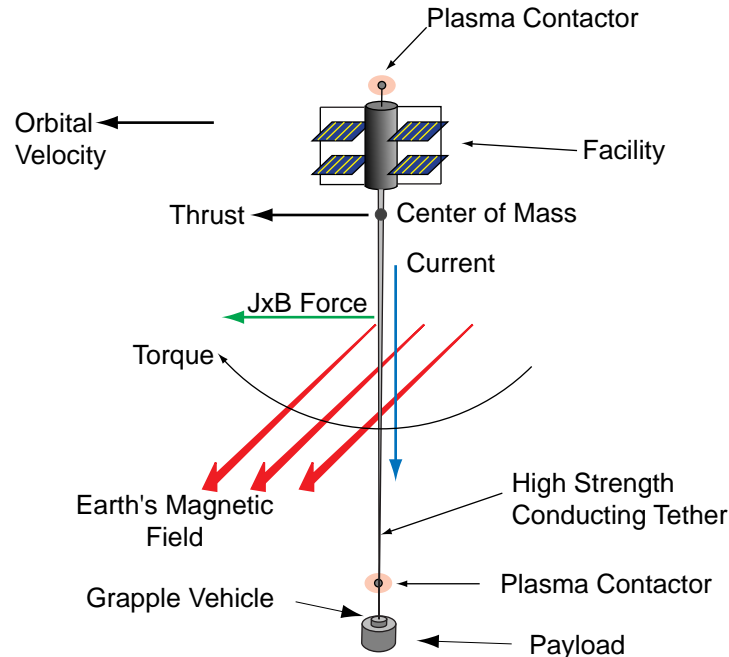


Figure 9. Schematic of a HEFT Facility.

Electrodynamic tether propulsion has the potential to provide propellantless propulsion in LEO.⁹ In this task, we investigated the possibility of combining electrodynamic tether propulsion with rotating tether techniques to provide a means of reboosting a tether facility without requiring propellant expenditure. This concept, called the “High-strength Electrodynamic Force Tether” (HEFT),¹⁰ is illustrated in Figure 9. The tether facility would include a power supply and a means of making electrical contact with the ionospheric plasma. The high-strength tether would be built with a conducting core, so that the power supply can be used to drive current along the length of the tether. This current will interact with the Earth’s magnetic field to generate electrodynamic forces on the tether. By properly varying the direction of the current, these forces can be used to either “spin-up” the tether or boost its orbit.

Using both analytical methods and numerical modeling with the TetherSim program, we have studied the HEFT concept applied to deployment of LEO and MEO constellation satellites and reboosting of the Earth-orbit tether facility in the Cislunar Tether Transport System. Figure 10 shows results of a simulation of reboost of the Cislunar Tether facility described in Appendix A. With a power supply of only 11 kW, the 26,250 kg tether facility can restore its orbit within 85 days after boosting a payload to the Moon; faster reboost rates could be possible with larger power supplies. These analyses of the HEFT concept are described in more detail in Appendix G.

By combining electrodynamic propulsion with momentum-exchange tether principles, the HEFT design will enable the first stage of the Cislunar and MERITT tether transport systems to send payloads to the Moon and Mars *before* tether facilities are deployed at those locations. Thus the Earth-orbit tether facility could be used to help send materials to the Moon and Mars to set up bases on those bodies. In addition, it will enable the Earth-orbit tether facility to perform other useful missions such as boosting communications satellites or solar power stations to geostationary transfer orbits. Thus the HEFT concept provides a means for building a Cislunar or Earth-Mars tether system in an incremental fashion, by enabling the first stage to perform useful tasks and earn revenue to help fund the design and deployment of tether facilities at the Moon and Mars.

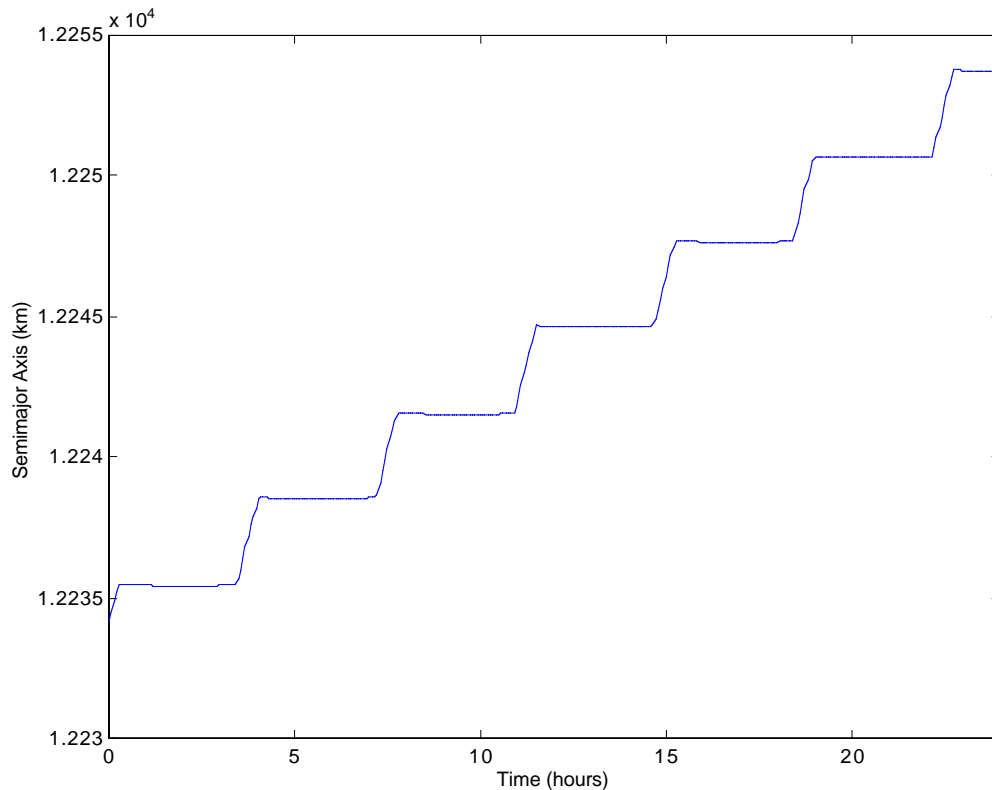


Figure 10. Electrodynamic boosting of the Earth-orbit tether in the Cislunar Tether Transport System.

III.C. TETHER SYSTEMS FOR EARTH \Leftrightarrow MARS TRANSPORT

In addition to our planned efforts to study the feasibility of using tethers to create a transport architecture for lunar traffic, we also conceived and studied concepts for using tethers to enable affordable transport to and from Mars. In the designs described below and in Appendices H and I, these concepts, the MERITT system and the MarsHEFT facility, are treated as separate systems from the Cislunar Tether Transport System. However, with further work it would be possible to combine these concepts into a single architecture able to handle transport between LEO, the Moon, Mars, and other planets.

III.C.1. Mars-Earth Rapid Interplanetary Tether Transport System

(Appendix H)

Routine travel to and from Mars will demand a means for providing efficient, rapid, and low cost round trip transport to the red planet. As a part of this Phase I effort, we have developed a preliminary architecture for a tether transport system to meet that need. The Mars-Earth Rapid Interplanetary Tether Transport (MERITT) system, illustrated conceptually in Figure 11, consists of two rapidly rotating tethers in highly elliptical orbits; EarthWhip around Earth and MarsWhip around Mars. A payload capsule is launched out of the atmosphere of Earth into a suborbital trajectory. The payload is picked up by the EarthWhip tether as the tether nears perigee and is tossed a half-rotation later, slightly after perigee. The ΔV given the payload deep in the gravity well of Earth is sufficient to send the payload on a high-speed trajectory to Mars with no onboard propulsion needed except for midcourse guidance. At Mars, the incoming payload is caught by the MarsWhip tether in the vicinity of periapsis and the payload is released later at a velocity and altitude which will cause it to reenter the Martian atmosphere. The MERITT system works in both directions, is reusable, and the only major payload propellant requirement is that needed to raise the payload out of the planetary atmosphere and put it into the appropriate

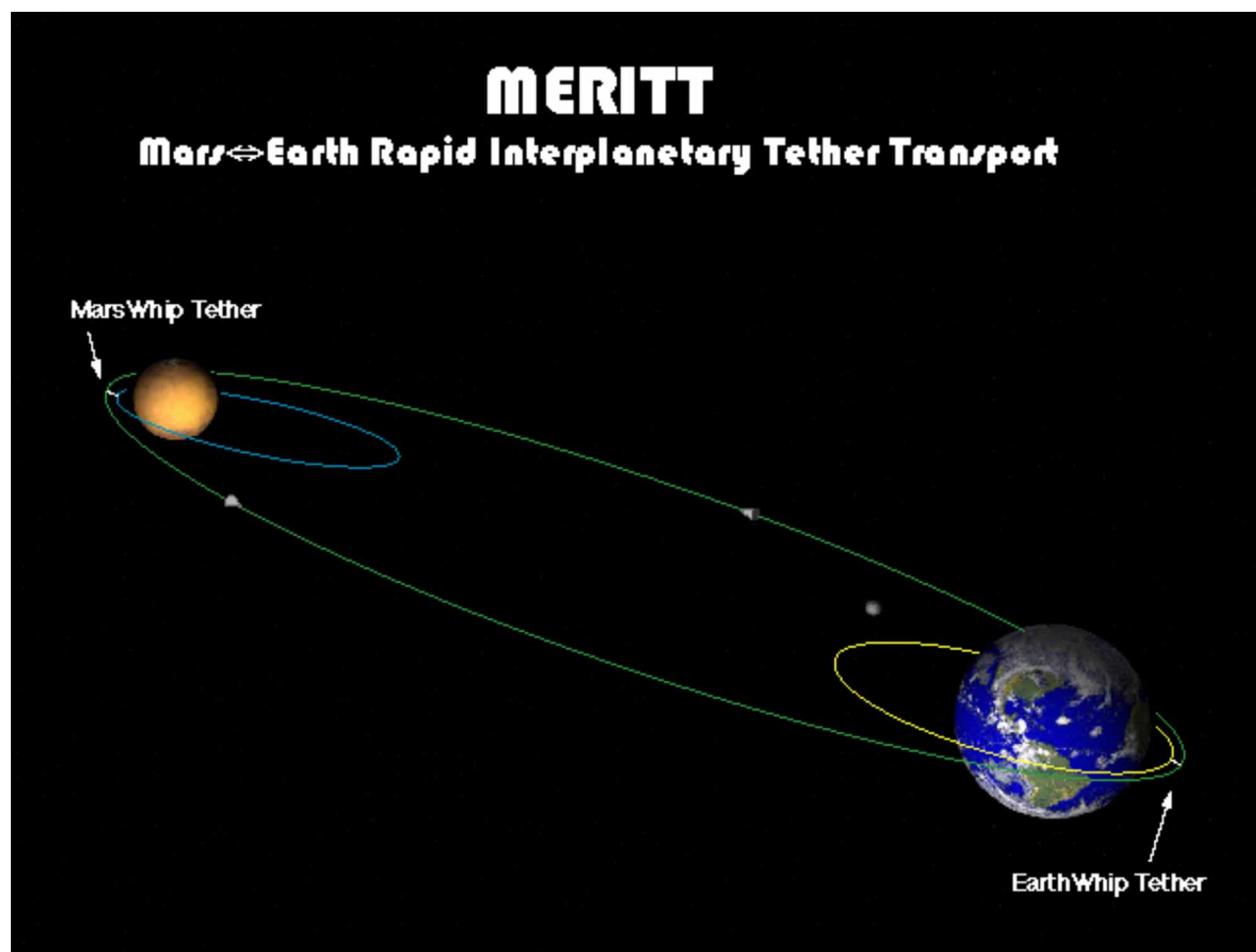


Figure 11. Conceptual illustration of the MERITT System.

suborbital trajectory. Tethers with tip velocities of 2.5 km per second can send payloads to Mars in as little as 90 days if aerobraking is allowed to dissipate some of the high relative velocity on the Mars end. Tether-to-tether transfers without aerobraking may be accomplished in about 130 to 160 days. The mass of each tether system, using commercially available tether materials and reasonable safety factors, including the mass of the two tether arms, grapple tips, and central facility, can be as little as 15 times the mass of the payload being handled. Unlike rocket propellant mass ratios, which can only launch one payload, the tether mass can be reused again and again to launch payload after payload. Further detail on the feasibility study of the MERITT system is presented in Appendix H.

III.C.2. Tether Boost Facility Design for the Human Mars Mission

(Appendix I)

On April 8, 1999, we presented the MERITT and Cislunar Tether Transport System concepts to Larry Kos, Hank Kirschmeyer, and the NASA/MSFC team developing the design for the Human Mars Mission, targeted for flights during the 2011 and 2013/14 Mars opportunities.

At their request, we have developed a preliminary architecture for a “MarsHEFT” tether facility designed to handle cargo payloads for the Human Mars Mission. This facility will impart a total ΔV of 2.5 km/s to the payloads, boosting them from LEO holding orbits to high-energy elliptical orbits in preparation for Trans-Mars-Injection rocket burns. The orbital architecture of this system is illustrated in Figure 12. Our analyses indicate that the total system mass required, using currently available tether materials and reasonable safety factors, would be approximately 4.6 times the payload mass, or 391 mt of facility mass for a 85 mt payload. Economically, this system would compare very favorably to a SEP boost stage if it is used for repeated missions. The system would provide rapid transfer times,

comparable to chemical rocket transfer times, yet require no propellant resupply. The system could also provide direct Mars transfer insertion for 15 mt payloads, and handle significant traffic to GEO and the Moon.

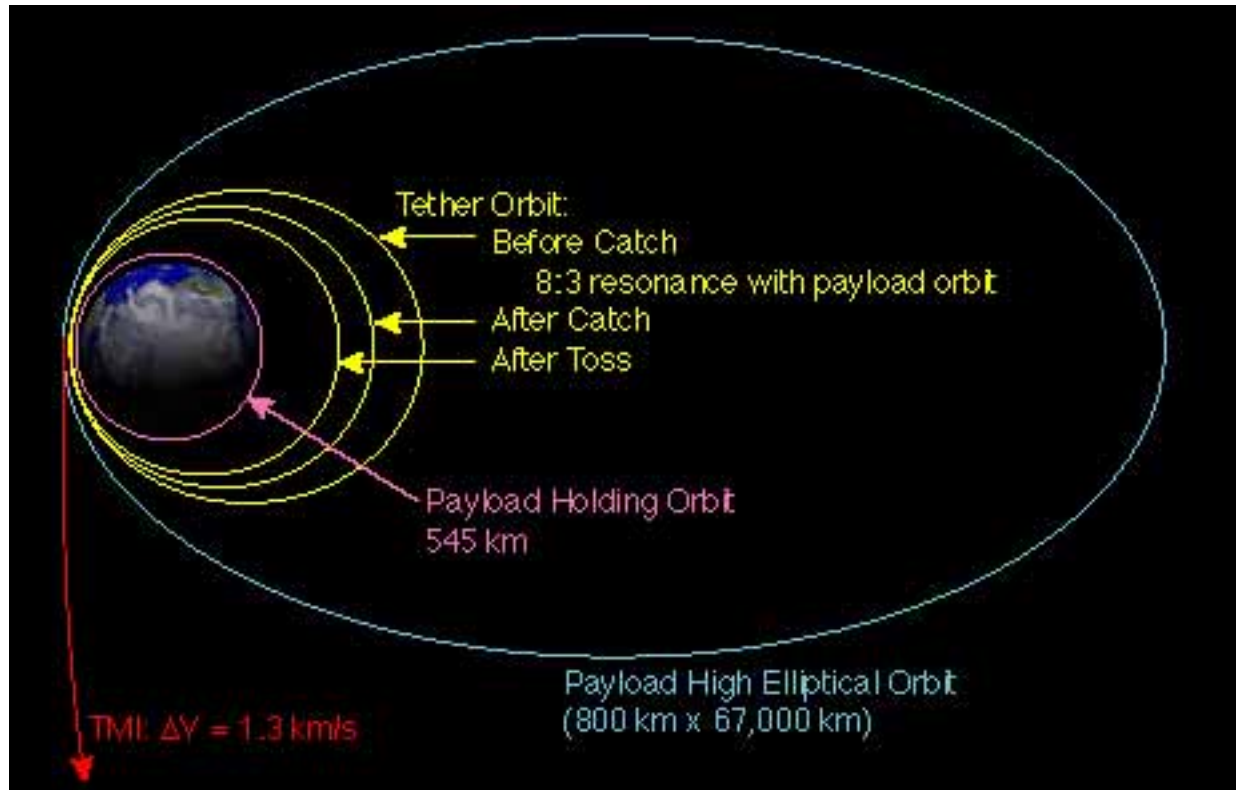


Figure 12. Orbital architecture of the MarsHEFT system.

III.D. COMPARISON TO COMPETING TECHNOLOGIES

Comparison to Solar Electric Propulsion Systems

Solar electric propulsion (SEP) systems such as Hall thrusters, MPD thrusters, and ion engines can provide very high specific impulses, and thus could potentially transport payloads from LEO to lunar orbit with relatively low propellant requirements. As a result, for a single mission to the Moon, SEP would require a lower on-orbit mass than a tether transport system, and thus could have lower initial costs. However, the drawback of SEP's excellent fuel economy is the relatively low thrusts that these systems can generate. As a result, using SEP for transport to the Moon requires that payloads be boosted slowly on spiral trajectories that can take many months or even years. This would not be acceptable for crewed missions, and would be problematical for cargo and scientific missions as well due to the radiation doses the cargo would experience during the slow spiral out through the radiation belts. Furthermore, EP thrusters have limited lifetimes, and the solar panels and electronics of the SEP system will degrade to unacceptable levels after just a few round trips through the radiation belts.¹¹ As a result, any SEP system for Cislunar transport would have a maximum service life of about five trips.

In comparison, a tether system could provide very short (several day) transit times and handle many more payloads. Consequently, although a tether system would involve a larger up-front development and deployment cost than a SEP transfer vehicle, for frequent and sustained round trip travel to the Moon, the Cislunar Tether Transport System could provide significantly lower transport costs. For a more detailed discussion of the economic comparison of a tether system to a SEP system, see the comparison of the MarsHEFT tether design to SEP in Appendix I.

Comparison to Chemical and Nuclear Thermal Rockets

Because transit time is a critical factor in the cost of transporting payloads to the moon, the real competition for the Cislunar Tether Transport System will be high-thrust rocket systems such as chemical rockets and proposed nuclear-thermal rockets. Travelling from LEO to the surface of the Moon and back requires a total ΔV of more than 10 km/s. To perform this mission using storable chemical rockets, which have an exhaust velocity of roughly 3.5 km/s, the standard rocket equation requires that a rocket system consume a propellant mass equal to 16 times the mass of the payload for each mission. The Cislunar Tether Transport System would require an on-orbit mass of less than 28 times the payload mass, but it would be able to transport many payloads. In practice, the tether system will require some propellant for trajectory corrections and rendezvous maneuvers, but the total ΔV for these maneuvers will likely be less than 100 m/s. Thus a simple comparison of rocket propellant mass to tether system mass indicates that the fully reusable tether transport system could provide significant launch mass savings after only a few round trips. Although the development and deployment costs associated with a tether system would present a larger up-front expense than a rocket based system, for frequent, high-volume round trip traffic to the Moon, a tether system could achieve large reductions in transportation costs by eliminating the need to launch large quantities of propellant into Earth orbit.

If nuclear-thermal rockets become available, their higher specific impulses will reduce the amount of propellant needed for transport to the Moon. However, nuclear-thermal rockets will still require propellant resupply as well as resupply of their nuclear fuel, so for frequent travel to and from the Moon, a fully reusable architecture like the Cislunar Tether Transport System can significantly reduce the total costs of transport. Moreover, the political and environmental concerns associated with the use of nuclear materials in space will continue to present a significant obstacle to their deployment.

III.E. HIGH-STRENGTH TETHER MATERIALS

The mass required for a rotating tether depends strongly on the ratio of the tip speed to the characteristic velocity of the tether material. Consequently, the viability of using momentum-exchange tethers for missions requiring large ΔV 's depends upon whether materials are available with sufficiently large strength-to-weight ratios to make the tether masses practical. Over the past two decades, there has been steady improvement in the field of high-strength fibers.¹² Because of the strong dependence of the required tether mass on the tip speed to characteristic velocity ratio, even a small increase in the characteristic velocity of tether materials can greatly reduce the tether mass. To determine the effects of improved fiber strengths on the viability of the Cislunar Tether Transport System and the MERITT concept, we contacted several fiber companies to assess the current state-of-the-art in high strength tether materials.

Spectra 2000

Currently, the material with the best strength to weight ratio commercially available in large quantities is Spectra 2000™, a form of highly oriented polyethylene fabricated by AlliedSignal. Spectra 2000™ fiber has a density of 970 kg/m³, and is now being produced in 75 denier yarns with an average tenacity of 41 g/denier. Spectra 2000's tenacity of 41 g/denier translates to a tensile strength of 3.6 GPa. High-quality specimens of Spectra 2000™ have been produced with tenacities as high as 46 g/denier, which translates to a tensile strength of 4 GPa.

Low Temperature Spectra

Spectra fiber has a very low absorption coefficient for solar spectrum light. As a result, clean Spectra tethers in Earth orbit will have rather low temperatures, on the order of 180-200 K.¹³ Joe Carroll of Tether Applications has found that when Spectra 1000 is cooled to 190 K, its strength increases by 21%. If it is placed under a load of approximately 1% of the breaking strength before and during the cooling, the strength increase improves to 41%.¹⁴

A 41% increase in tenacity of a 46 g/denier high-quality Spectra 2000 translates to a tensile strength of 5.6 GPa. If these low-temperature results for Spectra 1000 hold true for the newer Spectra 2000, this could further reduce the mass requirements for momentum-exchange tethers.

Dyneema

A fiber similar to Spectra, called Dyneema 66, is available in Europe from DSM High Performance fibers. It also is highly oriented polyethylene, but it is made by a slightly different process. Dyneema 66 is advertised as having a tenacity of 37 g/denier. Word-of-mouth has it that some of the fiber sold in the US by AlliedSignal as Spectra 2000 is, in fact, Dyneema 66.

One disadvantage to Spectra and Dyneema is that they have a relatively low melting temperature. They are not useful for applications where it will reach temperatures over about 423 K (150 C). Because the Lunavator™ will be exposed to significant thermal radiation from the sun-lit portions of the Moon's surface, use of other materials may be necessary for the bottom sections of a Lunavator™.

PBO/Zylon

Poly(p-phenylene-2,6-benzobisoxazole) (PBO) is a rigid-rod isotropic crystal polymer marketed under the brand name Zylon™ by the Toyobo company in Japan. PBO has a tenacity of 42 g/denier (5.8 GPa) and a density of 1.56 g/cc. PBO also has excellent temperature resistance properties, maintaining nearly full strength to temperatures near 500 C.¹⁵

Summary

The mass ratios required for the two tethers in the Cislunar Tether Transport System using these currently available materials, calculated using Moravec's equation (see Appendix A), are shown below in Table 1. [Note: These ratios are calculated for a perfectly tapered tether attached to an infinitely massive central facility. The larger tether masses ratios in the more detailed designs presented in Appendices A & B were calculated for stepwise-tapered tether attached to a finite-mass facility, and represent more realistic mass estimates.] Because launch costs to place mass in orbit are so high, the competitiveness of an in-space propulsion system is measured largely by the mass required for the system. Our analyses have shown that by using *currently available materials* such as Spectra 2000 and PBO, tethers for significant propulsion missions such as Cislunar transport will require total tether masses of less than 5 times the payload mass.

Table 1. TETHER MASS RATIO FOR CURRENT TETHER MATERIALS

Material	Spectra 2000		PBO/Zylon
	300 K	190K	
Tensile Strength (g/denier)	46	64 ?	42
Tensile Strength (GPa)	4	5.6	5.8
Density (g/cc)	0.97	0.97	1.56
Characteristic Velocity, km/s with Safety Factor F = 3.5	1.53	1.81	1.45
Earth-Orbit Tether $V_{tip} = 1.5$ km/s Safety Factor F = 3.5	3.73	2.18	4.5
Lunavator™ $V_{tip} = 1.6$ km/s, Safety Factor F = 3.5	4.7	2.7	5.7

III.F. HIGH-SURVIVABILITY TETHER STRUCTURES

For a tether transport system to be economically advantageous, it must be capable of handling frequent traffic for many years despite degradation due to impacts by meteorites and space debris. Fortunately, a survivable tether design exists, called the Hoytether™, which can balance the requirements of low weight and long life.¹⁶ As shown in Figure 13, the Hoytether™ is an open net structure where the primary load bearing lines are interlinked by redundant secondary lines. The secondary lines are designed to be slack initially, so that the structure will not collapse under load. If a primary line breaks, however, the secondary lines become engaged and take up the load.

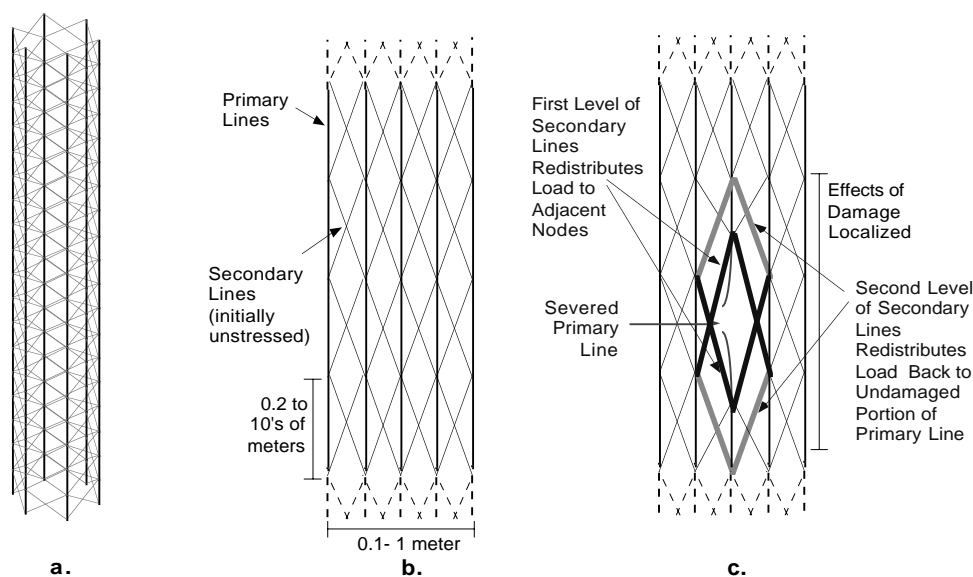


Figure 13. The Hoytether™ design and its response to a cut line.

Note that four secondary line segments replace each cut primary line segment, so that their cross-sectional area need only be 0.25 of the primary line area to carry the same load. Typically, however, the secondary lines are chosen to have a cross-sectional area of 0.4 to 0.5 of the primary line area, so as to better cope with multiple primary and secondary line cuts in the same region of the tether. This redundant linkage enables the structure to redistribute loads around primary segments that fail due to meteorite strikes or material failure. Consequently, the Hoytether™ structure can be loaded at high stress levels, yet retain a high margin of safety.² The Hoytether™ is described in more detail in Appendix J.

III.G. KEY FEASIBILITY ISSUE: PAYLOAD RENDEZVOUS & CAPTURE WITH A ROTATING TETHER

Of the technologies that must be developed in order to build a tether transportation system, the most significant challenge will be the hardware and techniques needed to achieve rendezvous between a payload and the tether. In a conventional docking, such as between the Space Shuttle and the ISS, rendezvous is achieved by slowly matching the orbits of the two spacecraft over a period of many minutes or even many hours. In a tether rendezvous, however, the payload must meet up with the grapple vehicle at the tip of a rotating tether. The orbits of the tether and payload must be set up in advance so that the grapple and tether will meet at a particular time with the same position and the same velocity. Furthermore, because the grapple vehicle is under constant acceleration due to the tether tension, it follows a non-Keplerian trajectory, and the “window” for rendezvous will typically be only several seconds.

Although the rendezvous will be a significant challenge to accomplish, it is not as difficult as it might seem at first glance. Essentially, what the payload must do is rendezvous with a “virtual” spacecraft to place it in the proper orbit to meet up with the tether tip at a later time. To illustrate the rendezvous task,

the relative separation and velocity of a payload and a tether-tip grapple vehicle during rendezvous are shown in Figure 14 and Figure 15, respectively. This rendezvous scenario is for the 25 km long HEFT Facility discussed in Appendix G. This tether rotates with a tip velocity of 0.4 km/s, and the tether tip acceleration level is approximately 0.7 gees. As the figures show, the relative motion is nearly along the local vertical direction. From the perspective of the payload, the tether tip drops down, approaching very quickly at first, but slowing down under nearly constant deceleration as it nears the payload. From the perspective of the grapple vehicle, the rendezvous scenario is roughly equivalent to a situation where a man stands on a balcony and his friend tosses a ball up to him; at first the ball rises quite quickly, but it decelerates constantly under the force of gravity, and when it reaches the level of the balcony it is, it is moving very slowly and the man can reach out and catch it at the apex of its trajectory.

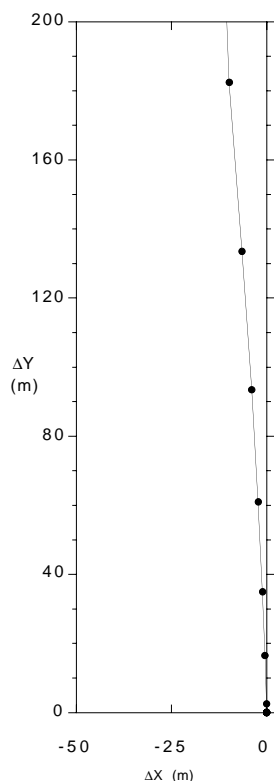


Figure 14. Relative separation between payload and tether tip during rendezvous approach. For a 25 km, rotating with tip velocity of 0.4 km/s. Markers show one-second intervals.

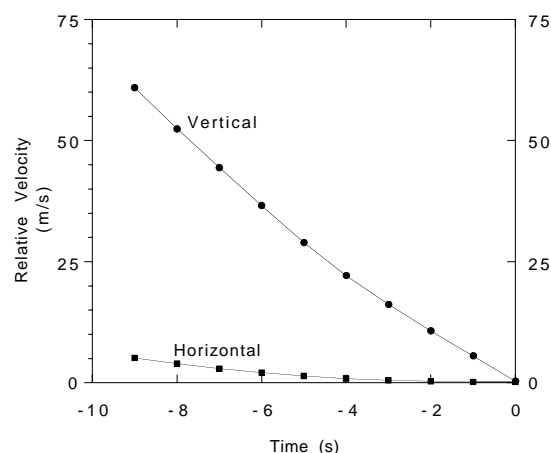


Figure 15. Relative velocity between payload and tether tip during rendezvous approach. For a 25 km, rotating with tip velocity of 0.4 km/s. Markers show one-second intervals.

The rendezvous challenge thus will be twofold: First, to arrange the orbits of the payload and tether very carefully, so that they will meet at a certain time, with the right velocity and the right position, and second, to enable the payload and grapple vehicle to maneuver to achieve a docking within a “window” of several seconds.

Orbital Rendezvous

The first task will require that both the payload and the tether system measure their positions and velocities very accurately with GPS or similar instrumentation and use high-fidelity orbital dynamics modeling to predict their trajectories. The two systems will then maneuver to set their trajectories up for a rendezvous. It will be necessary for the payload and grapple vehicle to work collaboratively to minimize the propellant expenditure needed to assure the rendezvous. Stuart¹⁷ has studied the

rendezvous between payloads and a hanging tether and developed optimal rendezvous strategies in which the tether tip grapple vehicle reels a length of the tether in and out to minimize the ΔV required for rendezvous and maximize the rendezvous window. Although the situation for a rapidly rotating tether is more challenging, the hanging tether studied by Stuart is, in fact, rotating once per orbit, so similar rendezvous strategies should be useful for the Cislunar and MERITT systems.

Tether Deployment to Extend Docking Time

In the rendezvous scenario shown in the figures above, the period where the payload and grapple are close and moving slowly relative to each other is only a few seconds. However, this “window” for docking can be extended considerably if the grapple vehicle contains a small tether reel and can deploy a length of tether at very low tension. If the grapple vehicle allows the tether to deploy, the grapple will cease to experience the acceleration due to the tether tension and will move along a Keplerian trajectory. Thus the grapple vehicle can meet up with a payload and then pay out tether so that it will move along with the payload for as long as the tether lasts. With several hundred meters of tether on the reel, the docking window could be extended to a period of ten seconds or more. This ability to pay out tether will also enable the grapple vehicle to “soften” the tension spike that the tether will experience when it captures the payload.

Prospects for Tether Automated Rendezvous and Capture

During this Phase I effort, we met with the Automated Rendezvous and Capture team at NASA/MSFC to discuss the prospects for using the AR&C technologies under development at NASA to enable a rotating tether facility to capture a payload. We briefed them on the requirements a tether system would place upon AR&C technologies, and they expressed the following opinion:

The Automated Rendezvous and Capture (AR&C) Project Office at Marshall Space Flight Center (MSFC) has been briefed on the AR&C requirements for the capture of a payload by a grapple vehicle at the end of a tether with a one-gee acceleration tip environment. MSFC has been working AR&C for over six years and has a great deal of experience in this area. It is our opinion that the present Shuttle-tested [STS-87 & STS-95] Video Guidance Sensor (VGS) hardware, and Guidance, Global Positioning System (GPS) Relative Navigation, and Guidance, Navigation and Control (GN&C) software, should, with sufficient funding, be able to be modified for this tether application.

Dallias S. Pearson
AR&C Chief Engineer
NASA MSFC Code EE41
dallias.pearson@msfc.nasa.gov
Pager: 800-946-4646 (#4923304)
Ph/Fax: 256-544-6621/5840
Building 4202 Room 222A

III.H. INCREMENTAL SYSTEM DESIGN

To create a tether transport system with facilities at the Earth, the Moon, and Mars will certainly be a very significant undertaking. To reduce the initial financial hurdles that will be faced in developing the system, it would be very desirable for the system architecture to be amenable to incremental deployment to enable the development to be broken up into smaller, more affordable pieces. Because a tether transport system will be based upon technologies and techniques that the established aerospace community will likely perceive as unconventional and unproven, it will also likely be necessary to demonstrate the momentum-exchange tether technologies on a small scale before the full system development can be financed. Moreover, it would be very desirable for the first components to be capable of performing useful tasks before the rest of the system is deployed, so that the early components can earn revenues to help fund the development of the rest of the system.

The architecture of the Cislunar Tether Transport System has been designed with such an incremental development approach in mind. A key to this will be the combination of propellantless electrodynamic propulsion and momentum-exchange tether techniques of the HEFT Facility concept, which will enable the Earth-orbit tether facility to boost payloads to the Moon, GTO, and Mars without requiring propellant or return traffic to restore the tether's orbital energy. This will allow the first component of the system to facilitate the development of bases on the Moon and Mars, and to earn revenue to pay for development of a Lunavator™ and a Mars tether facility. A possible incremental pathway for development of a Tether Transport System would be:

III.H.1. Tether Transport System Technology Design Effort

Under Phase II funding on this NIAC effort, Tethers Unlimited, Inc. and its partners would work on two tasks:

First, we would develop designs for the technology components needed for a Tether Transport System, including:

- Tether Facility: tether deployer, power generation and conversion systems, tether guidance and dynamics control systems
- Tether: High-strength, conducting, survivable tether structures
- Grapple vehicle and payload interface unit
- Tether Rendezvous and capture techniques and technologies

Second, we would develop a design for an affordable flight experiment to begin demonstrating the technologies and techniques utilized in the Cislunar Tether Transport System. A probable candidate for such a demonstration mission might be:

III.H.2. STOTS: Spinning Tether Orbital Transfer System Experiment

The purpose of the initial mission would be to demonstrate that we can deploy a payload at the end of a 20 km long tether, induce the tether system to rotate in a controllable manner, and then accurately toss the payload into a higher orbit. This experiment could be performed in an economical manner by building upon some of the technologies already developed and demonstrated in the SEDS and OEDIPUS tether experiments. For example, like the SEDS experiment, the STOTS mission could be launched as a piggyback experiment on a Delta II upper stage or other launch vehicle; the spent upper stage would provide the "ballast mass" for the tether facility. A deployer similar to the SEDS deployer could then be used to deploy a tether with a small endmass system. This endmass would contain a small tether reel similar to that developed for the OEDIPUS sub-orbital tether experiments, which would be used to induce the tether to rotate. The endmass would also contain a small payload as well as a mechanism for releasing that payload. This payload might be as simple as a baseball-sized mass with corner cubes to facilitate optical or radar tracking. Once the tether is rotating at the desired rate, the endmass would release the payload at the top of the tether's swing, injecting the payload into a higher orbit. Because this experiment would utilize some of the tether hardware that has already been developed, it could be performed relatively inexpensively. Based upon NASA/MSFC's experiences with the ProSEDS experiment, we estimate that a STOTS mission could be performed for between \$6M - \$10M, and thus might be a candidate for the next round of FUTURE-X proposals.

III.H.3. TORQUE: Tether Orbit-Raising Qualification Experiment

Once the STOTS experiment has demonstrated that we can controllably toss a payload from a rotating tether, we would build upon those technologies by developing the capability for a rotating tether to rendezvous with and capture a payload in a lower orbit. The TORQUE experiment would use the same tether architecture as the STOTS mission to keep it affordable, but add technologies to enable the tether endmass to catch and throw payloads. The rendezvous and capture will be a significant engineering challenge. It will require development of subsystems to allow the tether endmass and the payload to determine their positions and velocities with excellent accuracy and then maneuver into

trajectories that will intersect with the same velocity and the same position. The tether grapple vehicle and the payload will have to work collaboratively so as to achieve the rendezvous with minimum propellant expenditure.

A possible scenario for the TORQUE mission would be for the tether system to deploy the grapple vehicle and payload at the end of a tether, and then spin up the tether system. The grapple vehicle would then release the payload, injecting it into a higher orbit. The payload's orbit would be chosen to be resonant with the tether system's orbit, so that several orbits later the payload and tether will meet up. The grapple vehicle would then catch the payload, demonstrating the tether rendezvous and capture techniques. This toss and catch could be repeated a number of times to demonstrate the reliability of the technologies.

III.H.4. Earth-orbit Tether Boost Facility

By combining the technologies developed and the lessons learned in the STOTS, TORQUE, ProSEDS, and other tether experiments, we would then design and deploy the first component of the Cislunar Tether Transport System, the Earth-orbit HEFT Facility. This facility would be used to send payloads to the Moon and Mars to help set up and support bases on those bodies. In addition, the same facility could earn revenues by boosting GEO satellites and materials for building solar power stations into geostationary transfer orbits.

III.H.5. Lunavator™ Facility

Once a lunar base has been established and the round-trip traffic volume justifies the expense, the lunar tether facility would be deployed. As described in Appendix B, it is possible to send the Lunavator™ to the moon with a relatively low ballast mass and as payloads are sent to it from the Earth-orbit tether, the Lunavator™ can pick up lunar mass to build up its ballast mass and payload capacity.

III.H.6. MarsWhip Tether Facility

The Earth-orbit tether boost facility could also be used to send components of the Mars tether facility on trans-Mars trajectories. Once this facility is completed, it will support rapid round-trip travel between Earth and Mars.

IV. SUMMARY

Our analyses have concluded that the optimum architecture for a tether system designed to transfer payloads between LEO and the lunar surface will utilize one tether facility in an elliptical, equatorial Earth orbit and one tether in low lunar orbit. We have developed a preliminary design for a 80 km long Earth-orbit tether boost facility capable of picking payloads up from LEO and injecting them into a minimal-energy lunar transfer orbit. Using currently available tether materials, this facility would require a total mass of 10.5 times the mass of the payloads it can handle. After boosting a payload, the facility can use electrodynamic propulsion to reboost its orbit, enabling the system to repeatedly send payloads to the Moon without requiring propellant or return traffic. When the payload reaches the Moon, it will be caught and transferred to the surface by a 200 km long lunar tether. This tether facility will have the capability to reposition a significant portion of its "ballast" mass along the length of the tether, enabling it to catch the payload from a low-energy transfer trajectory and then "spin-up" so that it can deliver the payload to the Moon with zero velocity relative to the surface. This lunar tether facility would require a total mass of less than 17 times the payload mass. Both equatorial and polar lunar orbits are feasible for the Lunavator™. Using two different numerical simulations, we have tested the feasibility of this design and developed scenarios for transferring payloads from a low-LEO orbit to the surface of the Moon, with only 25 m/s of ΔV needed for small trajectory corrections. Thus it appears feasible to construct a Cislunar Tether Transport System with a total on-orbit mass requirement of less than 28 times the mass of the payloads it can handle, and this system could greatly reduce the cost of round-trip travel between LEO and the surface of the Moon by minimizing the need for propellant expenditure.

References

- ¹. Forward, R. L., "Tether Transport from LEO to the Lunar Surface," AIAA paper 91-2322, July 1991.
2. Hoyt, R.P., and Forward, R.L., *LEO-Lunar Tether Transport System Study*, Tethers Unlimited Final Report on Subcontract S06-34444 on NASA Grant P3776-5-96 to the Smithsonian Astrophysical Observatory, April 1997
3. Lewis, J.S., *Physics and Chemistry of the Solar System*, Academic Press, 1997.
4. Danby, J.M.A., *Fundamentals of Celestial Mechanics*, Willmann-Bell, 1992.
5. Kim, E., Vadali, S.R. "Modeling Issues related to Retrieval of Flexible Tethered Satellite Systems," *Journal of Guidance, Control, and Dynamics*, 18(5), 1995, pp 1169-76.
6. Uphoff, C., "Mission Analysis Evaluation and Space Trajectory Optimization Program", Final Report on NASA Contract NAS5-11900, March 1973.
7. Hoyt, R.P., Uphoff, C.W., "Cislunar Tether Transport System", AIAA Paper 99-2690.
8. Nordley, G. and Forward, R.L., "Mars-Earth Rapid Interplanetary Tether Transport (MERITT) System: I - Initial Feasibility Analysis", AIAA Paper 99-2151.
9. Johnson, L., "Propulsive Small Expendable Deployer System Mission (ProSEDS)", *OAST Advanced Propulsion Workshop*, JPL, Pasadena, CA, 20-22 May 1997.
10. *Failure Resistant Multiline Tether*, Robert L. Forward and Robert P. Hoyt, PCT/US97/05840, filed 22 April 1997.
11. Fitzgerald, A. "The Effect of Solar Array Degradation in Orbit-Raising with Electric Propulsion," AIAA Paper IEPC-93-207, 23rd *International Electric Propulsion Conference*, Sept. 1993.
12. Lorenzini, E.C., "Tethers for LEO to GEO: Materials and comparison data" Smithsonian Astrophysical Observatory Presentation, December 2, 1996.
13. Carroll, J.A., personal communication, 1/11/95.
14. Carroll, J.A., personal communication, email 4/8/97.
15. Information on PBO is available on the web at <http://www.toyobo.co.jp/e/seihin/kc/pbo/index.htm>
16. Forward, R.L., and Hoyt, R.P., "Failsafe Multiline Hoytether Lifetimes", AIAA paper 95-2890, 31st *AIAA/SAE/ASME/ASEE Joint Propulsion Conference*, San Diego, CA, July 1995.
17. Stuart, D.G., "Guidance and Control for Cooperative Tether-Mediated Orbital Rendezvous," *J. Guidance*, 13(6) pp. 1102-8, Nov-Dec 1990.

DESIGN OF EARTH-ORBIT TETHER FACILITIES FOR LUNAR TRANSFER ORBIT INJECTION

Robert P. Hoyt
Tethers Unlimited, Inc.

Abstract

In this work, we develop an architecture for a tether boost facility designed to exchange payloads between low-LEO orbits and Lunar Transfer Trajectories. We find that a tether system utilizing one tether facility in an elliptical orbit will provide the lowest system mass and complexity. Analysis of the system architecture indicates that a facility massing just 10.5 times the payload mass can inject payloads into minimum-energy lunar transfer trajectories. After boosting a payload, the facility can use propellantless electrodynamic tether propulsion near its perigee in LEO to rapidly reboost its orbit so that it can boost additional payloads. As a reference design, a tether facility massing 26 mt, with a power supply of 11 kW, can boost a 2.5 mt payload to the moon once every 95 days. We also find that apsidal precession of the tether's orbit can be handled either using tether reeling maneuvers or by selecting the tether's orbit so that the orbit's precession rate is resonant with the lunar orbital period.

Introduction

In this section, we develop a design for the first stage of a Cislunar Tether Transport System, a tether boost facility in elliptical Earth orbit capable of picking payloads up from low-LEO orbits and tossing them to the Moon. The objective of the design study was to determine a system architecture with minimum system mass, minimum system complexity, and minimum system propulsion requirements. To determine an optimum system configuration, we must balance the need to minimize the required masses of the tethers and facilities with the need to make the orbital dynamics of the system as manageable as possible.

The Mission:

The mission of the Earth-orbit portion of the Cislunar Tether Transport System is to pick up a payload from low-Earth orbit and inject it into a near-minimum energy Lunar Transfer Orbit (LTO). Although some previous studies have considered systems intended to capture payloads from suborbital trajectories and transfer them to the moon,¹ for this study we will focus on system designs intended to transfer payloads between low-LEO orbits and lunar transfer trajectories.

The desired lunar transfer trajectories have a C_3 of approximately -1.9 (km/s)^2 . The C_3 of a trajectory is defined as twice the vis-viva energy of the orbit: $C_3 \equiv V^2 - 2\mu/r$. A payload originating in a circular orbit at 350 km altitude has an initial velocity of 7.7 km/s and a C_3 of -60 (km/s)^2 . To impulsively inject the payload in to a trajectory with a C_3 of -1.9 would require a ΔV of approximately 3.1 km/s.

Design Considerations***Tether System Staging***

From an operational standpoint, the most convenient design for the Earth-orbit portion of a Cislunar Tether Transport System would be a single HEFT tether facility in circular low-Earth orbit. The facility would rendezvous with a payload, deploy the payload at the end of a long tether, and then use propellantless electrodynamic tether propulsion to spin up the tether until the tip speed reached 3.1 km/s. However, because the tether transfers some of its orbital momentum and energy to the payload when it boosts it, a tether facility in circular orbit would require a very large ballast mass so that its orbit would not drop into the upper atmosphere after it boosts a payload. Furthermore, the strong dependence of the required tether mass on the tether tip speed will likely make this approach impractical with current material technologies. The required mass for a tapered tether depends upon the tip mass M_p and the ratio of the tip velocity ΔV to the tether material's critical velocity V_c according to the relation derived by Moravec:

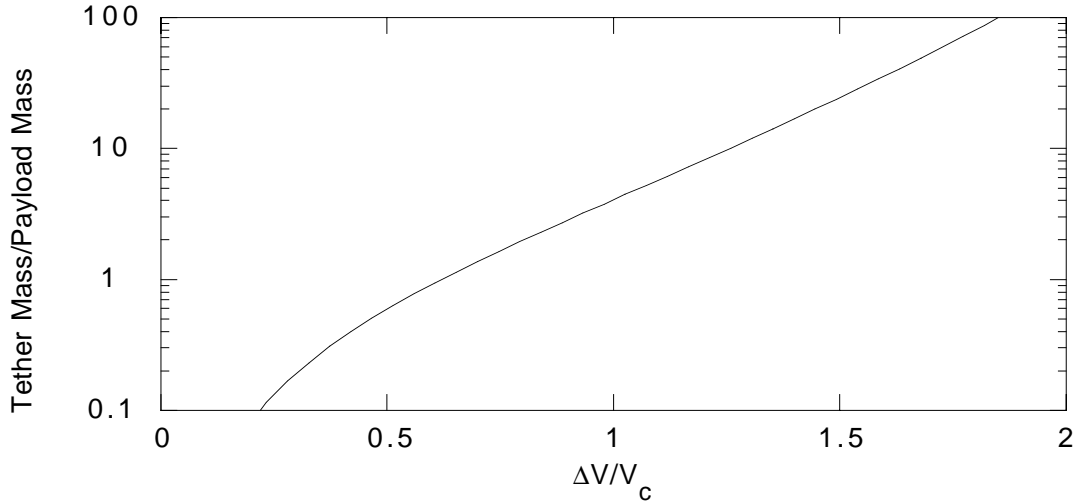


Figure 1. Tether mass ratio as a function of ratio of tip velocity to tether material critical velocity.

$$M_T = M_p \sqrt{\pi} \frac{\Delta V}{V_c} e^{\frac{\Delta V^2}{V_c^2}} \operatorname{erf} \left\{ \frac{\Delta V}{V_c} \right\}, \quad (1)$$

where $\operatorname{erf}()$ is the error function. The critical velocity of a tether material depends upon the tensile strength T , the material density d , and the design safety factor F according to:

$$V_c = \sqrt{\frac{2T}{Fd}}. \quad (2)$$

The exponential dependence of the tether mass on the *square* of the velocity ratio, results in a very rapid increase in tether mass with this ratio. Figure 1 shows a graph of the ratio of the required tether mass to the payload mass as a function of the ratio of the ΔV to the tether critical velocity.

Currently, the best commercially-available tether material is Spectra 2000, a form of highly oriented polyethylene manufactured by AlliedSignal. High-quality specimens of Spectra 2000 have a room temperature tensile strength of 4 GPa, and a density of 0.97 g/cc. With a safety factor of 3, the material's critical velocity is 1.66 km/s. Using Equation (1), an optimally-tapered Spectra tether capable of sustaining a tip velocity of 3.1 km/s would require a mass of over 100 times the payload mass. While this might be technically feasible for very small payloads, such a large tether mass probably would not be economically competitive with rocket technologies. In the future, very high strength materials such as "buckytube" yarns may become available with tensile strengths that will make a 3 km/s tether feasible; however, we will show that different approaches to the system architecture can utilize currently available materials to perform the mission with reasonable mass requirements.

As Figure 1 shows, the tether mass can be reduced to reasonable levels if the $\Delta V/V_c$ ratio can be reduced to levels near unity or lower. In the Cislunar system, we can do this by breaking the 3.1 km/s ΔV up into two or more smaller boost operations.

Architectures Considered:

In our design study, we investigated a number of different scenarios, including:

- A two-tether system with one tether in circular LEO that accelerates and throws a payload up to a second tether in a circular MEO orbit, which catches and then throws the payload to the moon.
- A two-tether system with one tether in a low elliptical orbit that accelerates and throws a payload up to a second tether in a higher elliptical orbit, which catches and then throws the payload to the moon.

- A single tether facility in an elliptical orbit, rotating with a tip velocity of approximately 1.5 km/s, that can catch a payload from a circular low-LEO orbit, giving it a ~1.5 km/s boost, and then throw it into a LTO, giving it another ~1.5 km/s boost.

Our analyses resulted in the conclusion that the system using a single tether in elliptical orbit is the most favorable architecture for reasons of mass minimization and system complexity minimization. A system with two circular orbit tethers was ruled out for two reasons: first, the ballast mass necessary to prevent the facilities' orbits from dropping into the atmosphere would be prohibitive, and second, performing the second boost operation in a high circular orbit is less efficient than performing the ΔV operation deeper in the Earth's gravity well, and thus a two circular orbit facility system required a larger total ΔV for LTO injection than did the other two architectures. The system with two tethers in elliptical orbit would break the total ΔV up into 4 pieces, moving the $\Delta V/V_c$ ratios to the left on Figure 1, and thus could achieve the lowest total *tether* mass. However, when the required ballast mass is factored in to the total system mass, the two-tether system would not have a significant mass advantage compared to the single tether system, and the added complexity of operating and scheduling two tether facilities would likely outweigh any benefits from lowering the tether mass. Consequently, we focused our design efforts on the single elliptical orbit tether architecture.

Behavior of Orbits in the Earth's Gravitational Field

One of the major challenges to designing a workable tether transportation system using elliptical orbits is motion of the orbit due to the oblateness of the Earth. The Earth's oblateness will cause the plane of an orbit to regress ("nodal regression") relative to the Earth's spin axis at a rate equal to:

$$\dot{\Omega} = -\frac{3}{2} J_2 \frac{R_e^2}{p^2} \bar{n} \cos(i) \quad (3)$$

And the line of apsides (ie. the longitude of the perigee) to precess or regress relative to the orbit's nodes at a rate equal to:

$$\dot{\omega} = \frac{3}{4} J_2 \frac{R_e^2}{p^2} \bar{n} (5 \cos^2 i - 1) \quad (4)$$

In equations (3) and (4), \bar{n} is the "mean mean motion" of the orbit, defined as

$$\bar{n} = \sqrt{\frac{\mu_e}{a^3}} \left[1 - \frac{3}{4} J_2 \frac{R_e^2}{p^2} \sqrt{1-e^2} (1 - 3 \cos^2 i) \right], \quad (5)$$

and p is the orbit parameter $p=a(1-e^2)$. For an equatorial orbit, the nodes are undefined, but we can calculate the rate of apsidal precession relative to inertial space as the sum $\dot{\Omega} + \dot{\omega}$ of the nodal and apsidal rates given by Eqns. (3) and (4).

The expression for the nodal regression of orbits reveals that the planes of orbits with different semimajor axes will precess relative to each other. Thus, if the orbits are inclined, the planes of their orbits will rotate around the Earth's spin axis at different rates, coinciding only infrequently.

Consequently, in the Cislunar Tether Transport System, we will place two constraints on our system design to make the orbital mechanics problem tractable:

- First, the orbits of the tether facility will be equatorial, so that $i=0$ and the nodal regression given by Eqn. (3) will not be an issue.
- Second, the tether system will throw the payload into a lunar transfer trajectory that is in the equatorial plane. This means that we can perform transfer operations when the moon is crossing either the ascending or descending node of its orbit.

Nonetheless, we still have the problem of precession of the line of apsides of an orbit. If the tether orbits are circular, this is not an issue, but it is an issue for systems that use elliptical orbits. In an elliptical orbit system we wish to perform all catch and throw operations at or near perigee. As

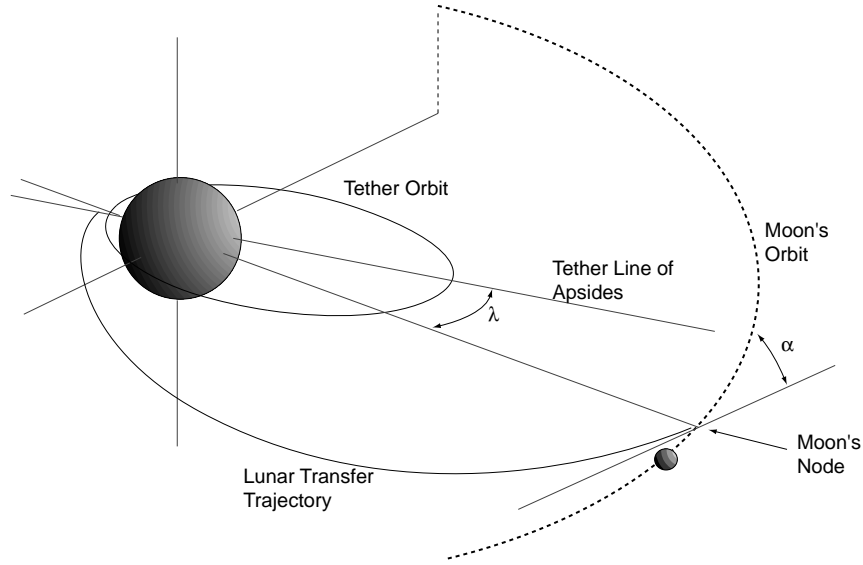


Figure 2. Geometry of the tether orbit and the moon's orbit.

illustrated in Figure 2, for the payload to reach the Moon's radius at the time when the moon crosses the Earth's equatorial plane, the payload must be injected into an orbit that has a line of apsides at some small angle λ from the line through the moon's nodes. If the orbit experiences apsidal precession, the angle λ will have the proper value only periodically. Consequently, in our designs we will seek to choose the orbital parameters such that the apsidal precession of the orbit will have a convenient resonance with the moon's orbit.

Elliptical-Orbit Tether Boost System

In the Cislunar Tether Transport System, the transfer of payloads between a low-LEO orbit and lunar transfer orbits is performed by a single rotating tether facility in an elliptical orbit which performs a catch and throw maneuver to provide the payload with two boosts of approximately 1.5 km/s each. To enable the tether to perform two "separate" ΔV operations on the payload, the facility is placed into a highly elliptical orbit with its perigee in LEO. First, the tether rotation is arranged such that when the facility is at perigee, the tether is swinging vertically below the facility so that it can catch a payload moving more slowly than the facility. After it catches the payload, it waits for one orbit and adjusts its rotation slightly (by reeling the tether in or out) so that when it returns to perigee, the tether is swinging above the facility and it can throw the payload into a trajectory moving faster than the facility.

In order to determine the feasibility of this system, we must determine the tether length, rotation rate, and orbit characteristics that will permit the tether to rendezvous with the payload and throw it into the desired lunar transfer trajectory.

In this analysis, the payload of mass M_p begins in a circular orbit with radius r_{IPO} . The payload orbits with a velocity of

$$V_{p,0} = \sqrt{\frac{\mu_e}{r_{IPO}}}. \quad (6)$$

The facility is placed into an elliptical orbit with a perigee above the payload's orbit, with the difference between the facility's initial perigee and the payload orbital radius equal to the distance from the tether tip to the center of mass of the facility and tether:

$$r_{p,0} = r_{IPO} + (L - l_{cm,unloaded}), \quad (7)$$

where L is the tether's total length, and $l_{cm,unloaded}$ is the distance from the facility to the center of mass of the system before the payload arrives (this distance must be calculated numerically for an tapered tether).

The tether tip velocity is equal to the difference between the payload velocity and the facility's perigee velocity:

$$V_{t,0} = V_{p,0} + V_{IPO} \quad (8)$$

In order to ensure that a payload will not be "lost" if it is not caught by the tether on its first opportunity, we choose the semimajor axis of the facility's orbit such that its orbital period will be some rational multiple N of the payload's orbital period:

$$P_{f,0} = NP_{IPO} \Rightarrow a_{f,0} = N^{2/3} r_{IPO} \quad (9)$$

For example, if $N=5/2$, this condition means that every two orbits the facility will have an opportunity to rendezvous with the payload, because in the time the facility completes two orbits, the payload will have completed exactly five orbits.

An additional consideration in the design of the system are the masses M_f and M_t of the facility and tether, respectively. A significant facility mass is required to provide "ballast mass." This ballast mass serves as a "battery" for storing the orbital momentum and energy that the tether transfers to and from payloads. If all catch and throw operations are performed at perigee, the momentum exchange results primarily in a drop in the facility's apogee. A certain minimum facility mass is necessary to keep the post catch and throw apogees of the facility orbit above the Earth's upper atmosphere. Most of this "ballast mass" will be provided by the mass of the tether deployer and winch, the facility power supply and power processing hardware, and the mass of the tether itself. If additional mass is required, it could be provided by waste material in LEO, such as spent upper stage rockets and shuttle external tanks.

The tether mass will depend upon the maximum tip velocity and the choices of tether material and design safety factor, as described by Eqn. 1. For a tapered tether, the tether's center-of-mass will be closer to the facility end of the tether. This can be an important factor when the tether mass is significant compared to the payload and facility masses. In the calculations below, we have used a model of a tether tapered in a stepwise manner to calculate tether masses and the tether center-of-mass.

By conservation of momentum, the perigee velocity of the center of mass of the tether and payload after rendezvous is:

$$V_{p,1} = \frac{V_{p,0}(M_f + M_t) + V_{IPO}M_p}{(M_f + M_t) + M_p} \quad (10)$$

When the tether catches the payload, the center-of-mass of the tether system shifts downward slightly as the payload mass is added at the bottom of the tether:

$$r_{p,1} = \frac{r_{p,0}(M_f + M_t) + V_{IPO}M_p}{(M_f + M_t) + M_p} \quad (11)$$

In addition, when the tether catches the payload, the angular velocity of the tether does not change, but because the center-of-mass shifts closer to the tip of the tether when the tether catches the payload, the tether tip velocity decreases. The new tether tip velocity can be calculated as

$$V'_t = V_t \frac{(L - l_{cm,loaded})}{(L - l_{cm,unloaded})} \quad (12)$$

At this point, it is possible to specify the initial payload orbit r_{IPO} , the payload/facility mass ratio χ , the facility/payload period ratio N , and the desired LTO C_3 , and derive a system of equations from which one particular tether length and one tether tip velocity can be calculated that determine an "exact" system where the tether tip velocity need not be adjusted to provide the desired C_3 of the payload lunar trajectory. However, the resulting system design is rather restrictive, working optimally for only one

particular value of the facility and tether masses, and results in rather short tether lengths that will require very high tip acceleration levels. Fortunately, we can provide an additional flexibility to the system design by allowing the tether facility to adjust the tip velocity slightly by reeling the tether in or out a few percent. If, after catching the payload, the facility reels the tether in by an amount ΔL , the tip velocity will increase due to conservation of angular momentum:

$$V_t'' = \frac{V_t'(L - l_{cm,loaded})}{(L - l_{cm,loaded}) - \Delta L} \quad (13)$$

Then, when the facility returns to perigee, it can throw the payload into a lunar transfer trajectory with perigee characteristics:

$$r_{p,LTO} = r_{p,1} + (L - l_{cm,loaded}) - \Delta L \quad V_{p,LTO} = V_{p,1} + V_t' \quad (14)$$

Using the equations above, standard Keplerian orbital equations, and equations describing the shift in the system's center-of-mass as the payload is caught and released, we have calculated a design for a single-tether system capable of transferring picking up payloads from a circular LEO orbit and throwing them to a minimal-energy lunar trajectory. During its initial period of operation, while a lunar facility is under construction and no return traffic exists, the tether system will use electrodynamic tether propulsion to reboost itself after throwing each payload. Once a lunar facility exists and return traffic can be used to conserve the facility's orbital momentum, the orbit of the tether will be modified slightly to permit round trip traffic. The orbital design is illustrated in Figure 3, and the system parameters are listed below.

Initial System Design: Outbound Traffic Only

Payload:

- mass $M_p = 2500 \text{ kg}$
- altitude $h_{IPO} = 308 \text{ km}$
- velocity $V_{IPO} = 7.72 \text{ km/s}$

Tether Facility:

- tether length $L = 80 \text{ km}$
- tether mass $M_t = 15,000 \text{ kg}$ (Spectra 2000 fiber, safety factor of 3.5)
- tether center-of-mass $L_{t,com} = 17.6 \text{ km from facility}$
- central facility mass $M_f = 11,000 \text{ kg}$
- grapple mass $M_g = 250 \text{ kg}$ (10% of payload mass)
- total system mass $M = 26,250 \text{ kg}$
 $= 10.5 \times \text{payload mass}$
- facility power $P_{wr} = 11 \text{ kW avg}$ (with storage to provide 75 kW during perigee)
- initial tether tip velocity: $V_{t,0} = 1530 \text{ m/s}$
- High Energy [Pre-Catch] Orbit:
 - perigee altitude $h_{p,0} = 378 \text{ km}$,
 - apogee altitude $h_{a,0} = 11,498 \text{ km}$
 - eccentricity $e_0 = 0.451$
 - period $P_0 = 5/2 P_{IPO}$ (rendezvous opportunity every 7.55 hrs)
- rendezvous acceleration $g_{tip} = 3.36 \text{ gees}$
- post-catch orbit (COM):
 - perigee altitude $h_{p,1} = 371 \text{ km}$,
 - apogee altitude $h_{a,1} = 9687 \text{ km}$
 - eccentricity $e_1 = 0.408$
- after catching the payload, the facility reels in 2950 m of tether, increasing the tip velocity to 1607 m/s,

- Low Energy [post-throw] orbit:
 perigee altitude $h_{p,2} = 365 \text{ km}$,
 apogee altitude $h_{a,2} = 7941 \text{ km}$
 eccentricity $e_2 = 0.36$

Lunar Transfer Trajectory:

- perigee altitude $h_{p,lto} = 438.7 \text{ km}$
- perigee velocity $V_{p,lto} = 10.73 \text{ km/s}$
- trajectory energy parameter $C_3 = -1.9 \text{ km}^2/\text{s}^2$

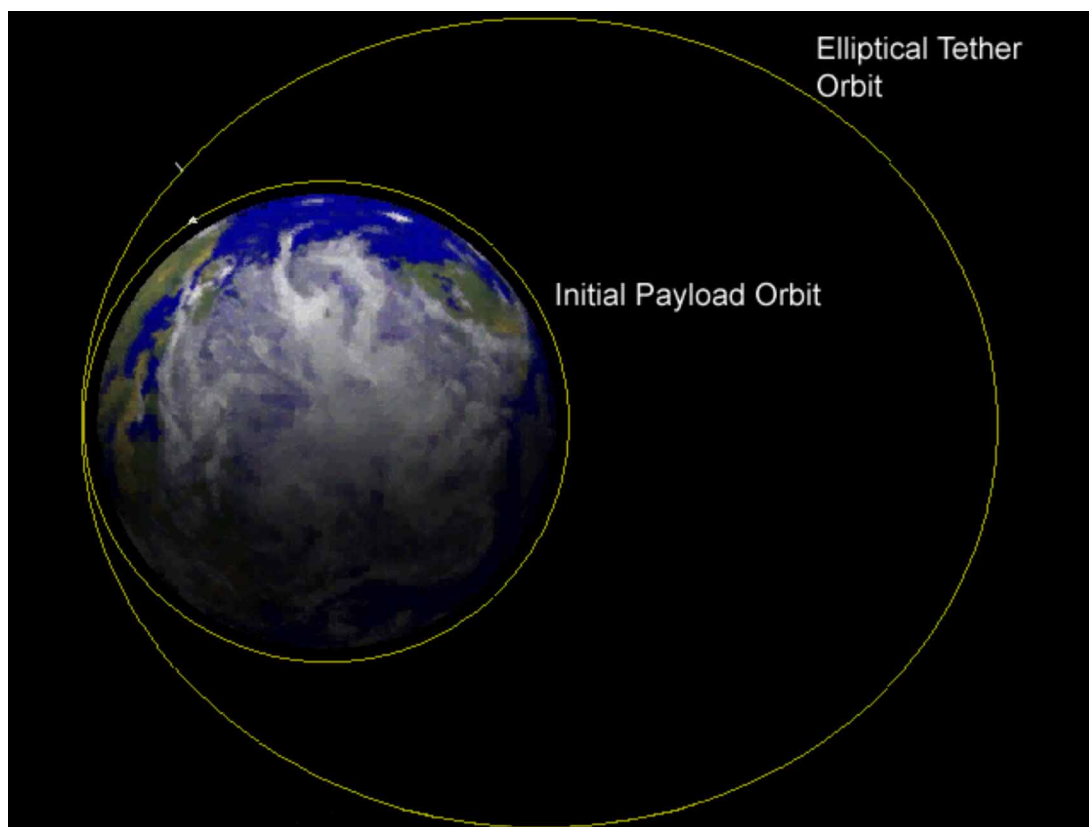


Figure 3. The circular initial payload orbit and the initial tether orbit, shown to scale.

Note that for a particular system design, the tether and facility mass will scale roughly linearly with the payload mass, so an equivalent system designed for sending 250 kg payloads to the moon could be constructed with a tether mass of 1,500 kg and a facility mass of 1,100 kg. Note also that the tether mass is not dependent upon the tether length, so longer tethers can be used to provide lower tip acceleration levels with no mass penalty.

Apsidal Precession

As noted earlier, the oblateness of the Earth will cause the line of apsides of the tether facility's elliptical orbit to precess. In the Cislunar Tether Transport System, we can deal with this issue in two ways. First, we can utilize tether reeling maneuvers to counteract the apsidal precession; this technique is described in more detail in Appendix F. Second, we can deal with apsidal precession by choosing the tether orbits such that their precession rates are nearly harmonic with the Moon's orbital rate, so that the line of apsides lines up with the Moon's nodes once every several months. Furthermore, we can use propellantless electrodynamic tether propulsion to "fine-tune" the precession rate, either by raising/lowering the orbit or by generating thrust perpendicular to the facility's velocity.

In the design given above, the mass and initial orbit of the tether facility was chosen such that after throwing a payload to the moon, the tether enters a lower energy elliptical orbit which will precess at a rate of 2.28 degrees per day. The initial, high-energy orbit has a slower precession rate of approximately 1.58 degrees per day. These orbits were chosen so that in the 95.6 days it takes the Moon to orbit 3.5 times around the Earth, the tether facility can reboost itself from its low-energy orbit to its high-energy orbit using propellantless electrodynamic propulsion, and, by properly varying the reboost rate, the apsidal precession can be adjusted so that the line of apsides will rotate exactly 180° , and the tether orbit will be lined up properly to catch and throw another payload to the moon. The orientation and precession of the orbits is illustrated in Figure 4.

System Design for Round-Trip Traffic

Once a lunar base is established and begins to send payloads back down to LEO, the orbit of the tether system can be modified slightly to enable frequent opportunities for round-trip travel. First, the facility's orbit will be raised so that its high-energy orbit has a semimajor axis of 12577.572 km, and an eccentricity of 0.41515. The tether will then pick up a payload from a circular, 450 km orbit and toss it to the moon so that it will reach the moon as the moon crosses its ascending node. The facility will then drop to a lower energy orbit. At approximately the same time, the return payload will be released by the lunar tether and begin its trajectory down to LEO. When the return payload reaches LEO, the Earth-orbit tether facility will catch it at perigee, carry it for one orbit, and then place it into the 450 km initial payload orbit. Upon dropping the return payload, the facility will place itself back into the high-energy orbit. The perigee of this orbit will precess at a rate such that after 4.5 lunar months (123 days) it will have rotated

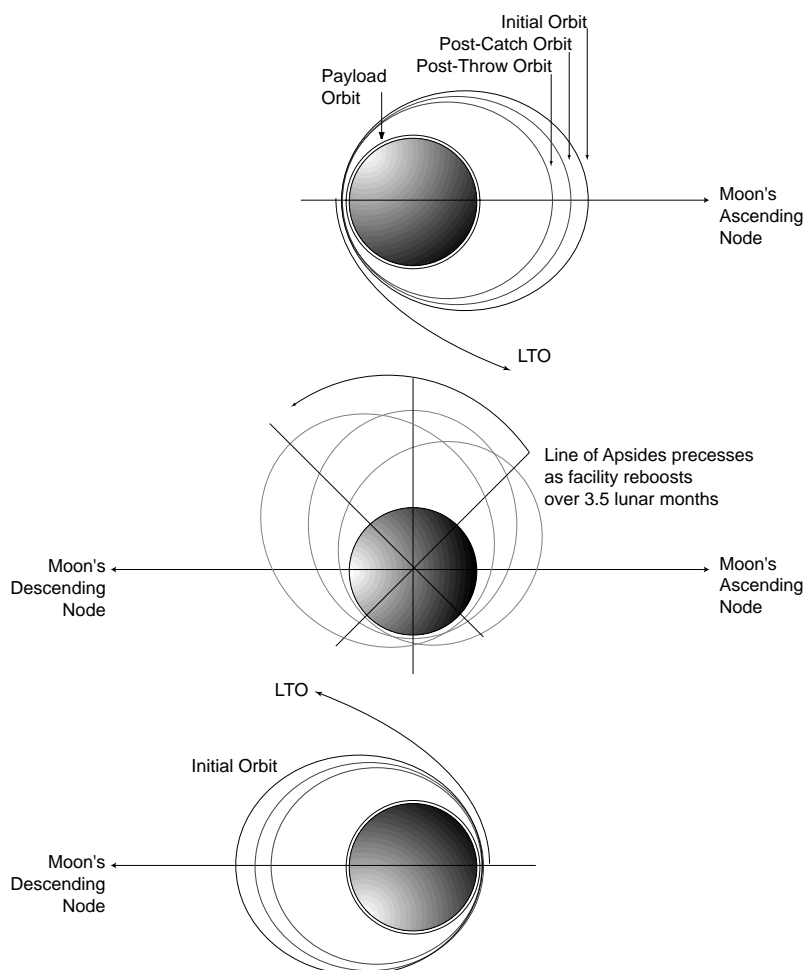


Figure 4. Schematic of the evolution of the orbits of the HEFT facility. Orbits are shown to scale.

180°, and the system will be ready to perform another payload exchange, this time as the moon crosses its descending node.

If more frequent round-trip traffic is desired, additional tether facilities can be placed into similar orbits that are rotated at intervals of 40° from the orbit of the original tether. Up to nine tethers could be put into place to accommodate round-trip travel every half lunar month (13.66 days).

HEFT System Reboost

After boosting the payload, the HEFT facility will be left in a lower energy elliptical orbit with a semimajor axis that is approximately 1780 km less than its original orbit. It can then use electrodynamic propulsion to reboost its apogee by driving current through the tether when the tether is near perigee. Because the tether is rotating, the direction of the current must be alternated as the tether rotates to produce a net thrust on the facility. Modeling of reboost of HEFT tether systems indicate that the system could reboost its semimajor axis at a rate of 50 km·mt / day·kW. Thus if the 26.25 mt facility has an 11 kWe power supply, it can reboost its orbit within about 85 days. Higher power levels would provide faster reboost.

Summary:

Our analyses have concluded that the optimum architecture for a tether system designed to transfer payloads from LEO to lunar trajectories will utilize one tether facility in an elliptical orbit. The system

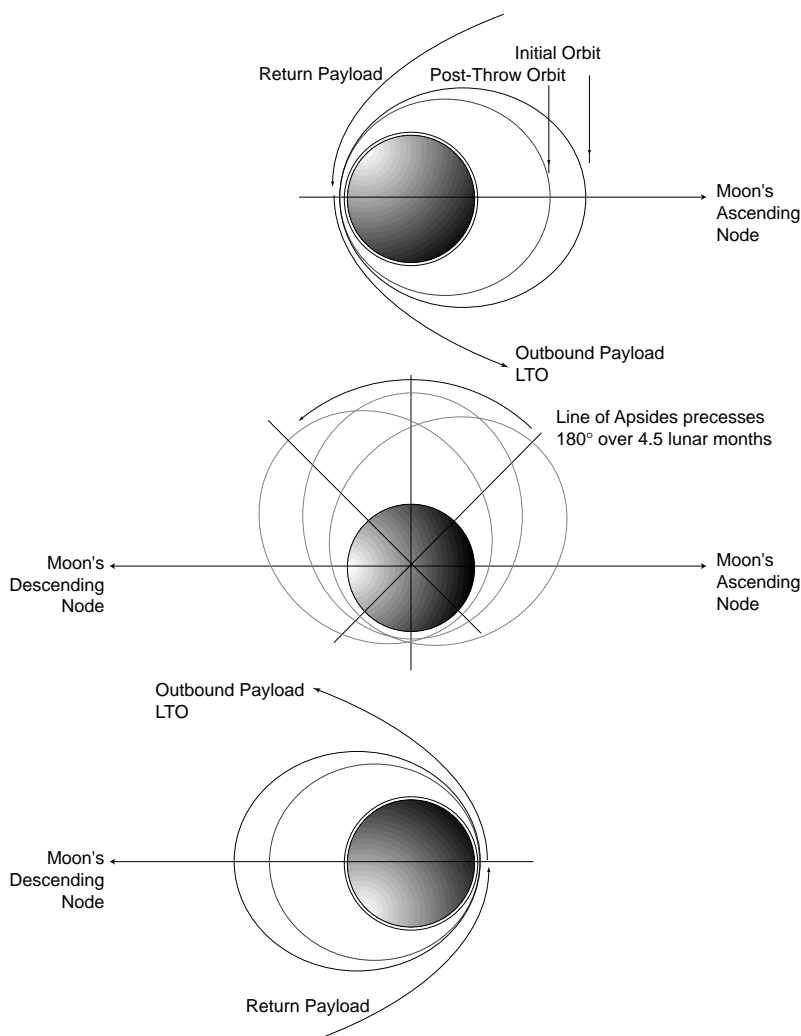


Figure 5. Schematic of orbit evolution for the Round-Trip Cislunar Transport System.

described above, composed of a single HEFT tether facility massing 26 metric tons and having a power supply of 11 kW, will be able to throw a 2,500 kg payload to the moon once every 3.5 synodic months, with no propellant expenditure and no return traffic required. Once a lunar base is established and return traffic begins, a slight modification of the facility's orbit enables the tether system to exchange payloads with a lunar tether once every 123 days. If more frequent exchanges are desired, up to nine identical tether facilities can be fielded to provide round-trip travel between LEO and the lunar surface roughly once every two weeks.

References

1. Hoyt, R.P., Forward, R.L., "Tether System for Exchanging Payloads Between Low-Earth-Orbit and the Lunar Surface", AIAA Paper 97-2794.

LUNAVATOR TETHER AND ORBITAL DESIGN FOR THE CISLUNAR TRANSPORT SYSTEM

Robert P. Hoyt
Tethers Unlimited, Inc.

Abstract

In this section we develop a design for a tether system capable of capturing payloads sent from the Earth to the Moon on minimal-energy trajectories and transferring them to the lunar surface. The challenge addressed is the need to enable a low-lunar-orbit tether facility that has an orbital velocity of 1.6 km/s to catch a payload from a hyperbolic lunar trajectory with a perigee velocity of 2.3 km/s (catch ΔV of ~ 0.7 km/s) and then deposit the payload on the surface of the moon with zero relative velocity (drop ΔV of 1.6 km/s). To enable this maneuver, we have invented a tether system in which the tether ballast mass is divided between a counterbalance at one end of the tether and a central facility that can adjust its position along the tether. Using this method, we have designed a Lunavator system massing under 42 tons that can exchange 2500 kg payloads between low-energy lunar transfer orbits and the lunar surface. This facility can be sent to the moon with a relatively low initial mass and build up its "ballast mass", and thus its payload capacity, by picking up lunar materials. Perturbations of the Lunavator's orbit can be stabilized using modest tether reeling operations.

Background*1978 Moravec "Lunavator"*

In 1978, Moravec proposed that it would be possible to use existing material such as Kevlar to construct a tether rotating around the Moon that would periodically touch down on the lunar surface.^{1,2} The lunar rotovator, or "Lunavator," that Moravec proposed is illustrated in Figure 1. In Moravec's design, two long tapered tethers would be extended from a massive central facility in orbit around the

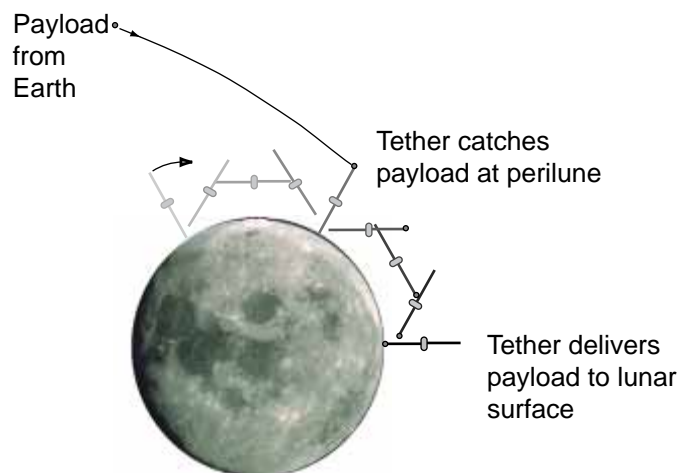


Figure 1. Time-lapse schematic of the double-arm Moravec Lunavator capturing a payload from Earth and depositing it on the lunar surface.

moon. The Lunavator would rotate in the same direction as its orbit with a tether tip velocity equal to the orbital velocity of the tether's center-of-mass. If the length of each tether arm were equal to the altitude of the central facility, the tether tips would periodically touch down on the moon with zero velocity relative to the surface (to visualize this, imagine the tether as a spoke on a giant bicycle wheel rolling around the Moon). Moravec found that the mass of the tether would be minimized if the tether had an arm length equal to one-sixth of the diameter of the Moon, rotating such that each of the two arms touched down on the surface of the moon three times per orbit.

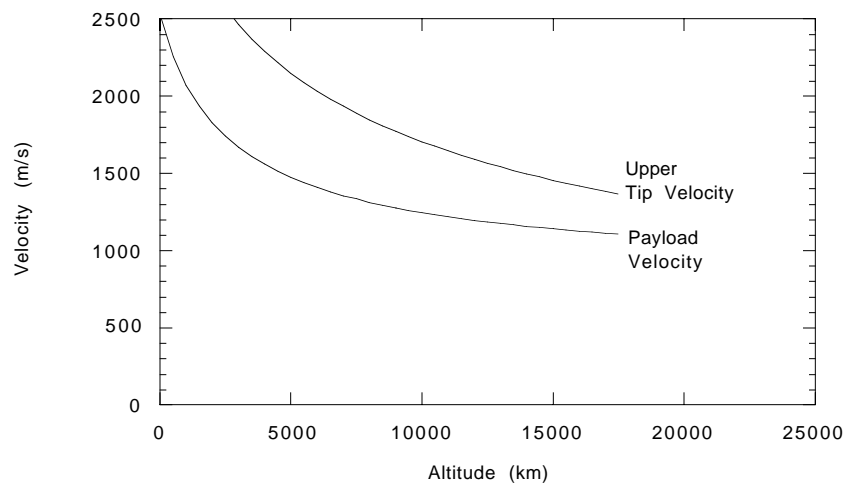


Figure 2. Comparison of payload velocity versus perilune altitude for a minimum-energy-LTO and the total velocity of the upper tip of a Moravec Lunavator.

As it rotates and orbits around the moon, the Lunavator could capture payloads from Earth as they passed perilune and then set them down on the surface of the moon. The Lunavator could pick up payloads to be returned to Earth at the same time, and then throw them on an Earth-return trajectory at some later time.

Using data for the best material available in 1978, Kevlar, which has a density of 1.44 g/cc and a tensile strength of 2.8 GPa, Moravec found that a two-arm Lunavator with a design safety factor of $F=2$ would have to mass approximately 13 times the payload mass. Each arm of the tether would be 580 km long, for a total length of 1160 km, and the tether center-of-mass would orbit the Moon every 2.78 hours in a circular orbit with radius of 2,320 km. At that radius, the orbital velocity is 1.45 km/s, and so the tether would rotate with a tip velocity of 1.45 km/s.

1996 LEO-Lunar Study Lunavator Design

In our 1996 study of a LEO-Lunar Tether Transport System, we used a Lunavator design very similar to that proposed by Moravec to catch payloads sent from the tethers in Earth orbit and deliver them to the lunar surface, with the only significant change being the choice of using only one tether arm to minimize the system mass.³ Using only one tether arm, and using modern tether materials such as Spectra or PBO, we found that the tether mass could be reduced to as little as 3 times the payload mass. A facility mass of >20 times the payload mass would be necessary to keep the tether from escaping from lunar orbit after catching the high-velocity payload.

Using Moravec's minimal-mass solution, however, requires not only a very long tether but also requires that the payload have a very high velocity relative to the moon at its perilune. Because the Lunavator in Moravec's design has an orbital velocity of 1.45 km/s and the tether tips have a velocity of 1.45 km/s relative to the center-of-mass, the payload's perilune velocity would need to be 2.9 km/s in order to match up with the tether tip at the top of their rotation. In order to achieve this high perilune velocity, the outbound lunar transfer trajectory would have to be hyperbolic rather than elliptical. This presented several drawbacks, the most significant being that if the Lunavator failed to capture the payload at perilune, it would continue on and leave Earth orbit on a hyperbolic trajectory. This high lunar trajectory energy also placed extra ΔV demands on the Earth-orbit tethers, driving us to use a complex two-tether system in Earth orbit to keep the system mass reasonable.

Design of a Lunavator Compatible with Minimal-Energy Lunar Transfers

In order to minimize the ΔV requirements placed upon the Earth-orbit portion of the Cislunar Tether Transport System and thereby permit the use of a single Earth-orbit tether with a reasonable mass, we have developed a method for a single lunar-orbit tether to capture a payload from a minimal-

energy lunar transfer orbit and deposit it on the tether surface with zero velocity relative to the surface.

Moon-Relative Energy of a Minimum-Energy LTO

A payload that starts out in LEO and is injected into an elliptical, equatorial Earth-orbit with an apogee that just reaches the Moon's orbital radius would, in the absence of lunar gravity, have a velocity at apogee of approximately 190 km/s. The Moon orbits the Earth with an average velocity of 1.02 km/s, with an inclination to the Earth's equatorial plane that varies between 18-28°, with an average of about 23°. Through simple vector addition, we estimate that a payload in a minimum-energy LTO would have a C_3 relative to the moon of approximately $0.72 \text{ km}^2/\text{s}^2$. Figure 2 shows the perilune velocity as a function of perilune altitude for such a moon-relative C_3 compared to the total velocity of the tip of a Moravec Lunavator designed to capture a payload at the specified altitude. The figure shows that for "reasonable" (i.e.: several-hundred km) tether lengths, the upper tip of the Lunavator would be traveling almost 1 km/s faster than the payload at perilune, and even for extremely long tether lengths, the tether tip would still be travelling several hundred m/s too fast to rendezvous with the payload.

Consequently, the design of the lunar tether system must be modified to permit a tether orbiting the moon at approximately 1.5 km/s to catch a payload to at perilune when the payload's velocity is approximately 2.3 km/s, then increase both the tether length and the angular velocity so that the payload can be set down on the surface of the moon with zero velocity relative to the surface. Simply reeling the tether in or out from a central facility will not suffice, because reeling out the tether will cause the rotation rate to decrease due to conservation of angular momentum.

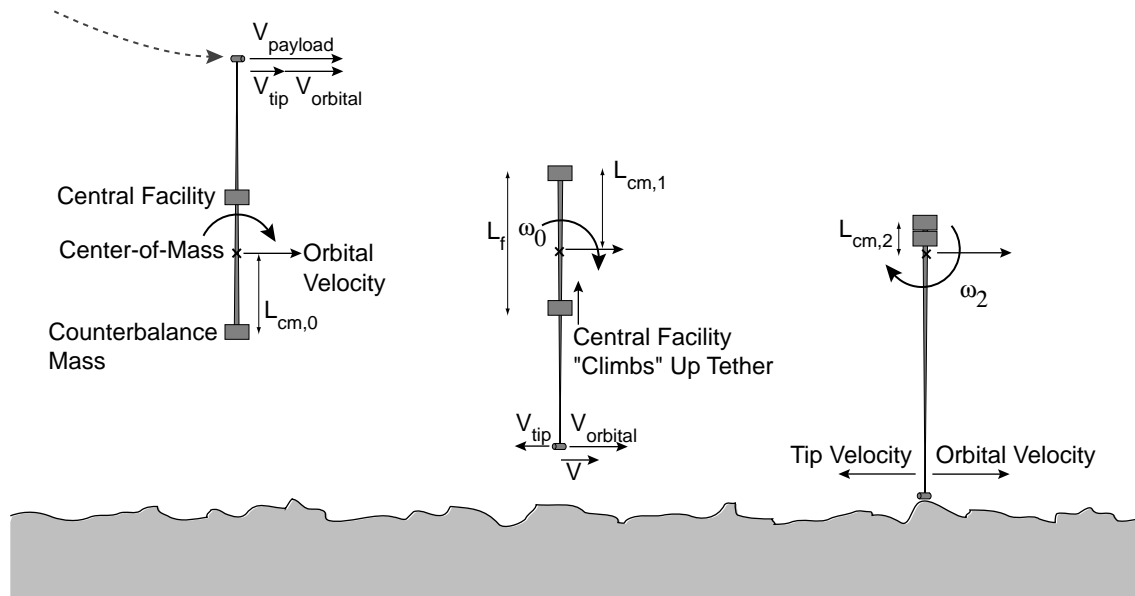


Figure 3. Method for a Lunavator to capture a payload from a minimal-energy LTO and deposit it on the lunar surface.

A method that can enable the tether to catch a payload and then increase the tether rotation rate while lowering the payload is illustrated in Figure 3. The tether system is composed of a long tether, a counterbalance mass at one end, and a central facility that has the capability to climb up or down the tether. Initially, the facility would locate itself near the center of the tether, and the system would rotate slowly around the center-of-mass of the system, which would be located roughly halfway between the facility and the counterbalance mass. The facility could then capture an inbound payload at its perilune. The facility would then use energy from solar cells or other power supply to "climb" up the tether towards the counterbalance mass. The center-of-mass of the system will remain at the same altitude, but the distance from the tether tip to the center-of-mass will increase, and conservation of

angular momentum will cause the angular velocity of the system to increase as the facility mass moves closer to the center-of-mass.

Analysis

A first-order design for the Lunavator can be obtained by calculating the shift in the system's center-of-mass as the central facility changes its position along the tether. We begin by specifying the payload mass M_p , the counterbalance mass M_c , the facility mass M_f , and the tether length L_t . The required tether mass cannot be calculated simply by using Moravec's tapered tether mass equation, because that equation was derived for a free-space tether. The Lunavator must support not only the forces due to centripetal acceleration of the payload and tether masses, but also the tidal forces due to the moon's gravity. The equations for the tether mass with gravity-gradient forces included are not analytically integrable, so the tether mass M_t must be calculated numerically.

Prior to capture of the payload, the distance from the counterbalance mass to the center-of-mass of the tether system is

$$L_{cm,0} = \frac{M_f L_f + M_t L_{cm,t}}{M_c + M_f + M_t}, \quad (1)$$

where L_f is the distance from the counterbalance to the facility and $L_{cm,t}$ is the distance from the counterbalance to the center-of-mass of the tether. $L_{cm,t}$ must be calculated numerically for a tapered tether.

If the Lunavator is initially in a circular orbit with altitude h_0 and semimajor axis $a_0 = R_m + h_0$, it will have a center-of-mass velocity of

$$v_{cm,0} = \sqrt{\frac{\mu_m}{a_0}}. \quad (2)$$

At the top of the tether swing, it can capture a payload from a perilune radius of

$$r_p = a_0 + (L_t - L_{cm,0}). \quad (3)$$

A payload sent from Earth on a near-minimum energy transfer will have a $C_{3,m}$ of approximately $0.72 \text{ km}^2/\text{s}^2$. Its perilune velocity will thus be

$$v_p = \sqrt{\frac{2\mu_m}{a_0 + (L_t - L_{cm,0})} + C_{3,m}}. \quad (4)$$

In order for the tether tip's total velocity to match the payload velocity at rendezvous, the velocity of the tether tip relative to the center of mass must be

$$v_{t,0} = v_p - v_{cm,0}, \quad (5)$$

and the angular velocity of the tether system will be

$$\omega_{t,0} = \frac{v_{t,0}}{L_t - L_{cm,0}}. \quad (6)$$

When the tether captures the payload, the center of mass of the new system, including the payload, is at perigee of a new, slightly elliptical orbit, as illustrated in Figure 4 (it was in a circular orbit and caught a payload going faster than the center-of-mass). The perigee radius and velocity of the center-of-mass are

$$v_{perigee,1} = \frac{v_{cm,0}(M_c + M_f + M_t) + v_p M_p}{M_c + M_f + M + M_p} \quad r_{perigee,1} = \frac{a_0(M_c + M_f + M_t) + r_p M_p}{M_c + M_f + M + M_p}, \quad (7)$$

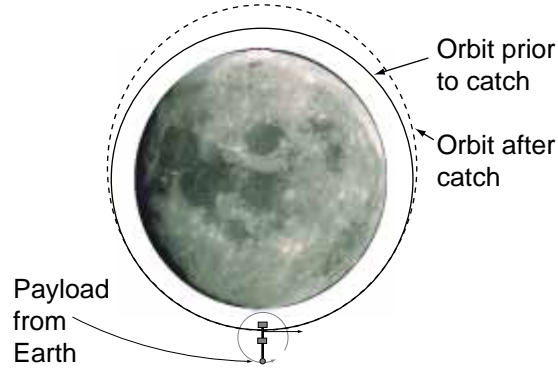


Figure 4. Lunavator orbits before and after payload capture. After capture, the Lunavator adjusts its rotation rate and delivers the payload to the surface when it returns to the perilune of its new orbit

and the new distance from the counterbalance mass to the system's center-of-mass of the system changes to

$$L_{cm,1} = \frac{M_f L_f + M_t L_{cm,t} + M_p L_t}{M_c + M_f + M_t + M_p}. \quad (8)$$

To increase the rotation rate of the tether system and increase the distance from the system's center of mass to the tether tip, the facility climbs up the tether to the counterbalance mass, reducing the distance from the counterbalance to the center-of-mass to

$$L_{cm,2} = \frac{M_t L_{cm,t} + M_p L_t}{M_c + M_f + M_t + M_p}. \quad (9)$$

By conservation of angular momentum, the angular velocity will increase to a new value of

$$\omega_2 = \omega_0 \frac{L_{cm,1} M_c + (L_f - L_{cm,1}) M_f + (L_{cm,t} - L_{cm,1}) M_t + (L_t - L_{cm,1}) M_p}{L_{cm,2} M_f + (L_{cm,t} - L_{cm,2}) M_t + (L_t - L_{cm,2}) M_p}, \quad (10)$$

and the payload will then have a velocity relative to the center-of-mass of

$$v_{t,2} = \omega_2 (L_t - L_{cm,2}). \quad (11)$$

If the initial orbit parameters, tether lengths, and facility and tether masses are chosen properly, then $v_{t,2}$ can be made equal to the perigee velocity of the tether system and the distance from the center of mass to the payload can be made equal to the perigee altitude. When the tether returns to its perigee it can then deposit the payload on the surface of the moon and simultaneously pick up a payload to be thrown back to Earth.

Using Lunar Material to Build Up the Lunavator's Payload Capacity

This design for the Lunavator enables the tether system to build up its ballast mass using material it picks up from the lunar surface, without requiring the use of any propellant. The system can be launched from Earth with low facility and counterbalance masses; for example, the tether might initially be launched to the moon with only a low-mass central facility containing a power supply and a mechanism for moving the facility along the tether, and the counterbalance mass could be provided by the upper stage of the launch vehicle used to send the tether to the moon. The system would thus start out with a relatively low payload capacity. Payloads would be sent to the Lunavator from the Earth-orbit tether, and the Lunavator would deposit them on the lunar surface. In catching and delivering the payload sent from Earth, the energy of the Lunavator's orbit will be increased, as illustrated in Figure 5. The Lunavator would then pick up an equal mass from the lunar surface. Rather than throwing the mass back to Earth, the central facility would translate down to the tip of the

tether and retrieve the mass, then translate up the tether and transfer half of the mass to the counterbalance. It would then repeat this maneuver, picking up a roughly equal mass and distributing it between the central facility and the counterbalance. After this second operation, the Lunavator's orbit will return to the original circular orbit. Thus, for every payload sent from the Earth, the Lunavator could increase its total ballast mass by twice the payload mass, which in turn will increase the payload capacity of the Lunavator.

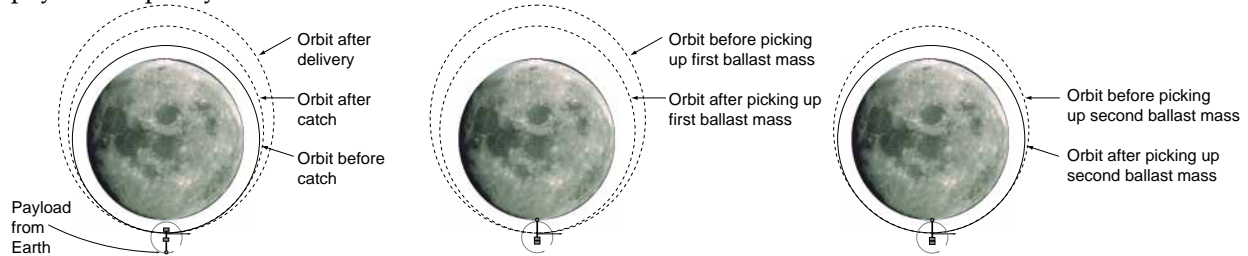


Figure 5. Method for building up ballast mass of the Lunavator using lunar resources in order to increase the Lunavator's payload capacity.

Example

Using the equations given above, we have found the following first-order design for a Lunavator capable of catching payloads from minimal-energy lunar transfer orbits and depositing them on the surface of the moon:

Payload Trajectory:

- mass $M_p = 2500 \text{ kg}$
- perigee altitude $h_p = 328.23 \text{ km}$
- Moon-relative energy $C_{3,M} = 0.719 \text{ km}^2/\text{s}^2$

Lunavator:

- tether length $L = 200 \text{ km}$
- initial central facility position $L_f = 155 \text{ km}$
- counterbalance mass $M_c = 15,000 \text{ kg}$
- facility mass $M_f = 15,000 \text{ kg}$
- tether mass $M_t = 11,765 \text{ kg}$ ($=4.7M_p$, Spectra fiber, safety factor of 3.5)
- initial tether tip velocity $V_{t,0} = 0.748 \text{ km/s}$
- initial rotation rate $\omega_0 = 0.00566 \text{ rad/s}$
- initial orbit:
 - center-of-mass altitude $h_{p,0} = 170.5 \text{ km}$
 - altitude of tether tip at the bottom of its rotation: 38.5 km
- post-catch orbit (COM):
 - perigee altitude $h_{p,0} = 178 \text{ km}$,
 - apogee altitude $h_{a,0} = 411.8 \text{ km}$
 - eccentricity $e_0 = 0.0575$

After catching the payload, the central facility climbs up the tether to the counterbalance mass, changing the rotation rate to:

- adjusted rotation rate $\omega_0 = 0.00929 \text{ rad/s}$
- adjusted tip velocity $V_{t,2} = 1.645 \text{ km/s}$

Payload Delivery:

- drop-off altitude $h = 1 \text{ km}$ (top of a lunar mountain)
- velocity relative to surface $v = 0 \text{ m/s}$

Maintenance of the Lunar Orbit Using Tether Reeling

In order to enable the Lunavator to service lunar bases anywhere on the surface of the moon, and in particular ice-mining facilities at the poles, it is desirable to place the Lunavator in a polar orbit around the moon. Polar lunar orbits, however, are notoriously unstable, in part due to orbital perturbations caused by the gravitational fields of the Earth and the Sun, and in part due to the nonspherical components of the lunar gravitational potential. Early results from the Lunar Prospector mission indicate that low lunar orbits will require up to 200 m/s of ΔV per year to maintain the orbit. Fortunately, the techniques of orbital propulsion using tether reeling, pioneered by Landis,⁴ Martínez-Sánchez, and Gavit,⁵ provide a means of stabilizing the Lunavator's orbit that does not require propellant expenditure. Tether reeling can add or remove energy from a tether's orbit by working against the non-linearity of a gravitational field. The basic concept of orbital modification using tether reeling is illustrated in Figure 6. When a tether is near the apoapsis of its orbit, the tidal forces on the tether are low. When it is near periapsis, the tidal forces on the tether are high. If it is desired to reduce the eccentricity of the tether's orbit, then the tether can be reeled in when it is near apoapsis, under low tension, and then allowed to unreel under higher tension when it is at periapsis. Since the tidal forces that cause the tether tension are, to first order, proportional to the inverse radial distance cubed, more energy is dissipated as the tether is unreeled at periapsis than is restored to the tether's orbit when it is reeled back in at apoapsis. Thus, energy is removed from the orbit. Conversely, energy can be added to the orbit by reeling in at periapsis and reeling out at apoapsis.

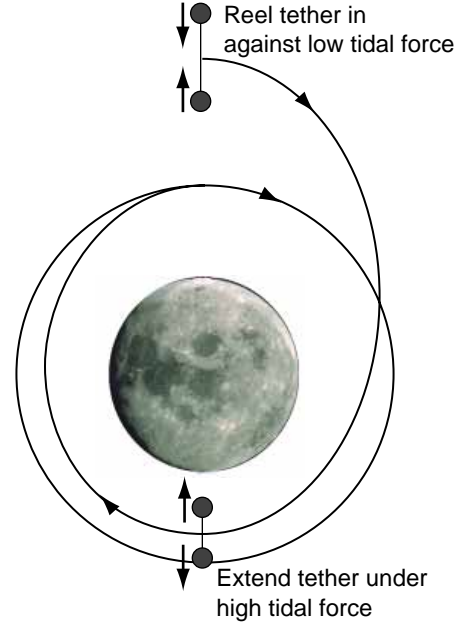


Figure 6. Schematic of tether reeling maneuver to reduce orbital eccentricity.

Although energy is removed (or added) to the orbit by the reeling maneuvers, the orbital angular momentum of the orbit does not change. Thus the eccentricity of the orbit can be changed.

Hanging Tether Analysis

Landis has developed equations for estimating the rate of eccentricity change that can be achieved by a hanging tether system.⁴ If two objects of mass m are placed at the ends of a massless tether of total length L , and the tether length is varied by a length ΔL , then the rate of change of eccentricity is

$$\frac{d}{dt}e = \frac{3}{8P_0} \frac{[L^2 - (L - \Delta L)^2]}{a_0^2} (6 + e^2)(1 - e^2)^{\frac{3}{2}}, \quad (12)$$

and the average rate at which energy is subtracted (or added) to the orbit is

$$\frac{dE}{dt} = \frac{3}{8P_0} (2m) \frac{\mu}{a_0^3} [L^2 - (L - \Delta L)^2] e (6 + e^2)(1 - e^2)^{\frac{3}{2}}, \quad (13)$$

where P_0 is the period of the circular orbit, a_0 is the semimajor axis of the circular orbit, and μ is the gravitational coefficient ($\mu = GM$) for the planet or moon that the tether is orbiting. Eqn. (13) shows that as the orbit becomes nearly circular, when e is very close to 0, the rate of energy transfer approaches zero. However, because eccentricity is highly sensitive to changes in the energy when e is near 0, tether pumping can still be quite effective.

In our baseline Lunavator design the counterbalance mass and the central facility mass are separated by a length L of 155 km. If the central facility climbs up and down the tether a distance $\Delta L = 1$ km each orbit (requiring a reeling rate of approximately 0.25 m/s), then the eccentricity of the

Lunavator's orbit can be changed by 0.0022/day. This corresponds to a periapse altitude change of over 4 km/day, or a ΔV of approximately 3.5 m/s per day.

Rotating Tether Analysis

The preceding analysis assumed that the tether is hanging (rotating once per orbit). The Lunavator, however, will be rotating faster than once per orbit, and thus the analysis of eccentricity damping by tether reeling must be extended to the case of a rotating tether.

As Landis has pointed out, a rotating tether system can take advantage of the fact that a non-vertical tether experiences forces on its center of mass that can produce net forces on the center of mass in both the radial and azimuthal directions, as illustrated in Figure 7.⁴ If no reeling is performed on the tether, this side force will average out to zero as the tether makes a complete rotation. If, however, the tether is retracted during a portion of its rotation and extended during the other portion of its rotation, the net force can be non-zero and energy can be either added to or subtracted from the tether's orbit.

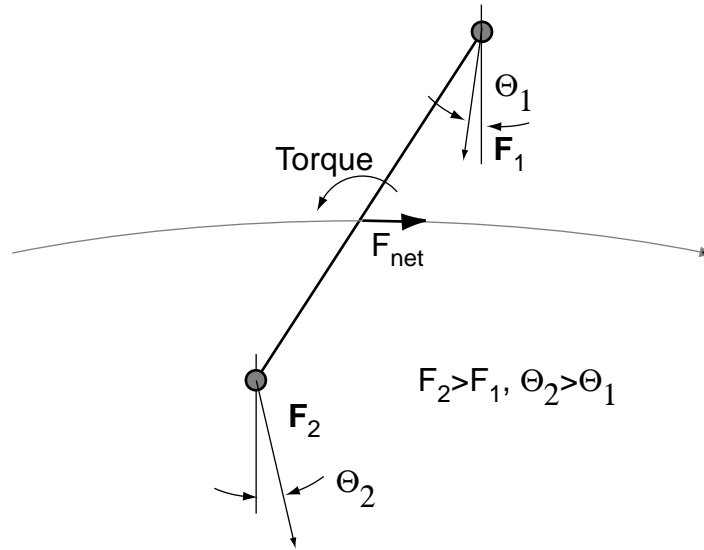


Figure 7. Forces on a non-vertical tether. Because $F_2 > F_1$ and $\Theta_2 > \Theta_1$, the tether experiences a net force in the plane of rotation.

For our analysis, we will assume that the tethered system consists of a massless tether of average length L connecting two masses, m_1 and m_2 . Martínez-Sánchez and Gavit have found that the forces on the tether system's center of mass are given by⁵

$$\begin{aligned} dF_r &= -\frac{3\mu}{r_G^2} m_{12} \left(\frac{L}{r_G} \right)^2 \left(1 - \frac{3}{2} \sin^2 \alpha \right) \\ dF_\theta &= +\frac{3\mu}{r_G^2} m_{12} \left(\frac{L}{r_G} \right)^2 \sin \alpha \cos \alpha \end{aligned} \quad (14)$$

where $r_G = p/(1 + e \cos \theta)$, $m_{12} = (m_1 + m_2)/m_1 m_2$ is the reduced mass of the system, α is the angle of tether rotation away from vertical, and μ is the gravitational coefficient (GM) of the planet or moon that the tether is orbiting.

The rate of change of the eccentricity of a satellite's orbit is

$$\dot{e} = \frac{p}{h} \left\{ a_r \sin \theta + a_\theta \left[\cos \theta + \frac{r_G}{p} (e + \cos \theta) \right] \right\}, \quad (15)$$

where $p = a(1-e^2)$ is the orbit's semiparameter, h is the orbital angular momentum of the satellite, a is the semimajor axis, θ is the angular position of the satellite in its orbit, measured from its periapse, and a_r and a_θ are the instantaneous acceleration of the satellite in the radial and azimuthal directions.

In the case of the Lunavator, we are interested in using tether pumping to maintain the circularity of the tether's orbit. Thus we can simplify Eqns. (14) and (15) by assuming that the eccentricity e is held essentially zero, so that $r_G=p=a$, and $\theta=\omega_{orb}t$. For simplicity, we will also assume that the tether length is varied by an amount ΔL that is small compared to the nominal length L so that the tether rotation rate ω_T is not significantly affected by the tether reeling operations. By dividing Eqns. (14) by the total mass of the tether system to obtain the accelerations and then inserting them into Eqn. (15), we find the rate of eccentricity change to be

$$\dot{e} = \frac{3\sqrt{\mu}}{a^{7/2}} \left[\frac{m_{12}}{m} \right] \left\{ 2 \cos(\omega_{orb}t) \sin(\omega_T t) \cos(\omega_T t) - \sin(\omega_{orb}t) \left(1 - \frac{3}{2} \sin^2(\omega_T t) \right) \right\} [L + \Delta L(t)]^2. \quad (16)$$

For the baseline Lunavator design, the tether orbits at an altitude of 170.5 km, and the tether has two equal masses (the central facility and the counterbalance mass) separated by 155 km of tether. For simplicity in our calculations, we will assume that the tether rotates an integral number of times per orbit; in the baseline design, the tether rotates approximately 6 times per orbit, so $\omega_T = 6\omega_{orb}$.

Using the orbital parameters for the baseline Lunavator design, the function in the brackets of Eqn. (16) is plotted in Figure 8. If the tether length L is held constant, then over an orbit the eccentricity change given by Eqn. (16) will average to zero. If, however, the tether length is varied once per orbit with a phasing as shown in Figure 9, we can produce a net change in the orbit eccentricity. Figure 10 shows the rate of eccentricity change over an orbit when the tether is reeled in and out by ± 2 km in a sinusoidal manner as shown in Figure 9. Integrating this curve results in a rate of eccentricity damping of 0.0011 per day, which for this orbit corresponds to a periapse shift of 2.2 km/day, or a ΔV of 1.85 m/s per day. This reeling operation would require a traverse rate of 1 m/s. During the half-orbit the facility is climbing up the tether against the centrifugal force it will require approximately 32 kW of power. However, while the facility is sliding back down the tether, nearly the same amount of power can be regenerated, so the net power requirement will be very small. In fact, if this reeling operation is performed to reduce the orbital eccentricity (and thus the orbital energy), then net power generation might be achieved.

Thus, provided the Lunavator system has the capability to adjust the position of the central facility along the tether (which it needs anyway in order to adjust the tip velocity to deliver the payload to the surface), it appears that modest tether reeling operations can provide the ΔV necessary to maintain the stability of the tether's polar lunar orbit, without requiring propellant expenditure.

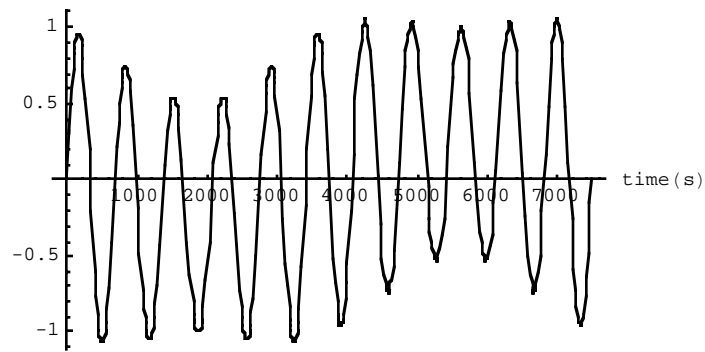


Figure 8. Function in the {} of Eqn. (16), plotted over one orbit for the baseline Lunavator.

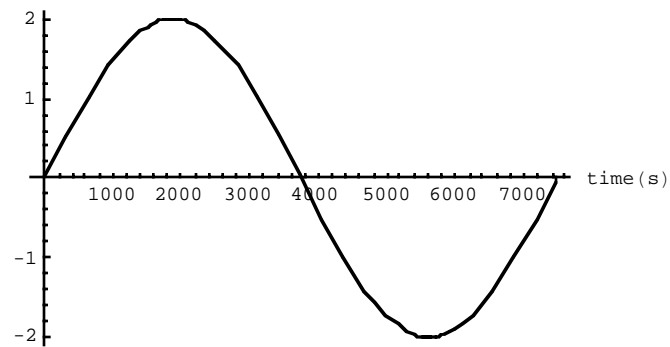


Figure 9. Tether reeling ΔL , in km.

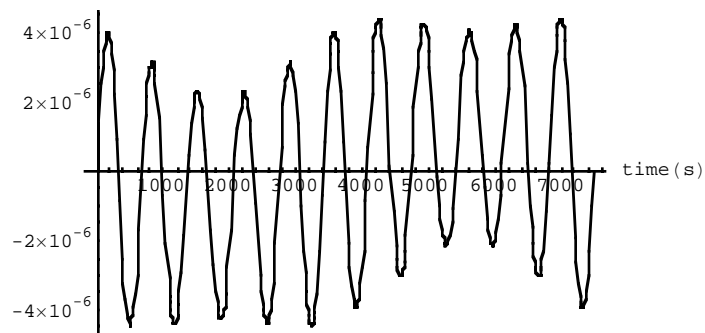


Figure 10. Rate of eccentricity change computed over one orbit according to Eqn. (16), with tether reeling as shown in Figure 9.

Summary

Dividing the “ballast” mass of a tether system into two parts, the first a “counterbalance” mass at the top of the tether and the second a central facility that can change its position along the tether, enables the tether system to simultaneously increase its rotation rate and the distance from the center of mass to the tether tip. This method works on the principle of conservation of angular momentum, and requires no propellant expenditure. Using this method, we have designed a Lunavator tether facility massing 16.7 times the payload mass that can catch payloads from a minimal-energy lunar transfer trajectory and deposit them on the surface of the moon. This same system could pick up payloads from a lunar base and throw them back down to low-Earth-orbit facilities. This facility design will also enable the Lunavator to be launched from Earth with a low initial mass and relatively low initial payload capacity and then build up its “ballast” mass using lunar resources in order to increase its payload capacity. The orbit of the Lunavator can be stabilized against perturbations using modest tether reeling operations that do not require propellant expenditure.

References

1. H. Moravec, “A Non-Synchronous Orbital Skyhook,” *J. Astro. Sci.*, 25(4), pp 307-322, 1977.
2. H. Moravec, “Non-Synchronous Orbital Skyhooks for the Moon and Mars with Conventional Materials,” unpublished paper, 1978.
3. Hoyt, R.P., Forward, R.L., *LEO-Lunar Tether Transport System Study*, Tethers Unlimited, Inc. Final Report on SAO Purchase Order S06-34444, April 28, 1997.
4. Landis, G.A., “Reactionless Orbital Propulsion using Tether Deployment,” *Acta Astronautica* 26(5), IAF Paper 90-254, 1992.
5. Martínez-Sánchez, M., Gavit, S.A., “Orbital Modifications using Forced Tether Length Variations”, *J. Guidance, Control, and Dynamics*, 10(3) May-June 1987, pp 233-241.

CISLUNAR SYSTEM DYNAMICS VERIFICATION THROUGH SIMULATION

Robert P. Hoyt
Tethers Unlimited, Inc.

Abstract

In order to validate the orbital mechanics and tether dynamics of the Cislunar Tether Transport System, we have developed a numerical simulation of the system that includes models for the full 3D orbital mechanics in the Earth-Moon system, tether dynamics, tether electrodynamics, and other physics. Using this code, we have designed and simulated a scenario for transferring a payload from low Earth orbit to the surface of the Moon.

Introduction

The operation of the tether facilities utilized in the Cislunar Tether Transport System involve many different interrelated phenomena, including orbital dynamics, tether librations and oscillations, interactions with the ionospheric plasma, day/night variations of the ionospheric density, solar and ohmic heating of the tether, magnetic vector variations around an orbit, and the behavior of electron emission devices. In order to enable accurate analyses of the performance and behavior of the this and other tether system, we have developed a numerical simulation of electrodynamic tethers called "TetherSim" that includes models for all of the aforementioned physical phenomena.

In the following sections we summarize the physics models used in the TetherSim program.

Tether Dynamics

The dynamics of the tether were modeled by approximating the continuous tether mass as a series

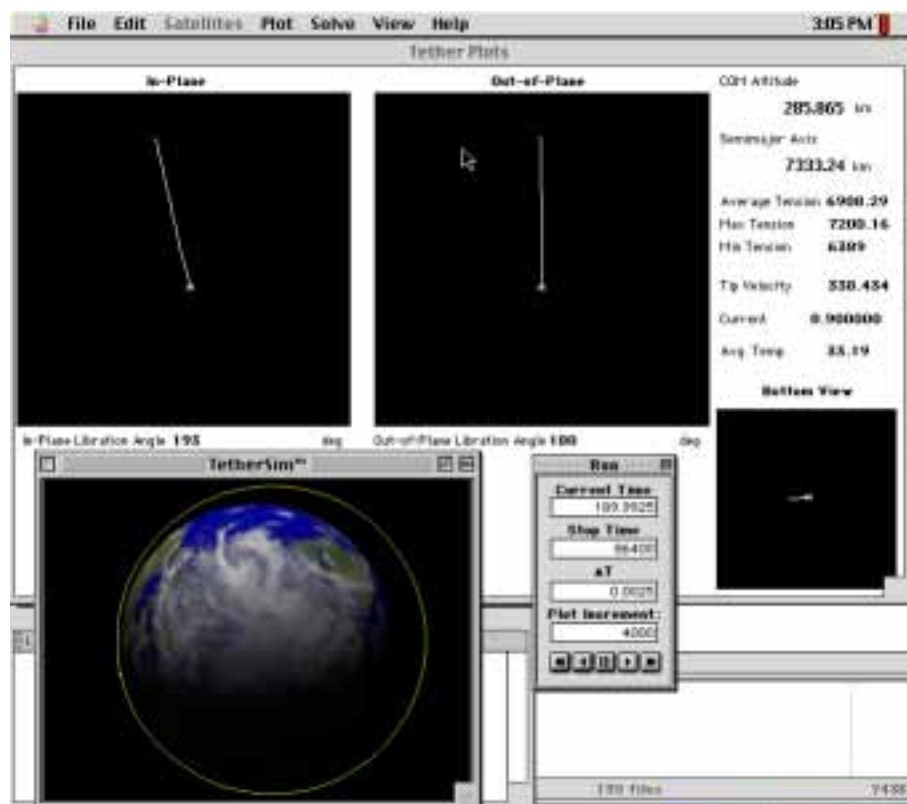


Figure 1. Screen shot of the TetherSim program simulating orbital reboosting of a 25 km HEFT Tether Facility.

of point masses linked by massless springs. This method is similar to that used by Kim and Vadali,¹ and also by Carroll's BeadSim.² Because the temperature of the tether can fluctuate significantly due to solar heating and ohmic dissipation, the simulation uses a temperature-dependent model for the stress-strain behavior of the aluminum tether. The model also assumes that the tether has no torsional or flexural rigidity.

Orbital Dynamics Model

The code calculates the orbital motion of the satellite, endmass, and tether elements using a 4th order Runge-Kutte algorithm to explicitly integrate the equations of motion according to Cowell's method.³ The program uses an 8th-order spherical harmonic model of the geopotential and a 1st order model for the lunar gravity. When a satellite enters the Moon's sphere of influence, the trajectory is updated using the lunar potential as the primary body and a 1st order model of the geopotential as a perturbing force.

Geomagnetic Field Model

The Earth's magnetic field is modeled as a magnetic dipole with the magnetic axis of the dipole tilted off from the spin axis by $\phi=11.5^\circ$, as illustrated in Figure 2. In this model, we have ignored the 436 km offset of the dipole center from the Earth's geometric center.

The magnetic field vector is given by

$$\mathbf{B} = \frac{B_E R_E^3}{r^3} \begin{bmatrix} 3xz/r^2 \\ 3yz/r^2 \\ 3z^2/r^2 - 1 \end{bmatrix},$$

where $B_E = 31 \mu\text{T}$ is the dipole moment of the Earth, R_E is the Earth's mean radius, and x , y , and z are cartesian coordinates expressed in a reference frame that has been rotated so that the z axis is aligned with the magnetic axis.

The geomagnetic field rotates with the Earth as it spins, so in calculations of $\mathbf{v} \times \mathbf{B}$ induced voltages experienced by the tether as it orbits the Earth, the local velocity of the geomagnetic field is subtracted from the tether's velocity before the cross product is calculated.

Ionospheric Plasma Density Model

The density of the ionospheric plasma is computed using data on electron density for average solar conditions provided by Enrico Lorenzini of the Smithsonian Astrophysical Observatory.⁴ The electron density is computed by determining if the tether is in sunlight or shade, and then interpolating the density on the appropriate curve shown in Figure 3.

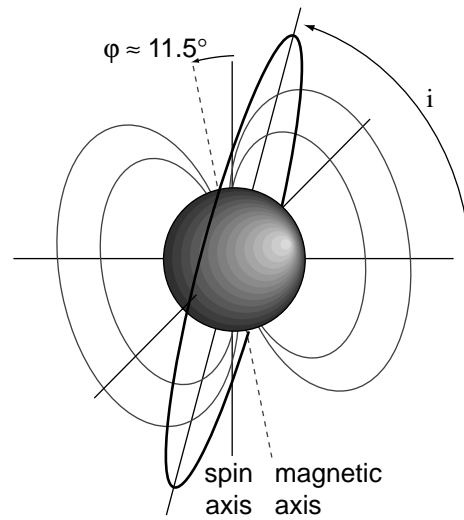


Figure 2. Tilted-dipole approximation to the geomagnetic field.

1. Kim, E., Vadali, S.R. "Modeling Issues related to Retrieval of Flexible Tethered Satellite Systems," J. Guid. Contr. & Dyn., 18(5), 1995, pp 1169-76
2. Carroll, J.A., Personal Communication.
3. Battin, R.H., *An Introduction to the Mathematics and Methods of Astrodynamics*, AIAA, 1987, p. 447.
4. Lorenzini, E., email 1/9/98.

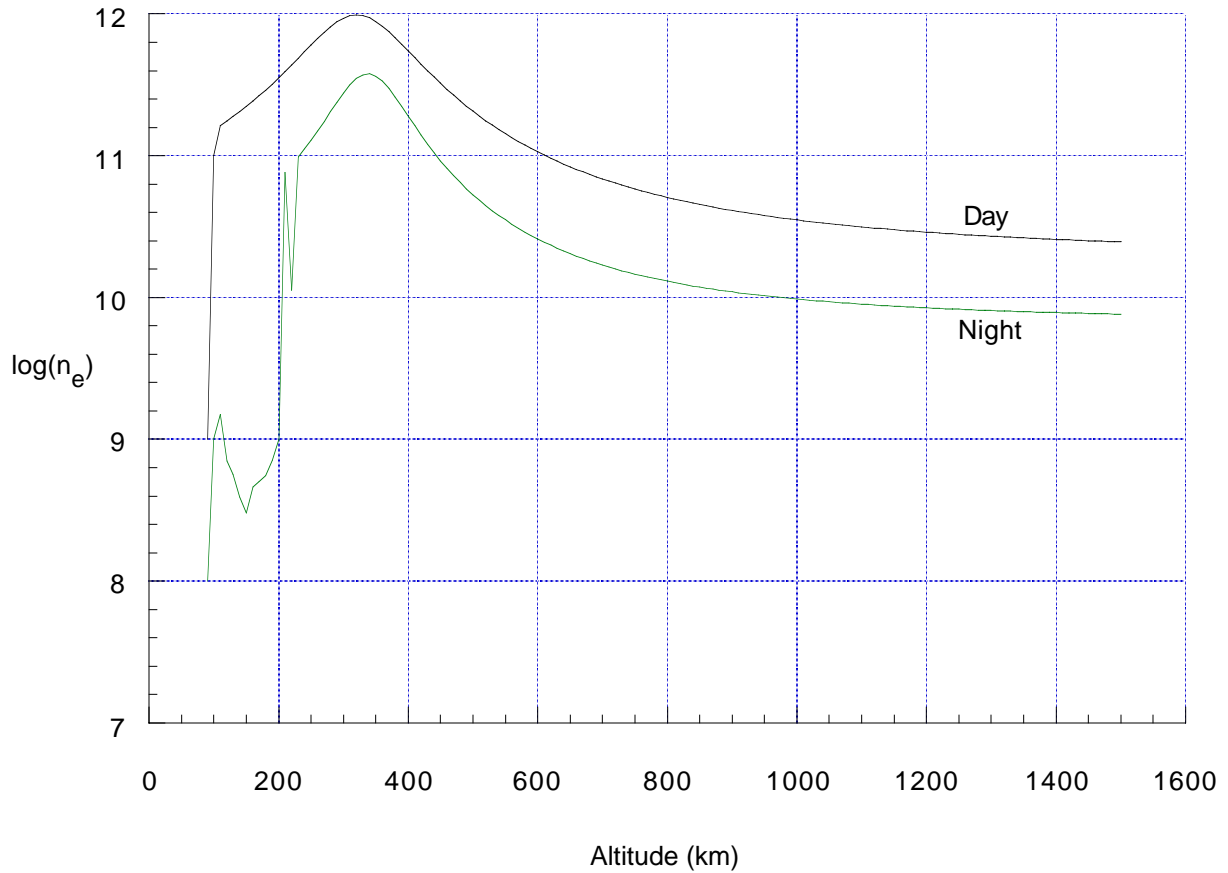


Figure 3. Average ionospheric plasma electron density as a function of altitude for sunlit and eclipse conditions.

Atmospheric Drag Model

At low altitudes, neutral particle drag on the tether may become a significant effect. The code thus calculates the neutral particle drag on the satellite, endmass, and tether elements according to

$$F_{drag} = \frac{1}{2} \rho C_D V_{rel}^2 A$$

where $C_D \approx 2.2$ is the coefficient of drag for a cylindrical tether in free-molecular flow, V_{rel}^2 is the relative velocity between the tether and the atmosphere (assumed to rotate with the Earth), A is the cross-sectional area the tether presents to the wind, and ρ is the neutral density, calculated according to the heuristic formula developed by Carroll:⁵

$$\rho = \frac{1.47 \times 10^{-17} T_{ex} (300 - T_{ex})}{1 + \frac{2.9(h - 200)}{T_{ex}}} \quad h > 200 \text{ km},$$

where h is the altitude and T_{ex} is the average exospheric temperature, 1100K.

5. Carroll, J.A., "Aerodynamic Drag", p 160 in *Tethers In Space Handbook*, 3rd Edition, Cosmo and Lorenzini, editors, Smithsonian Astrophysical Observatory, 1997.

TARGETING TO LUNAVATOR ORBITS

Chauncey Uphoff
Fortune-Eight Aerospace
 Robert Hoyt
Tethers Unlimited, Inc.

We have conducted studies of the Earth-Moon transfer to verify that the payload can be targeted to arrive at the Moon in the proper plane to rendezvous with the Lunavator™. This study was performed with the MAESTRO code,ⁱ which includes the effects of luni-solar perturbations as well as the oblateness of the Earth. In this work we studied targeting to both equatorial and polar lunar trajectories.

Transfer to Equatorial Lunar Trajectories

Transfer of a payload from an equatorial Earth trajectory to an equatorial lunar trajectory can be achieved without propellant expenditure, but this requires use of a one-month “resonance hop” transfer, as illustrated in Fig. 1. In a resonance hop maneuver, the payload is sent on a trajectory that passes the Moon in such a way that the lunar gravitational field slingshots the payload’s orbit into a one-month Earth orbit that returns to the Moon in the lunar equatorial plane. Using MAESTRO, we have developed a lunar transfer scenario that achieves this maneuver.

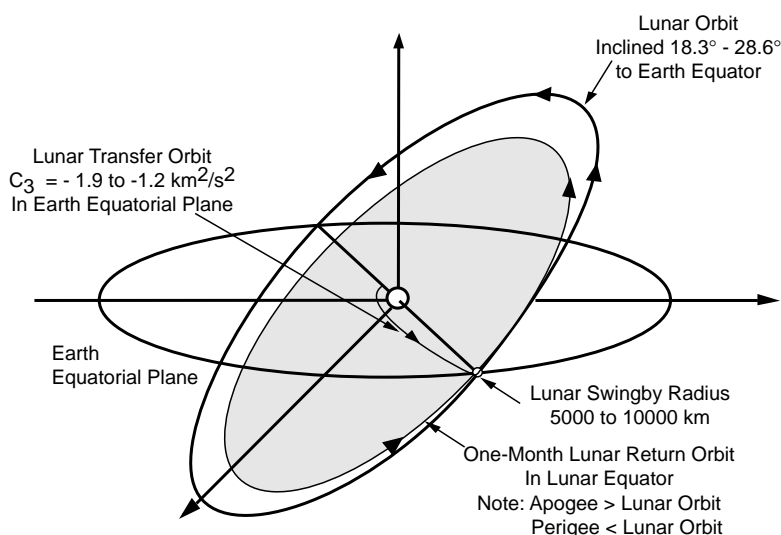


Figure 1. Schematic of one-month “resonance-hop” transfer to place payload in lunar equator without using propellant.

In order to avoid the one-month transfer time, we can instead use a small impulsive thrust as the payload crosses the lunar equator to bend its trajectory into the equatorial plane. A patched-conic analysis of such a transfer predicts that such a maneuver would require 98 to 135 m/s of ΔV . However, our numerical simulations of the transfer revealed that under most conditions, luni-solar perturbations of the payload’s trajectory will perform much of the needed bending for us, and the velocity impulse needed to place the payload in a lunar equatorial trajectory is only about 25 m/s. Fig. 2 shows the time-history of a transfer of a payload from the Earth-orbit tether boost facility to the Moon, projected onto the Earth’s equatorial plane. Fig. 3 shows this same transfer, projected onto the lunar equatorial plane in a Moon centered frame. The motion of the payload relative to the lunar equator can be observed in Fig. 4, which shows the trajectory projected onto the lunar x-z plane. The payload crosses the lunar equator approximately 10 hours before its closest approach to the Moon. Fig. 5, which plots the Moon-relative velocity of the payload, shows that the payload’s velocity at the time of lunar equatorial crossing is about 925 m/s. However, a plot of the declination of the payload’s velocity with respect to

the lunar equator, shown in Fig. 6, reveals that the declination of the Moon-relative velocity vector is only a few degrees, much less than the 18° - 29° value predicted by a simple zero-patched conic analysis; the Moon's (or Sun's) gravity has bent the velocity vector closer to the lunar orbit plane.

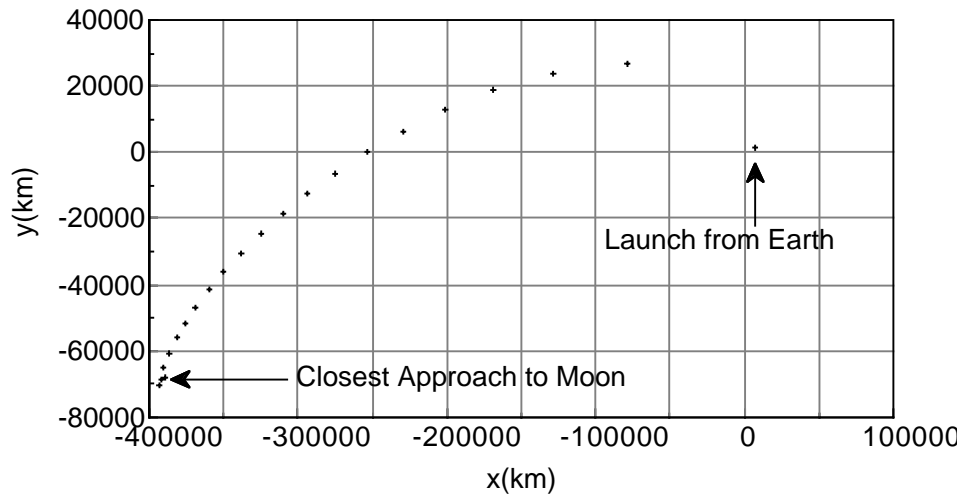


Figure 2. Transfer of payload to lunar equatorial trajectory, projected onto the True Earth Equator.

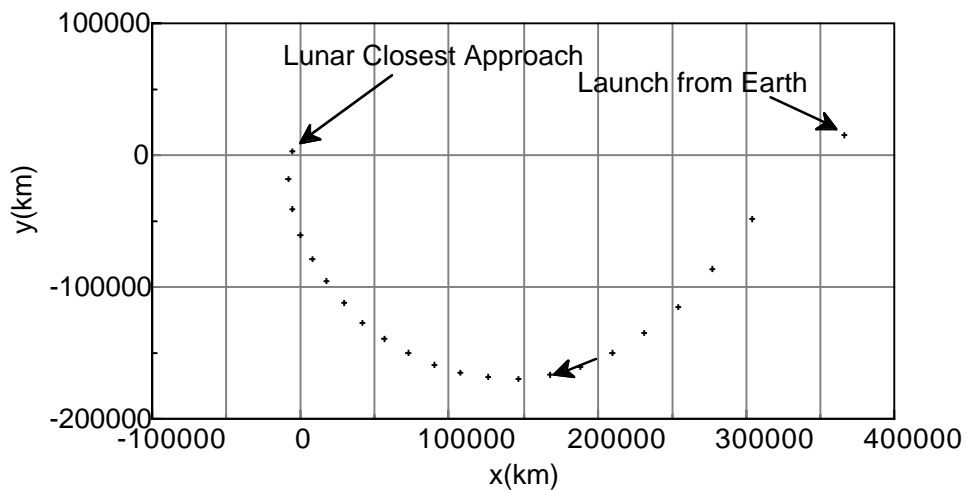


Figure 3. Projection of payload transfer onto Lunar Equatorial Plane (Moon centered frame).

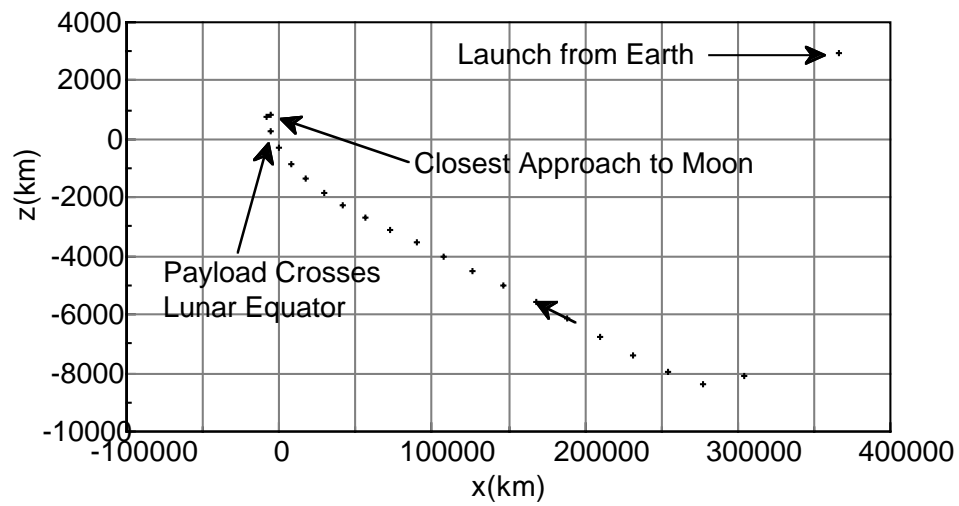


Figure 4. Projection of payload transfer onto Lunar x-z plane (Moon centered frame).

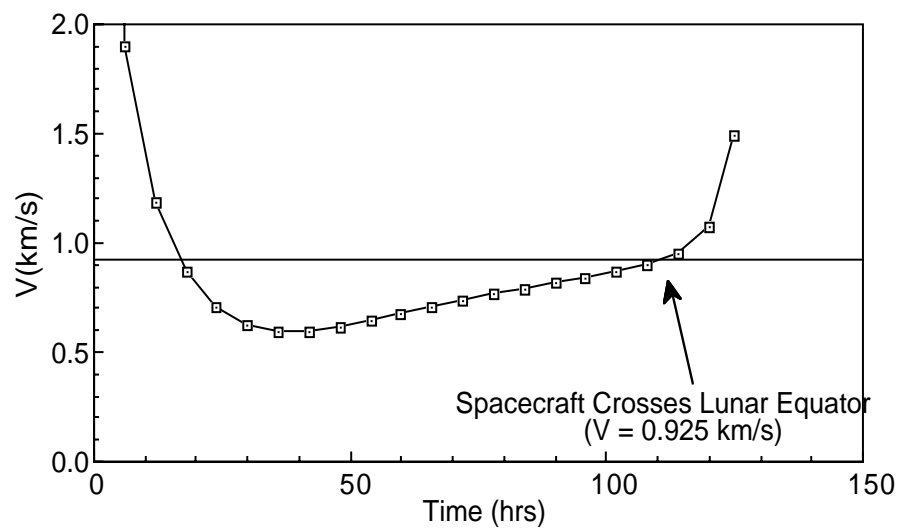


Figure 5. Moon-relative velocity of spacecraft.

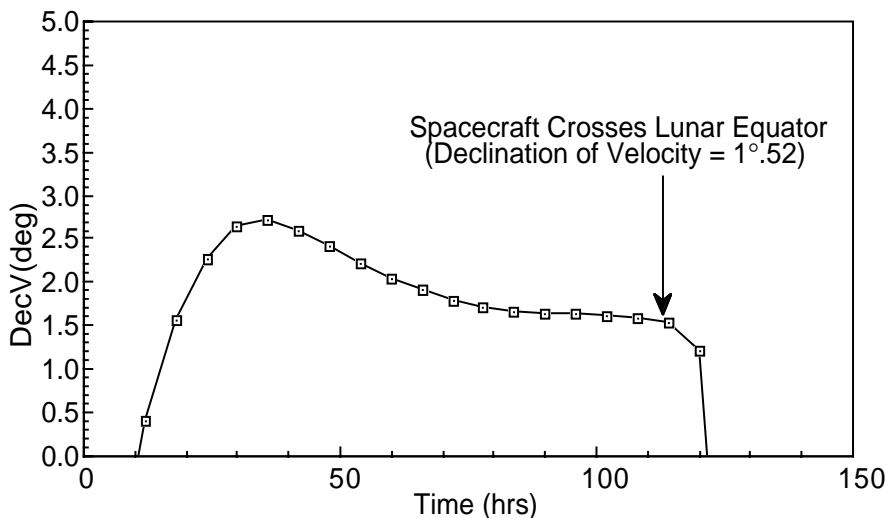


Figure 6. Declination of Moon-relative velocity vector with respect to Lunar Equator.

At the time when the payload's trajectory crosses the lunar equator, declination of the incoming velocity vector of only 1.52° . This dynamical situation permits us to bend the approach trajectory into the lunar equator with a very small amount of impulse supplied by the spacecraft propulsion system. In the case shown here, the amount of ΔV required is only 24.5 m/s, applied about 10 hours before closest approach to the Moon, as the spacecraft crosses the lunar equator.

In previous sections we have seen how the use of a polar Lunavator™ orbit can provide rapid transfer from the Earth Equatorial Tether Transport device to a capture into a lunar polar orbiting tether facility. It remains to be shown that the Earth-moon transfer can be done in such a way as to ensure that the lunar approach orbit is in the plane of the Lunavator™ at the time of capture by the Lunavator™. The following sections contain arguments and numerical verifications that these kinds of transfers can be achieved for a wide range of orientations (longitudes of the ascending node) of the Lunavator™ orbit on the lunar equator.

Transfer to Polar Lunar Trajectories

Fig. 7 shows a typical transfer from Earth to moon in the Earth's equator. The declination of the incoming asymptote at the moon (with respect to the lunar orbit plane) ranges from 18° to 31° , depending upon the orientation of the Earth's equator to the lunar orbit plane and, of course, the spacecraft encounters the moon near the intersection of the lunar orbit and the Earth's equator. The trajectory has been targeted to a lunar equatorial inclination of 90.06° and an ascending node (with respect to the lunar prime meridian) of 100.95° . The reference line for the ascending node is the geographical prime meridian of the lunar reference system. The x axis of this system points, closely, to the Earth. This trajectory, and all subsequent trajectories presented here, has been integrated using the MAESTRO code,ⁱⁱ which performs full-model numerical integration including the effects of lunar and solar gravitational perturbations, and the effects of the first four ($J_2 - J_5$) zonal harmonics of the Earth's gravity field.

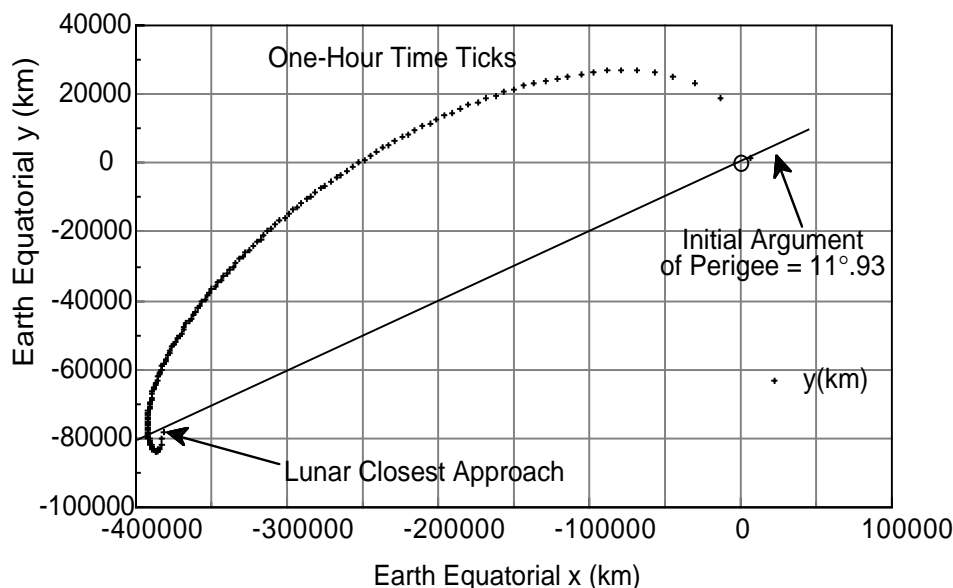


Fig. 7 Time History Earth-Moon Transfer for Lunar Node = - 100°.95

The astute reader will note that the trajectory shown in Fig. 7 is a Type II transfer with central angle on the initial orbit greater than 180° . Similar transfers can be generated with two-body central angles of less than 180° (Type I transfers). A good way to think of these transfers is to imagine that the spacecraft is launched on a trajectory that just reaches the moon's orbit. At that distance, the spacecraft is moving only a few hundred meters per second. Now the moon comes along, moving at about 1000 m/s and "captures" the spacecraft with its gravitational attraction. The first trick of targeting is to make sure that the incoming asymptote of the lunar approach trajectory takes the spacecraft directly over the lunar pole. The other trick is to ensure that the approach orbit encounters the moon at the proper angle for rendezvous with the LunavatorTM.

The Targeting/Control Variables

It is not obvious that the selection of a certain set of control variables at launch will result in the desired lunar orbit at pericyynthion. Selection of the control variables is a part of the "black art" of cislunar and interplanetary targeting. In the present situation, one is constrained to a launch orbit that is in the Earth's equatorial plane. For a given geometric situation, the only way to control the transfer is to choose the time of launch, the direction of launch, and the energy of the launch trajectory. In this case, we have not the luxury of selecting the inclination or the Equatorial longitude of the ascending node of the trans-lunar trajectory; here we must use the timing of launch, the position of the launch point, and the energy of the trans-lunar orbit. It is not obvious that this set of variables is sufficient to ensure transfer from an elliptical tether transport facility to a polar lunar orbit capture by a LunavatorTM in an orbit with an arbitrary orientation of its node on the lunar equator.

To test this targeting mechanism, we ran four fundamental trajectories with four separate initial values of argument of perigee for the initial launch trajectory from Earth. In each of the four cases, the launch time and launch energy were varied until the pericynthion radius and (lunar equatorial) inclination achieved the values desired for the capture of the spacecraft by the LunavatorTM. In each of the four cases, the incoming longitude of the ascending node on the lunar equator varied by about 4° and there were clearly no regions of the function space wherein the desired lunar orbit could not be achieved. Fig. 8 shows a moderately close-up view of the lunar approach trajectories for each of the four targeted trajectories.

It is clear, from the trajectories shown in Fig. 8, that the orientation of the final lunar orbit can be controlled by selection of the argument of perigee of the initial Earth-to-moon trajectory. For each of

the transfers, one can ensure the polar inclination (with respect to the lunar equator) and the radius of pericynthion. These latter conditions can be achieved by selecting the time of launch at Earth and the energy of the trans lunar trajectory.

The important thing about this selection of control variables is that there is no need for any nominal control out of the Earth's equatorial plane. Therefore, simply by selection of the time of launch (near the time when the Earth-orbit tether facility's line of apsides crosses the intersection of the Earth-moon plane and the Earth's equator) and the speed of release of the spacecraft from the top of the Earth-orbit tether, one can ensure a velocity match of the incoming spacecraft with the upper end of the Lunavator™ for a wide range of ascending node positions of the Lunavator™ orbit. The targeting examples of this memo (section) show that the range of viable nodal positions is at least $\pm 10^\circ$ from the normal to the Earth-moon line.

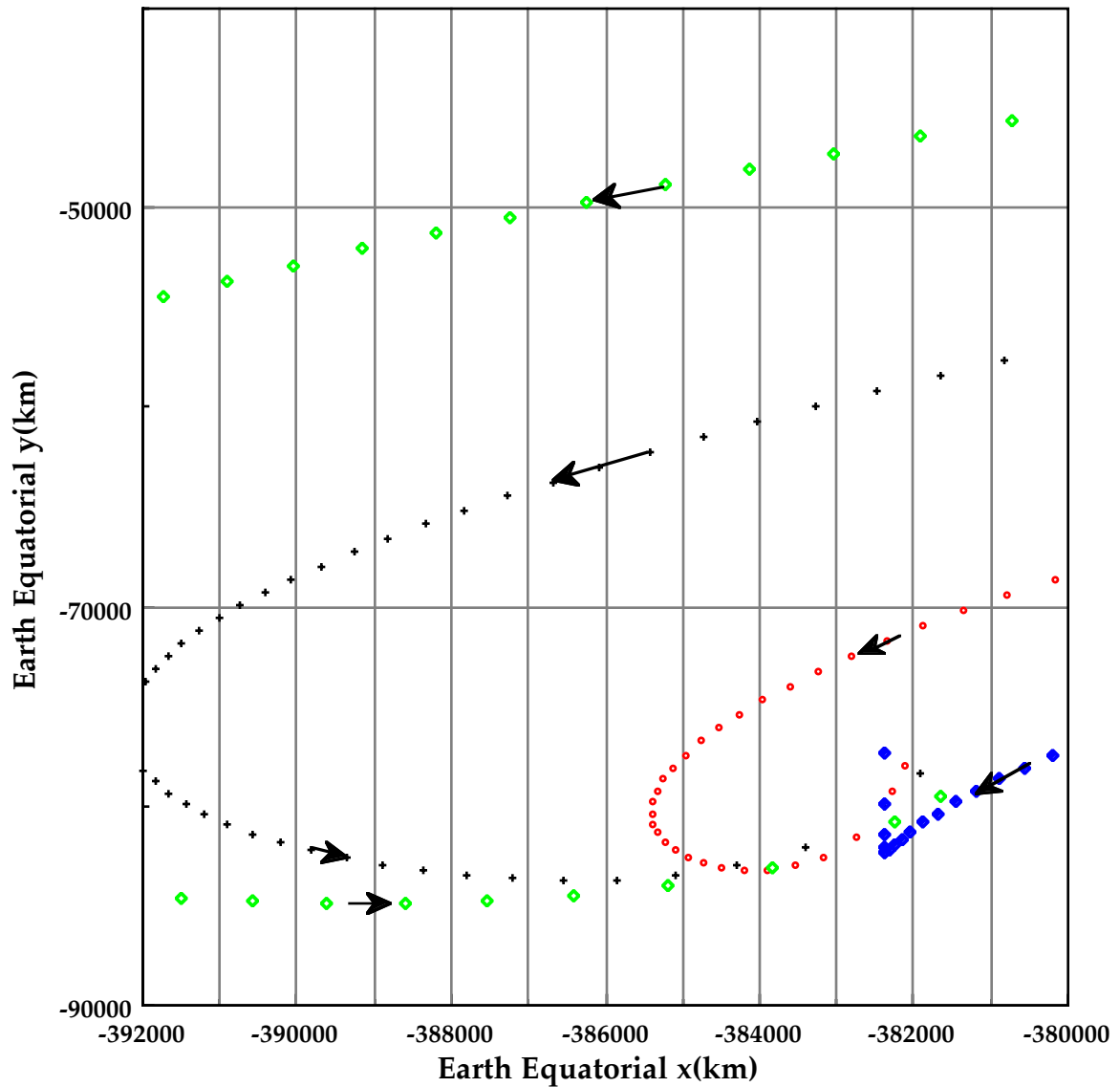


Fig. 8 Approach Trajectories to Polar Lunar Orbits with Variable Nodal Position

Fig. 9 shows a close-up Earth-centered view of the four targeted trajectories with lunar equatorial inclination of 90° and with ascending nodal values of -101° to -90° with respect to the moon-Earth line. Thus, by control of the orientation of the launch trajectory in the Earth's equator, the speed of release from the EEO tether, and the time of release, one can control the lunar approach trajectory within a wide range of nodal positions, ensuring matchup with the orbiting Lunavator™ at the desired time.

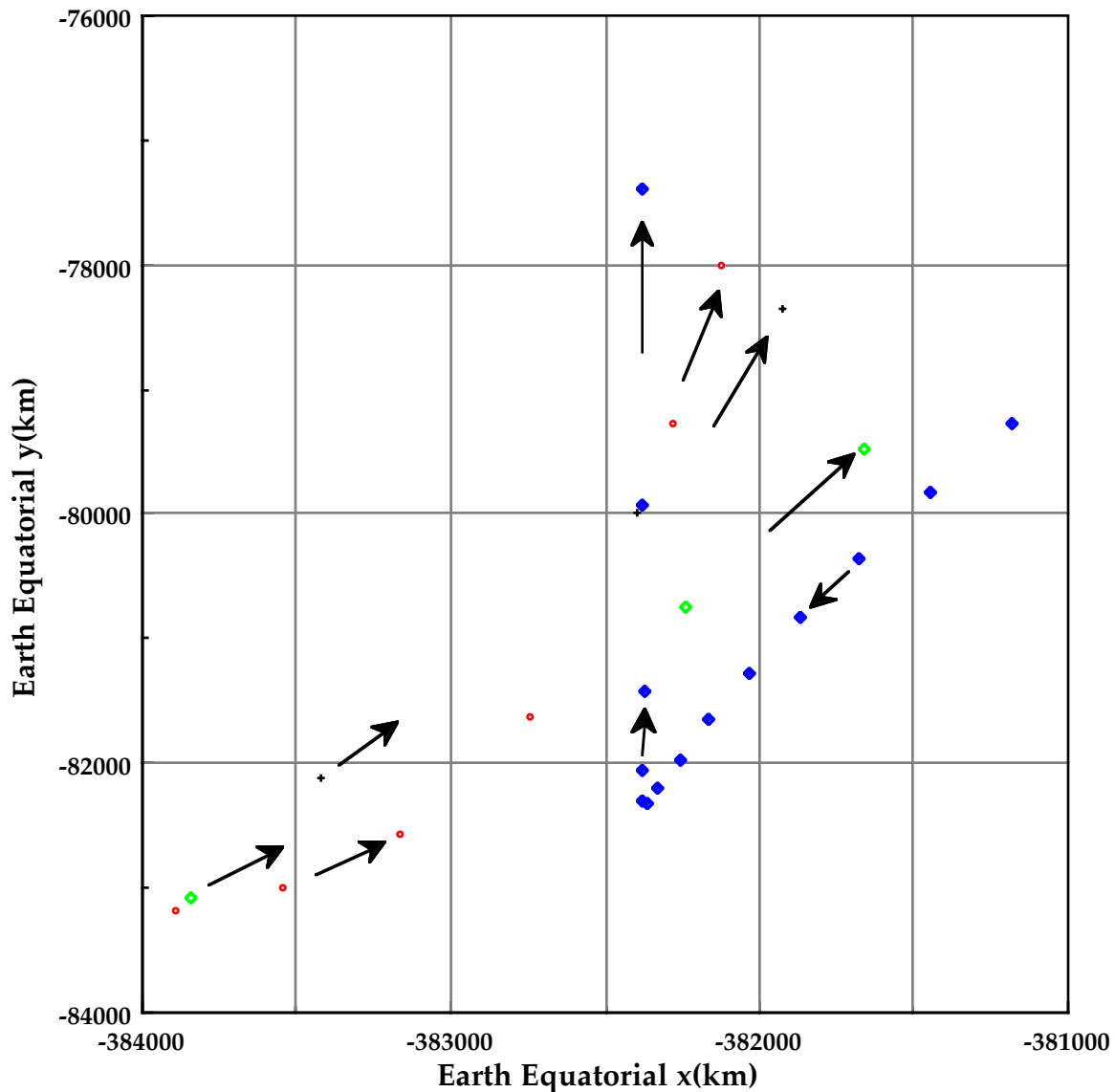


Fig. 9 Close-Up View of the Four Lunar Approach Trajectories of Fig. 8

- ⁱ. Uphoff, C., "Mission Analysis Evaluation and Space Trajectory Optimization Program", Final Report on NASA Contract NAS5-11900, March 1973.
- ⁱⁱ. Uphoff, C., "Mission Analysis Evaluation and Space Trajectory Optimization Program", Final Report on NASA Contract NAS5-11900, March 1973.

STUDIES OF LUNAR ORBITAL STABILITY

Chauncey Uphoff
Fortune-Eight Aerospace

Robert Hoyt
Tethers Unlimited, Inc.

There are two reasons why it is desirable for a Cislunar Tether Transportation System to utilize a Lunavator in a nearly polar orbit at the moon. The first reason is that one can target the Earth-moon transfer orbit directly to the Lunavator, without the need for an extra month of transfer time required for the Earth-equator to lunar-equator scheme. The second reason for having a polar Lunavator is that one can drop or pick up a payload at any point on the lunar surface, including the permanently-shadowed craters near the poles where a potentially significant amount of water is available.

Polar lunar orbits, however, are notoriously unstable and tend to drop the pericynthion below the lunar surface within a few months if the orbit is not adjusted to remove the large eccentricity variations. Fig. 1 shows the time history of the pericynthion radius for a typical polar lunar orbit with a semi-major axis of the baseline Lunavator design (circular orbit altitude = 190 km).

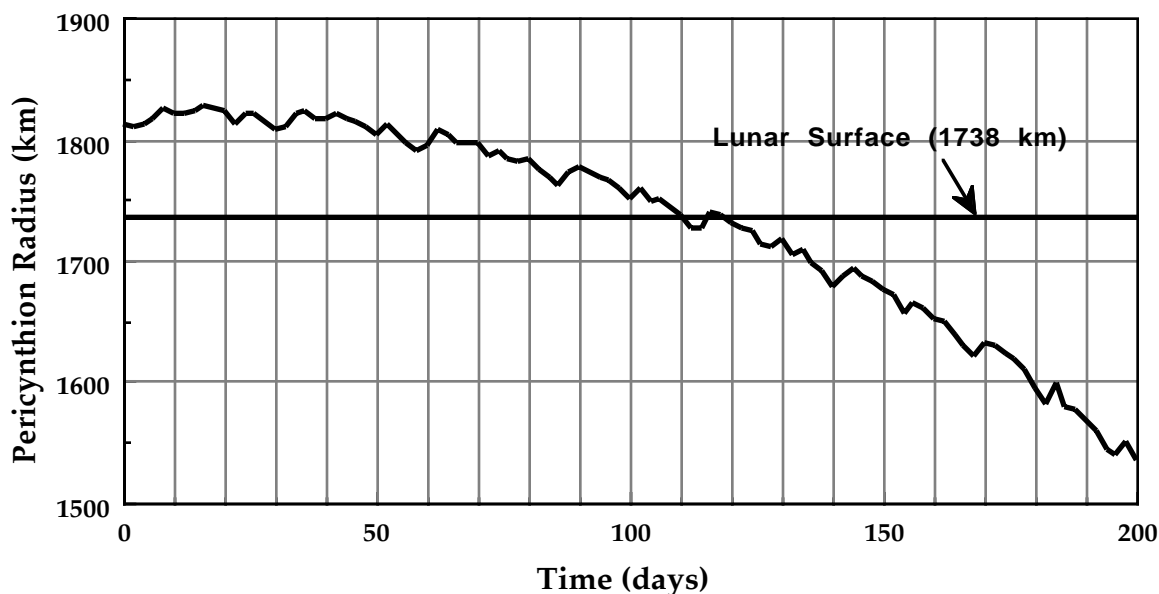


Fig. 1 Time History of Pericynthion Radius for Typical Polar Lunar Orbit

The time histories of the orbital parameters presented here have been obtained by numerical integration of the singly-averaged (by numerical quadrature over one revolution of the spacecraft in its orbit) equations of motion for the orbital elements p , $e \sin \omega$, $e \cos \omega$, $\omega + f$, i , and Ω where p is the semi-latus rectum of the osculating ellipse, e is the eccentricity, ω is the argument of pericynthion, i is the lunar equatorial inclination, and Ω is the longitude of the ascending node referred to an inertial frame.

Search for a Polar Frozen Orbit

If the significant zonal harmonics of the lunar gravity field were limited to J_2 , J_3 , J_4 , and J_5 , there would be an orbit whose eccentricity and argument of pericynthion are stable (frozen) at about $e = 0.03$ and $\omega = -90^\circ$. Unfortunately, the higher degree zonal harmonics of the lunar gravity disrupt this pleasant symmetry and cause the close polar orbiter to crash very quickly. A numerical search for a truly frozen polar orbit was conducted (using the JPL 15-8 field) and was unsuccessful. The "best" orbit, in the sense of minimum excursion of eccentricity during a time period of about 200 days is shown in

Fig. 2. Lunar Prospector data has provided a much more refined field, but the differences between the Lunar Prospector field and the 15-8 field are at much higher values of degree and order than are considered here. We do not expect to find a frozen orbit at the altitude of the LTT baseline design, no matter how many terms we use in the expansion. For this reason, we expect to use a tether length control device to maintain a near-circular, close lunar polar Lunavator.

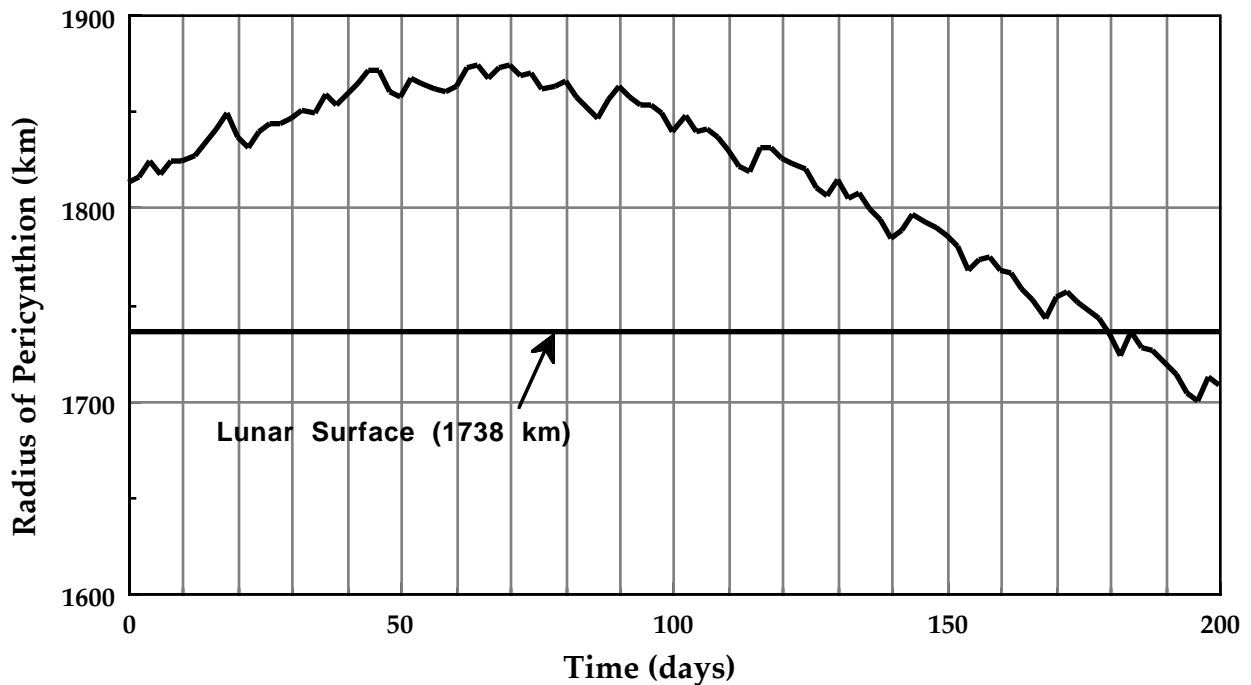


Fig. 2 Time History of Pericynthion Radius for "Nearly Frozen" Orbit

Looking at Fig. 2, one can imagine an orbit that lasts about 200 to 250 days between corrections to avoid crashing into the moon. Practical considerations will probably limit the time between corrections to about 200 days.

The reader should note that these simulations are for a point mass spacecraft and not for an extended tether that may or may not be rotating in the orbital frame. While we do not currently have the software to simulate the motion of a massive rotating tether in close lunar orbit, we are convinced that the point mass simulations are representative of the perturbations that will have to be dealt with in the polar lunar orbit case.

MAINTENANCE OF ROTATING TETHER ORBITS BY TETHER REELING

Robert P. Hoyt
Tethers Unlimited, Inc.

Abstract

The orbits of the tether facilities in a tether transportation system will inevitably experience perturbations due to third-body forces, nonspherical gravitational potentials, solar pressure, and other effects. Tether reeling maneuvers may provide a means to modify or maintain the orbits of tether facilities without requiring propellant consumption. Previous work has studied tether reeling maneuvers in hanging tether systems, but did not study rotating tether systems in depth. In this paper we develop analytical methods for determining the effectiveness of tether reeling maneuvers in rotating tether systems. These analyses indicate that modest tether reeling maneuvers can provide an effective method of dissipating the eccentricity perturbations that would threaten the long-term orbital stability of a lunar tether, and for modifying the rate of apsidal precession of Earth-orbit tether facilities.

Introduction

In tether transportation systems such as the Cislunar Tether Transport System¹ and the Mars-Earth Rapid Interplanetary Tether Transport (MERITT) System², maintenance of the shape and orientation of the tether facility orbits will be critical to enabling frequent opportunities for these systems to exchange payloads between Earth, the Moon, and Mars. The orbits of tether facilities around the Earth, the Moon, and Mars will experience perturbations due to the oblateness of the planetary bodies, lunisolar or geosolar gravity fields, solar pressure, atmospheric drag, and other effects. Although high-specific impulse thruster propulsion might be considered for orbital maintenance of the tether facilities, thrusters require propellant expenditure. If tether systems are to achieve their full potential for reducing the cost of in-space transportation, they must be able to operate with a minimum of propellant expenditure. Propellantless electrodynamic tether propulsion may provide a very effective means of performing some of the orbital maneuvers required for the low-Earth-orbit portions of the tether systems, but tether facilities around the Moon, Mars, and in high-Earth-orbit will not be able to avail themselves of electrodynamic tether propulsion due to the paucity of magnetic field and ambient plasma in those orbits.

Fortunately, the technique of orbital modification using tether reeling operations may provide a means of maintaining tether facility orbits without requiring propellant expenditure. The concept of orbital propulsion using tether reeling was pioneered by Landis,³ and by Martínez-Sánchez, and Gavit.⁴ Tether reeling can add or remove energy from a tether's orbit by working against the non-linearity of a gravitational field. The basic concepts of orbital modification using tether reeling are illustrated in Figs. 1 and 2.

Figure 1 illustrates use of tether pumping to change the eccentricity of a hanging tether's orbit [a "hanging tether rotates once per orbit, so that it is always aligned along the local vertical"]. When the tether is near the periapsis of its orbit, the tidal forces on the tether are high. When it is near apoapsis, the tidal forces on the tether are low. If it is desired to increase the eccentricity of the tether's orbit, then the tether can be reeled in when it is near periapsis, under high tension, and then allowed to unreel under lower tension when it is at apoapsis. Since the tidal forces that cause the tether tension are, to first order, proportional to the inverse radial distance cubed, more energy is required to reel the tether in at periapsis than is recovered at apoapsis, and so net energy is added to the tether's orbit by the reeling maneuver. Although energy is added to the orbit by the reeling maneuvers, the forces on the tether are always perpendicular to the orbit, and so the orbital angular momentum of the orbit does not change. This results in an increase in both the eccentricity and semimajor axis of the orbit, while the angular momentum h remains constant.

A rotating tether system, however, will experience forces that are parallel to the orbital velocity, as illustrated in Figure 2. If the tether length remains constant, these forces will average out to zero

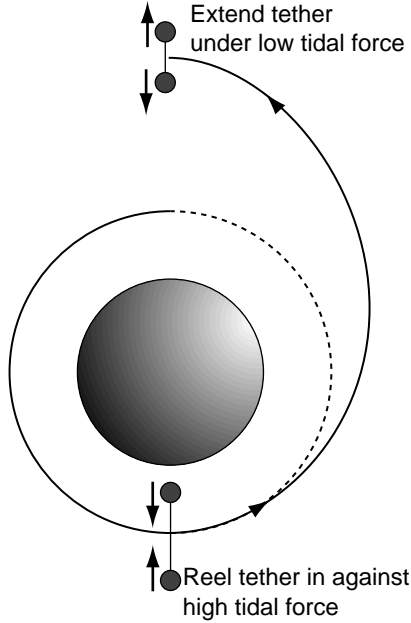


Figure 1. Schematic of tether pumping to increase orbital eccentricity.

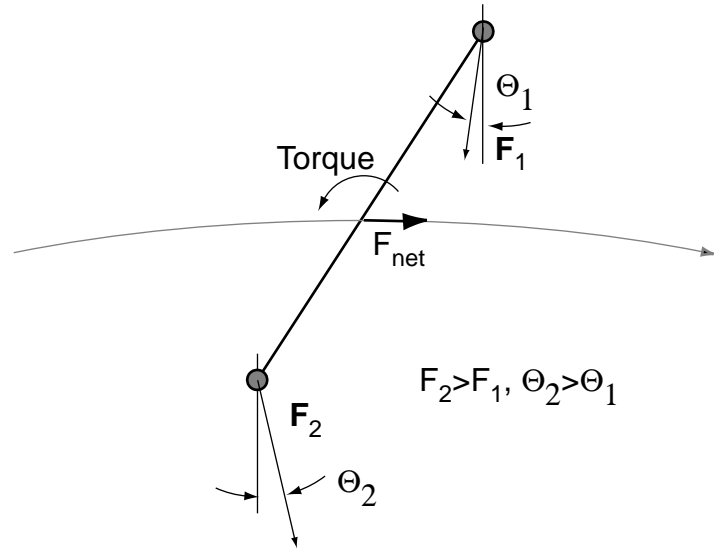


Figure 2. Forces on a non-vertical tether. Because $F_2 > F_1$ and $\Theta_2 > \Theta_1$, the tether experiences a net force in the plane of rotation.

over an orbit. If, however, the tether length is varied over a rotation and/or over an orbit, energy can be added or subtracted from the orbit, and angular momentum can be transferred between the rotation of the tether system and the orbit, resulting in a modification to the tether's orbit.

Rotating Tether Analysis

The analyses presented in 1987 by Martínez-Sánchez and Gavit⁴ and the analysis developed by Landis³ in 1992 assumed that the tether is hanging (rotating once per orbit). The tethers in the Cislunar and MERITT systems will, however, be rotating rather rapidly so that they can catch and throw payloads at high relative velocities. We will therefore extend the theory of Martínez-Sánchez and Gavit to enable us to study the effectiveness of tether reeling operations in rotating tether systems.

For our analysis, we will assume that the tethered system consists of a massless tether of average length L connecting two masses, m_1 and m_2 . Martínez-Sánchez and Gavit have found that the forces on the tether system's center of mass are given by

$$\begin{aligned} dF_r &= -\frac{3\mu}{r_G^2} m_{12} \left(\frac{L}{r_G} \right)^2 \left(1 - \frac{3}{2} \sin^2 \alpha \right) \\ dF_\theta &= +\frac{3\mu}{r_G^2} m_{12} \left(\frac{L}{r_G} \right)^2 \sin \alpha \cos \alpha \end{aligned} \quad (14)$$

where $r_G = p/(1 + e \cos \theta)$, $m_{12} = (m_1 + m_2)/m_2 m_1$ is the reduced mass of the system, α is the angle of tether rotation away from vertical, and μ is the gravitational coefficient (GM) of the planet or moon that the tether is orbiting.

In Gauss' form of Lagrange's orbital equations, the rate of change of the eccentricity of a satellite's orbit is⁵

$$\frac{d}{dt} e = \frac{p}{h} \left\{ a_r \sin \theta + a_\theta \left[\cos \theta + \frac{r_G}{p} (e + \cos \theta) \right] \right\}, \quad (15)$$

where $p = a(1-e^2)$ is the orbit's semiparameter, h is the orbital angular momentum of the satellite, a is the semimajor axis, θ is the true anomaly (the angular position of the satellite in its orbit, measured from its periapse), and a_r and a_θ are the instantaneous acceleration of the satellite in the radial and azimuthal directions.

Gauss' formulation also gives the rate of rotation of the line of apsides for an equatorial orbit as

$$\frac{d}{dt}(\Omega + \omega) = -\frac{1}{eh} \left\{ a_r \cos \theta - a_\theta [(p + r_G) \sin \theta] \right\}. \quad (16)$$

At this point, we will assume that the tether rotates several times per orbit, and that the changes in the rate of rotation due to reeling operations and due to variations in the gravity gradient around an orbit are negligible. The angle of the tether relative to vertical can thus be related to the time by

$$\alpha(t) = \omega_T t. \quad (17)$$

We then need to obtain a relationship between the time t and the true anomaly θ . This can be obtained by solving Kepler's equation

$$\omega_{orb} t = E - e \sin E. \quad (18)$$

for the eccentric anomaly E and then converting E to the true anomaly according to

$$\cos \theta = \frac{1}{e} \left[\frac{p}{a(1 - e \cos E)} - 1 \right]. \quad (19)$$

Example 1: Maintenance of a Circular Polar Lunar Orbit

In the Cislunar System, it would be desirable to place the lunar-orbit tether in a polar lunar orbit so that the tether can service bases on the entire surface of the moon, and in particular at the ice-rich poles. Polar lunar orbits, however, are unstable due to the nonuniformity of the lunar gravitational potential; as a satellite orbits the moon, the odd-order zonal harmonics of the moon's gravitational field cause the satellite's orbit to become more and more elliptical until eventually the satellite's perigee drops below the lunar surface. Thus, for stabilizing a polar Lunavator's orbit, we are interested in using tether pumping to maintain the circularity of the tether's orbit. For this case, we can simplify Eqns. (14) and (15) by assuming that the eccentricity e is held essentially zero, so that $r_G = p = a$, and $\theta = \omega_{orb} t$. For simplicity, we will also assume that the tether length is varied by an amount ΔL that is small compared to the nominal length L so that the tether rotation rate ω_T is not significantly affected by the tether reeling operations. By dividing Eqns. (14) by the total mass of the tether system to obtain the accelerations and then inserting them into Eqn. (15), we find the rate of eccentricity change to be

$$\frac{d}{dt}e = \frac{3\sqrt{\mu}}{a^{7/2}} \left[\frac{m_{12}}{m} \right] \left\{ 2 \cos(\omega_{orb} t) \sin(\omega_T t) \cos(\omega_T t) - \sin(\omega_{orb} t) \left(1 - \frac{3}{2} \sin^2(\omega_T t) \right) \right\} [L + \Delta L(t)]^2. \quad (20)$$

For the baseline Lunavator design, the tether orbits at an altitude of 170.5 km. Numerical simulations of a satellite in a polar lunar orbit at this altitude indicate that the non-uniformity of the lunar gravitational potential will cause the eccentricity of the satellite's orbit to increase at a rate of approximately 0.0088 per day.⁶ In the baseline Lunavator design, the tether system has two equal masses (the central facility and the counterbalance mass) separated by 155 km of tether. For simplicity in our calculations, we will assume that the tether rotates an integral number of times per orbit; in the baseline design, the tether rotates approximately 6 times per orbit, so $\omega_T = 6\omega_{orb}$.

Using the orbital parameters for the baseline Lunavator design, the function in the brackets of Eqn. (16) is plotted in Figure 3. If the tether length L is held constant, then over an orbit the eccentricity change given by Eqn. (16) will average to zero. If, however, the tether length is varied once per orbit with a phasing as shown in Figure 4, we can produce a net change in the orbit eccentricity. Figure 5 shows the rate of eccentricity change over an orbit when the tether is reeled in and out by ± 2 km in a sinusoidal manner as shown in Figure 4. Integrating this curve results in a rate of eccentricity damping

of -0.0011 per day. This eccentricity damping rate would be more than enough to counteract the 0.00088 eccentricity growth rate caused by the moon's gravity. The eccentricity change induced by the tether reeling maneuvers corresponds to a periapse shift of 2.2 km/day, or a ΔV of 1.85 m/s per day. The reeling operation would require a traverse rate of 1 m/s. During the half-orbit the facility is climbing up the tether against the centrifugal force it will require approximately 32 kW of power. However, while the facility is sliding back down the tether, nearly the same amount of power can be regenerated, so the net power requirement will be very small. In fact, if this reeling operation is performed to reduce the orbital eccentricity (and thus the orbital energy), then net power generation might be achieved.

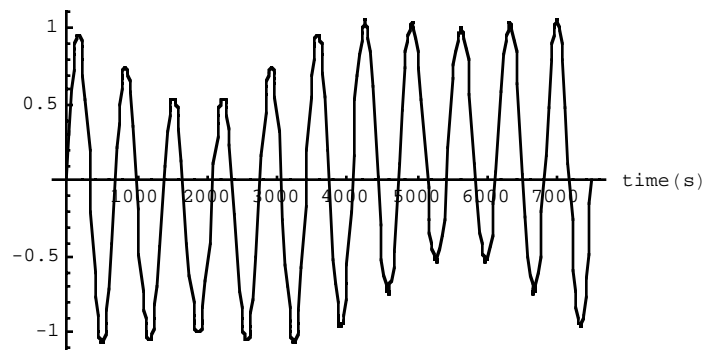


Figure 3. Function in the {} of Eqn. (16), plotted over one orbit for the baseline Lunavator.

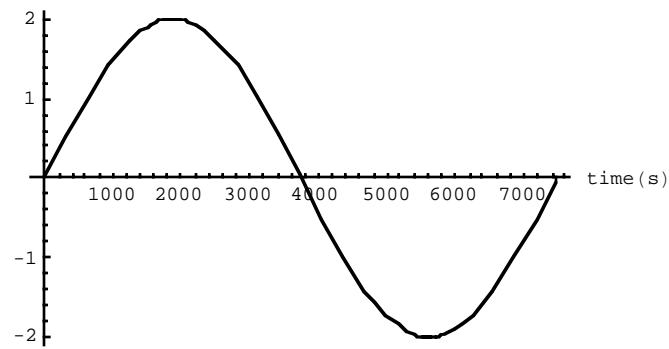


Figure 4. Tether reeling ΔL , in km.

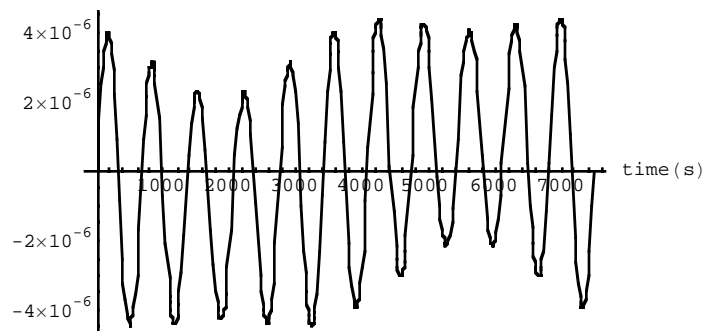


Figure 5. Rate of eccentricity change computed over one orbit according to Eqn. (16), with tether reeling as shown in Figure 4.

Thus, provided the Lunavator system has the capability to adjust the position of the central facility along the tether (which it needs anyway in order to adjust the tip velocity to deliver the payload to the surface), it appears that modest tether reeling operations can provide the ΔV necessary to maintain the stability of the tether's polar lunar orbit, without requiring propellant expenditure.

Example 2: Rotation of the Line-of-Apsides of an Elliptic-Earth-Orbit Tether Facility

In tether transportation systems such as the Cislunar system and the MERITT system, payloads are caught and thrown at or near the periaipse of the tether facility's orbit in order to maximize the energy transferred to the payload and to ensure that the facility's post-throw orbit does not drop into the planetary atmosphere. In order to permit payloads to be exchanged between the Earth and other planetary bodies, the tether system's orbit must be controlled so that the orbit's line of apsides points at or near the destination point. For example, in the Cislunar system, the Earth-orbit tether's line of apsides must point towards one of the moon's nodes so that it can throw a payload to the moon when it crosses its node. If the Earth were perfectly spherical, this would not be an issue, because the orbit orientation would remain fixed. However, the Earth's oblateness causes the line of apsides of elliptical orbits to precess (or regress for inclinations above 63.4°).

Using tether reeling, it may be possible to either counteract the apsidal precession to hold the line of apsides pointed at one lunar node, or to enhance it so that the apsides line up with one of the moon's nodes at the right time for a transfer to the moon. The rate of apsidal precession of an equatorial, elliptical orbit satellite is given by

$$(\dot{\Omega} + \dot{\omega}) = \left[-\frac{3}{2} J_2 \frac{R_e^2}{p^2} \bar{n} \cos(i) \right] + \left[\frac{3}{4} J_2 \frac{R_e^2}{p^2} \bar{n} (5 \cos^2 i - 1) \right] \quad (21)$$

where \bar{n} is the "mean mean motion" of the orbit, defined as

$$\bar{n} = \sqrt{\frac{\mu_e}{a^3}} \left[1 - \frac{3}{4} J_2 \frac{R_e^2}{p^2} \sqrt{1 - e^2} (1 - 3 \cos^2 i) \right], \quad (22)$$

For example, the Earth-orbit tether in the Cislunar system, with a semimajor axis of 12316 km and an eccentricity of 0.451, will experience an apsidal precession rate of approximately 1.57 degrees per day.

The baseline Earth-orbit tether designed to throw 2500 kg payloads to the moon would have a facility mass of 11,000 kg, an 80 km tapered tether massing 15,000 kg, and a tip grapple vehicle massing approximately 200 kg, rotating approximately 46 times per orbit. Because the tether mass in this system is not negligible, the simple dumbbell model for the tether system will produce only approximate results, but should be adequate for determining the order-of-magnitude of the orbital modifications that tether reeling can produce. For this purpose, the tether system will be modeled as a dumbbell consisting of a 11,000 kg facility, an 18.1 km long massless tether (calculated from the center-of-mass of the tapered tether and the payload, measured from the facility), and a 15,200 kg endmass.

If the tether length is held fixed, Eqn. (16) results a variation in the rate of apsides rotation around the orbit as plotted in Figure 6. Note that the values of $d(\omega + \Omega)/dt$ are more positive near perigee, and more negative near apogee. If Eqn. (16) is integrated over one orbit with no tether length variation, the net $d(\omega + \Omega)/dt$ is zero. If, however, the tether length is varied once per orbit so that the tether is shorter near perigee and longer near apogee, as shown in Figure 7, then the time-averaged value of $d(\omega + \Omega)/dt$ can be made negative. In this case, the tether length is varied ± 7.8 km in a sinusoidal manner [this would require a reeling rate of approximately 2.25 m/s]. A plot of the $d(\omega + \Omega)/dt$ calculated by Eqn. (16) with the tether length variation in Figure 7 is shown in Figure 8. Integrating this over one orbit results in an average apsidal regression rate of -1.57 degrees per day, sufficient to counteract the apsidal precession due to the Earth's oblateness. Thus tether reeling maneuvers can be used to hold the orientation of an elliptical tether facility fixed in the proper position to throw and catch payloads sent to and from one of the Moon's nodes, without expenditure of propellant. Alternatively, a more modest tether reeling operation could be used to "fine-tune" the apsidal

precession rate so that the tether orbit's line of apsides lines up with one of the two lunar nodes at the proper time for a transfer. In the MERITT system, tether reeling could be used to enable the EarthWhip and MarsWhip tethers to "track" their respective target planets as the Earth and Mars orbit the sun.

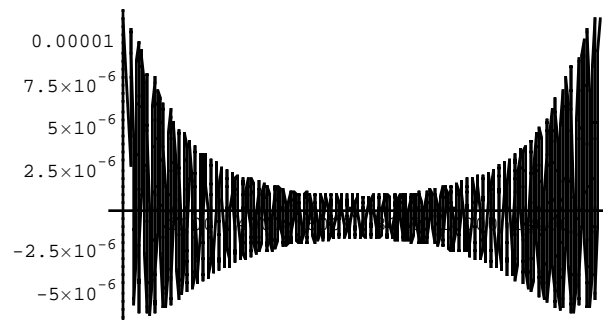


Figure 6. Plot of $d\omega/dt$ (in radians) over one orbit where the tether length is held constant.

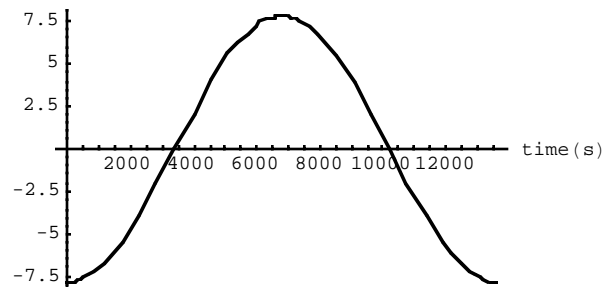


Figure 7. Sinusoidal tether length variation ΔL over one orbit, in kilometers.

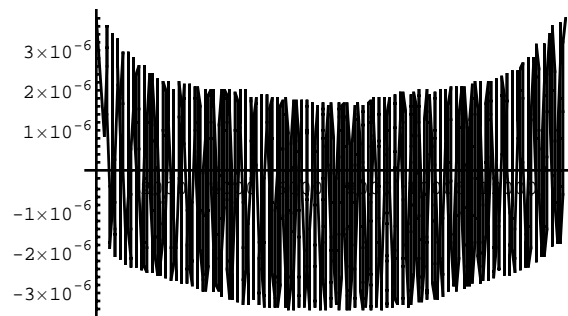


Figure 8. Plot of $d\omega/dt$ (in radians) over one orbit with the tether length variation shown in Figure 7.

Other Reeling Functions:

The plot of $d\omega/dt$ when the tether length is held constant in Figure 6 shows that the function has two characteristic frequencies: a once-per-orbit variation and a faster twice-per-rotation variation. The twice-per-rotation variation is easy to understand if one considers a "dumbbell" tether system with equal masses at both ends of the tether. In such a system, the forces on the tether would change direction twice per rotation. If the two masses were equal, the curve in Figure 6 would be symmetric about the time axis; the asymmetry in the curve is due to the fact that in the tether system analyzed, the two masses are different. In the tether reeling program investigated above, we used a once-per-

orbit variation in tether length to force the average $d\omega/dt$ to be negative. Alternatively, the tether system could reel the tether in and out a shorter distance twice per rotation. However, because the forces on the tether vary as the *square* of the tether length in Eqns. (14), it is more effective to reel a distance ΔL once per orbit than to reel a distance $\Delta L/n$, n times per orbit. Thus for a given reeling rate, a once-per-orbit reeling program is more effective at rotating the line of apsides than a twice-per-rotation program.

Summary

We have developed analytical methods for estimating the effectiveness of tether reeling operations for altering the eccentricity or orientation of the orbits of rotating tether facilities. Using these methods we have analyzed the possibility of using tether reeling to stabilize the orbit of a tether in low polar orbit around the Moon as well as to negate the apsidal precession of a tether facility in elliptical orbit around the Earth. These analyses indicate that for both applications, relatively modest tether reeling operations can provide the orbital modifications needed with no propellant expenditure required.

References

1. Hoyt, R.P., Uphoff, C.W., "Cislunar Tether Transport System", AIAA Paper 99-2690.
2. Nordley, G. and Forward, R.L., "Mars-Earth Rapid Interplanetary Tether Transport (MERITT) System: I - Initial Feasibility Analysis", AIAA Paper 99-2151.
3. Landis, G.A., "Reactionless Orbital Propulsion using Tether Deployment," *Acta Astronautica* 26(5), IAF Paper 90-254, 1992.
4. Martínez-Sánchez, M., Gavit, S.A., "Orbital Modifications using Forced Tether Length Variations", *J. Guidance, Control, and Dynamics*, 10(3) May-June 1987, pp 233-241.
5. Battin, R.H., *An Introduction to the Mathematics and Methods of Astrodynamics*, AIAA, NY, NY, 1987, p. 488.
6. Uphoff, C., "Studies of Polar Lunar Orbit Stability", Memo dated March 18, 1999.

HIGH-STRENGTH ELECTRODYNAMIC FORCE TETHER (HEFT) FACILITIES FOR PROPELLANTLESS IN-SPACE PROPULSION

Robert P. Hoyt
Tethers Unlimited, Inc.

Abstract

Tether facilities that combine rotating momentum-exchange tether principles with propellantless electrodynamic tether propulsion can provide a means to repeatedly boost satellites and other payloads from low-Earth-orbit to higher orbits, to the Moon, to Mars, and other planets without requiring propellant expenditure. We present a design concept for a “High-strength Electrodynamic Force Tether” (HEFT) Facility, and analyze two different scenarios for deploying satellites into LEO and MEO constellations. First, a HEFT facility in a circular low-LEO orbit can perform a standard slow rendezvous with a payload, then deploy the payload at the end of a tether, use electrodynamic forces to “spin-up” the tether over a period of several days, and then toss the payload into an elliptical transfer orbit. Alternatively, a rotating HEFT facility can capture a payload from a low-LEO orbit and then inject it directly into a circular high-LEO or MEO operational orbit. In addition, we examine the use of electrodynamic thrusting to restore the orbit of the Earth-orbit tether facility used to throw payloads to the Moon in the Cislunar Tether Transport System.

Introduction

In this work, we analyze the potential for combining the principles of rotating momentum-exchange tethers with the technology of propellantless electrodynamic tethers to create a system capable of repeatedly boosting payloads from Low-Earth Orbit (LEO) to higher orbits without requiring propellant expenditure. The primary purpose of this study is to identify a system architecture for a tether transfer facility that can provide an economically-competitive capability for deploying constellations of LEO satellites.

Background

A number of studies have concluded that rotating “momentum-exchange” tethers may provide a means for creating a reusable transportation system for transferring payloads between Low-Earth-Orbit (LEO) and higher orbits, and perhaps between LEO and the surface of the moon and other planetary bodies.^{1,2,3,4,5,6} However, when a rotating tether facility boosts the orbit of a payload, it does so by transferring some of its own orbital energy and momentum to the payload. Unless there is an equal mass of return traffic from higher orbits that the facility can catch and de-boost to restore its orbital energy, the facility will require some form of propulsion, such as high-Isp electric propulsion, to reboost itself in order to prepare for its next payload boost operation. A tether transport system, therefore, would enable payloads to be boosted from LEO to GEO or beyond with the fuel-economy of electric propulsion, but without the many-month transfer times normally required for high-Isp systems.

A recent study by Boeing, SAO, and NASA/MSFC concluded that a two-stage tether system for boosting communications satellites to geostationary orbit could significantly reduce the costs of launching payloads compared to the use of chemical upper stages.⁷ Nonetheless, because these satellites require onboard propulsion for station-keeping, and because electric propulsion is finding wide acceptance for this task, the technology that a tether transport system will be most likely to compete with in the future is onboard electric propulsion. If the tether system requires propellant for reboost after transfer operations, its mass and cost savings relative to onboard electric propulsion may not provide a benefit sufficient to outweigh the logistics complications and risks associated with using a tether system.

Electrodynamic tethers have the capability to provide propellantless propulsion in LEO.⁸ Electrodynamic tethers work by driving currents through a conducting tether; these currents interact with the geomagnetic field to produce a Lorentz $\mathbf{J} \times \mathbf{B}$ force which can propel the tether and the spacecraft

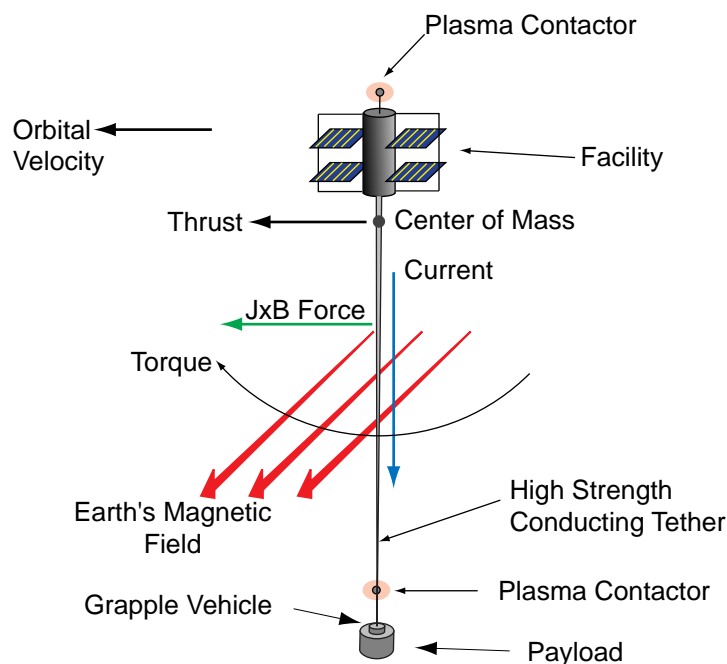


Figure 1. Schematic of a HEFT Facility.

to which it is attached. An electrodynamic tether uses the mass of the Earth, coupled through its magnetic field, as the “reaction mass” for propelling the spacecraft. Electrodynamic tethers, however, require a significant plasma density and magnetic field strength for efficient operation. Consequently, electrodynamic tether propulsion is normally thought to be useful only for propulsion missions within LEO altitudes.

A combination of electrodynamic tether propulsion with rotating tether techniques may provide a means not only for eliminating the need for propellant expenditure for reboosting a tether transfer facility, but also for enabling propellantless electrodynamic tether propulsion to be used for missions well beyond LEO altitudes.⁹ The basic concept is to place a payload at the end of a long high-strength, conducting tether and use electrodynamic “torque” to spin up the tether. In this manner, the payload can be accelerated at LEO altitudes and then thrown into a transfer trajectory beyond LEO. In addition, electrodynamic tether propulsion can simultaneously be used to boost the orbit of the tether facility, eliminating the need for propellant expenditure for reboost. This concept was originated in 1991 by Robert Hoyt, and later dubbed the “Hoyt Electrodynamic Force Tether (HEFT)” by Robert L. Forward.

HEFT Conceptual Design

A HEFT facility could serve as the first building block of modular LEO-GEO, LEO-Lunar, or LEO-Mars transport systems. The HEFT concept is illustrated in Figure 1.

In this configuration, the HEFT system would be composed of a central facility, a tether, and a grapple vehicle at the tether tip. The central facility would include a power supply and a tether deployer/reeling mechanism, and its mass would be larger than the mass of the payload. The tether would be composed primarily of high-strength fibers, such as Spectra 2000, braided in a multiline survivable structure such as the Hoytether™. A small fraction of its mass would be a conductor, such as aluminum wire, to allow it to conduct electrical current. At both ends of the tether, plasma contactors would provide electrical connection to the ionospheric plasma.

By using the power supply to apply a voltage between the two ends of the tether, current can be forced to flow along the tether; the contactors will transmit this current to the ionosphere, and the current

“circuit” will be completed by the ionospheric plasma. The action of the radial current \mathbf{I} flowing across the Earth’s magnetic field \mathbf{B} would create a Lorentz force $\mathbf{F} = \mathbf{I} \times \mathbf{B}$.

If the central facility has a mass different than the payload mass, the electrodynamic force distributed along the tether will result in both a net thrust on the tether system and a torque on the tether system around its center of mass. By properly varying the direction of applied current as the tether rotates and moves around in its orbit, the electrodynamic forces on the tether can be used to:

- Increase or decrease the tether spin rate
- Boost or deboost the orbit of the tether system
- Change the inclination of the tether system’s orbit
- Adjust the argument of perigee of the tether system’s orbit.

In this paper, we will analyze the potential performance of the HEFT concept for boosting payloads in two different scenarios: first, a tether facility in circular LEO that could spin-up and toss a payload into an elliptical transfer orbit; and second, a tether facility in elliptical orbit, with perigee in LEO, that could pick up a payload from a circular LEO holding orbit and deliver it to a higher, circular operational orbit.

Circular LEO-HEFT Facility

The first scenario is illustrated in Figure 2. A HEFT Facility in a circular, low-Earth orbit (thus the “cLEO” moniker), with its tether initially retracted and the system not spinning, could rendezvous with a payload and capture it. It would then attach the payload to the grapple vehicle at the tether tip and deploy the tether. Next, by properly controlling the direction of the tether current, it could cause the tether to librate back and forth until the tether system “turns over”, after which it would continue to drive the current so as to increase the tether spin rate. Once the tether tip reaches the desired velocity relative to the tether system’s center of mass, the payload could be released, injecting it into an elliptical transfer to its desired orbit. In this scenario, the payload would be required to perform a ΔV burn at apogee of its transfer orbit in order to circularize its orbit.

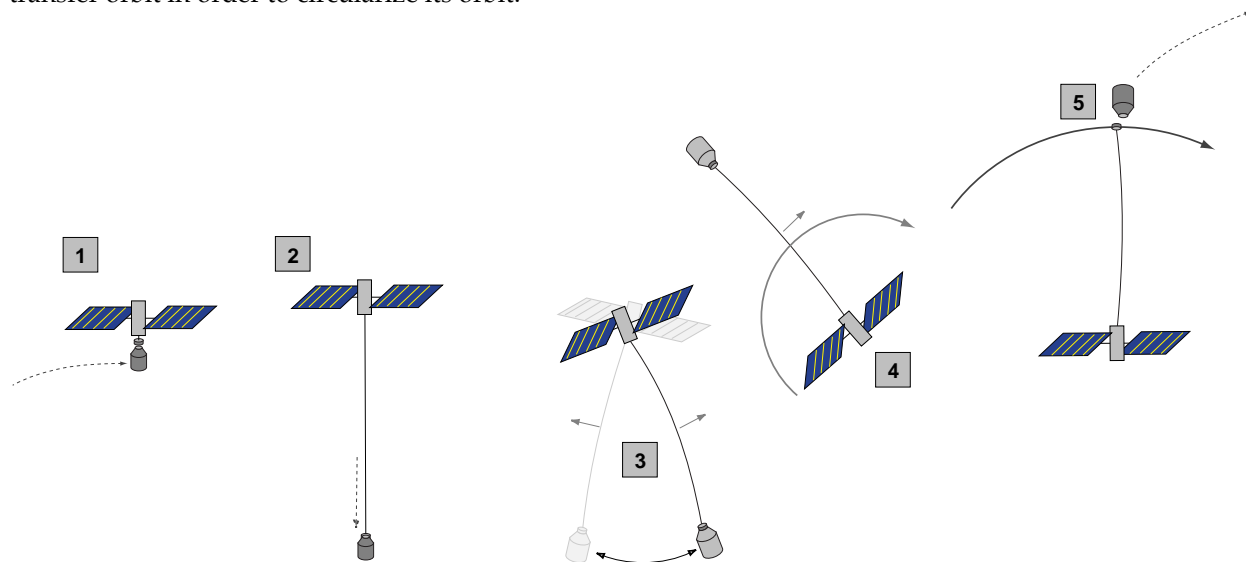


Figure 2. Schematic of method for boosting a payload using a LEO-HEFT Facility. 1) Payload rendezvous with facility. 2) Facility deploys payload at the end of the tether. 3) Facility pumps current along the tether, alternating the current direction with the swing, in order to pump the in-plane tether libration. 4) Once tether “turns over”, the facility continues to drive current through the tether to accelerate the rotation until the tip velocity equals the desired ΔV for the payload. 5) At the top of the tether swing, the tether releases the payload, tossing it into an elliptical transfer trajectory.

Analysis

To examine the feasibility and utility of the cLEO-HEFT concept, we will first calculate analytically the power and time required for a cLEO-HEFT system to spin-up and accelerate a 1000 kg payload to 1 km/s. For simplicity in the analysis, we will assume that the facility mass is large compared to the payload mass, so that the system's center-of-mass is located at the facility end of the tether. We will choose a tether length of $L = 50$ km. If the tether is constructed of Spectra 2000 high strength fiber, with tenacity of 4 GPa and density of 0.76 g/cc, and a design safety factor of 3.5, the required mass for the tether is 1.4 times the payload mass, or 1400 kg. At the tether tip, the grapple fixture and associated hardware are assumed to mass 100 kg. In addition to the high strength fibers to bear the load, the tether will have a conductor running along its length. We will choose this conductor to be an aluminum wire massing 100 kg, so the tether masses a total of 1500 kg. We will assume that the tether will simultaneously boost the payload and deorbit a spent rocket massing the same as the payload.

The rotational inertia of the payload and the grapple at the end of the 50 km tether are

$$\Theta_p = m_p L^2 = 2.75 \times 10^{12} \text{ kg} \cdot \text{m}^2. \quad (1a)$$

The taper on this tether is small, so the rotational inertia of the tether can be approximated by

$$\Theta_t \approx \frac{m_t}{L} \int_0^L x^2 dx = m_t \frac{L^2}{3} = 1.25 \times 10^{12} \text{ kg} \cdot \text{m}^2 \quad (1c)$$

The rotational inertia of the facility can be ignored, since it is at the center of the rotational system. The total rotational inertia of the tether system is thus

$$\Theta_{tot} = \Theta_t + \Theta_p = 4 \times 10^{12} \text{ kg} \cdot \text{m}^2. \quad (2)$$

The resistance of the tether is

$$R = \frac{\rho \delta L^2}{m_{conductor}} = 1850 \text{ } \Omega, \quad (3)$$

where $\rho = 27.4 \times 10^{-9}$ is the resistivity of aluminum and $\delta = 2700 \text{ kg/m}^3$ is its density.

The power supply is used to drive a current of $I = 2.5$ Amps through the tether. The cLEO-HEFT system will thus consume an ohmic power of

$$P_{ohm} = I^2 R = 11.5 \text{ kW}. \quad (4)$$

In addition to the ohmic power, the cLEO-HEFT facility will see a power consumption due to the voltage induced along the tether by its motion through the geomagnetic field; however, this voltage will vary sinusoidally as the tether rotates around the facility. To drive a constant current, the facility will thus require an average power of 11.5 kW, varying with the rotation between a maximum of 20 kW and a minimum of 3.5 kW. The variations in the power demand could be minimized by relaxing the requirement for constant current, but for simplicity in our analysis we will assume that the current is driven at a constant level.

For this analysis, we will assume that the current flows along the whole length of the tether. The action of the current I flowing across the Earth's magnetic field \mathbf{B} induces a Lorentz $I \times \mathbf{B}$ force on the tether. At 350 km altitude, $B \approx 2.64 \times 10^{-5} \text{ T}$. The net torque on the system is

$$\tau = \int_0^L F l \, dl = IB \frac{L^2}{2} = 82500 \text{ N} \cdot \text{m}^2. \quad (5)$$

To achieve a tip speed of 1 km/s at the end of the 50 km tether, the cLEO-HEFT facility must accelerate rotationally to an angular velocity of

$$\omega = V_{tip}/L = 0.02 \text{ rad/s}. \quad (6)$$

With a constant torque τ , the cLEO-HEFT will spin up at a constant angular acceleration α and the payload will reach the desired 1 km/s velocity in

$$T = \omega/\alpha = \omega\Theta/\tau \approx 11 \text{ days.} \quad (7)$$

If the current is held constant, the power going into the rotational energy of the system will vary linearly with the angular momentum, increasing from zero to a maximum of $P_{rot} = \tau\omega = 1.65 \text{ kW}$. Consequently, the “thrust efficiency” will increase from zero to approximately 14% as the facility spins up, with an average of around 7%. While this thrust efficiency is lower than the numbers usually quoted for some electric propulsion techniques, this measure of thrust efficiency is not a valid comparison. In electric propulsion, the propellant and power supply must be accelerated along with the payload; if one looks at the efficiency of thrust power going into the payload alone, the efficiency of standard electric propulsion techniques is very small. In the cLEO-HEFT, there is no propellant required, and the power supply is at the center of the system, so the power required to spin up its mass is negligible. Thus nearly all of the thrust power goes into accelerating the payload.

Thus this simple analytical approach predicts that a cLEO-HEFT facility in a 300 km orbit, with a 50 km, 1500 kg tether and a power supply of 11.5 kW, could accelerate a 1000 kg payload by 1 km/s within 11 days, and toss it into a $350 \times 5718 \text{ km}$ orbit.

Performance of a cLEO-HEFT Facility for Deploying Big-LEO Constellation Satellites.

To obtain a more detailed prediction of HEFT tether performance, we utilized a numerical tether dynamics simulation program to model a cLEO-HEFT facility in a 350 km circular, equatorial orbit deploying a constellation of 1000 kg satellites to a 2000 km operational orbit, as illustrated in Figure 3.

At 350 km altitude, the facility’s orbital velocity is 7.7 km/s. To inject the satellite into a Hohmann transfer with an apogee at 2000 km, the tether must be spun up to a tip speed of approximately 0.39 km/s.

For this analysis, we assumed that the facility consisted of:

- A central facility with a total mass of 35,000 kg (e.g., a 5 ton facility, including power supply and tether deployer, with a 30 ton Shuttle External Tank for ballast).
- A grapple/rendezvous vehicle on the end of the tether with a mass of 100 kg.
- A 25 km long tether. The tether was chosen to include a conductor mass of 200 kg of aluminum, which at room temperature will have a resistance of 231 ohms. For a ΔV of 0.4 km/s and a total tip mass of 1100 kg, a tapered tether constructed of Spectra 2000 with a safety factor of 4 will mass approximately 108 kg; including the conductor, the tether will thus mass only 308 kg. At the final tip velocity of 0.4 km/s, the acceleration at the tip of the 25 km tether will be 0.65 gees

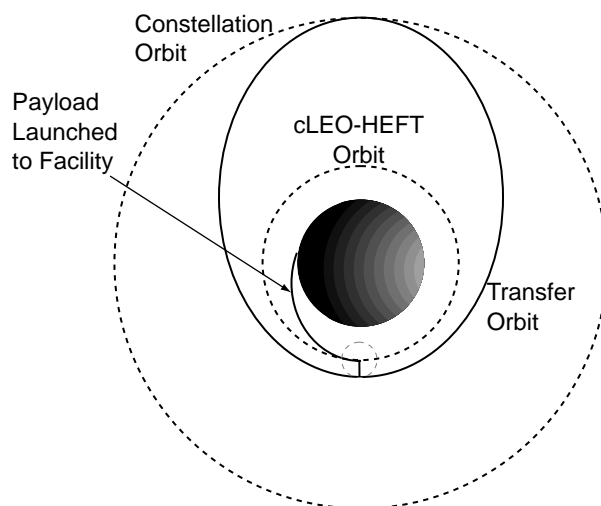


Figure 3. cLEO-HEFT Orbital Scenario.

To study the potential performance of this concept for deploying constellations of satellites, we utilized the TetherSim program to simulate the rotational acceleration of the tether and payload. (In this simulation, deorbit of the upper stage was not performed). TetherSim is a numerical simulation program that includes models for the orbital mechanics, tether dynamics, Earth's magnetic field and ionosphere, atmospheric drag, as well as electrodynamics and plasma interaction physics. For this first study, we assumed that the system was capable of driving a maximum current of 6.5 Amps through the tether (consuming 10 kW in ohmic power). For this simulation, we assumed that electrical contact to the ionospheric plasma was provided by hollow-cathode plasma contactors at each end of the tether with an effective contact resistance of $10\ \Omega$ each.

Figure 4 shows the velocity of the tether tip (and attached payload) relative to the center of mass of the system. The graph shows that the tether tip swings back and forth with a growing amplitude for about half a day, and then the tether “turns over” and begins to rotate. Once it begins to rotate, the tip velocity increases at a steady rate of $0.0017\ \text{m/s}$, or $147\ \text{m/s}$ per day. At that rate of rotational acceleration, the tether system provides the 1000 payload an equivalent “thrust” of $1.7\ \text{N}$. It reaches the desired tip speed of $0.39\ \text{km/s}$ in less than three days. Including the ΔV imparted to the payload when it catches it from its initial orbit, this HEFT facility would provide a total ΔV of $0.43\ \text{km/s}$ to the satellite.

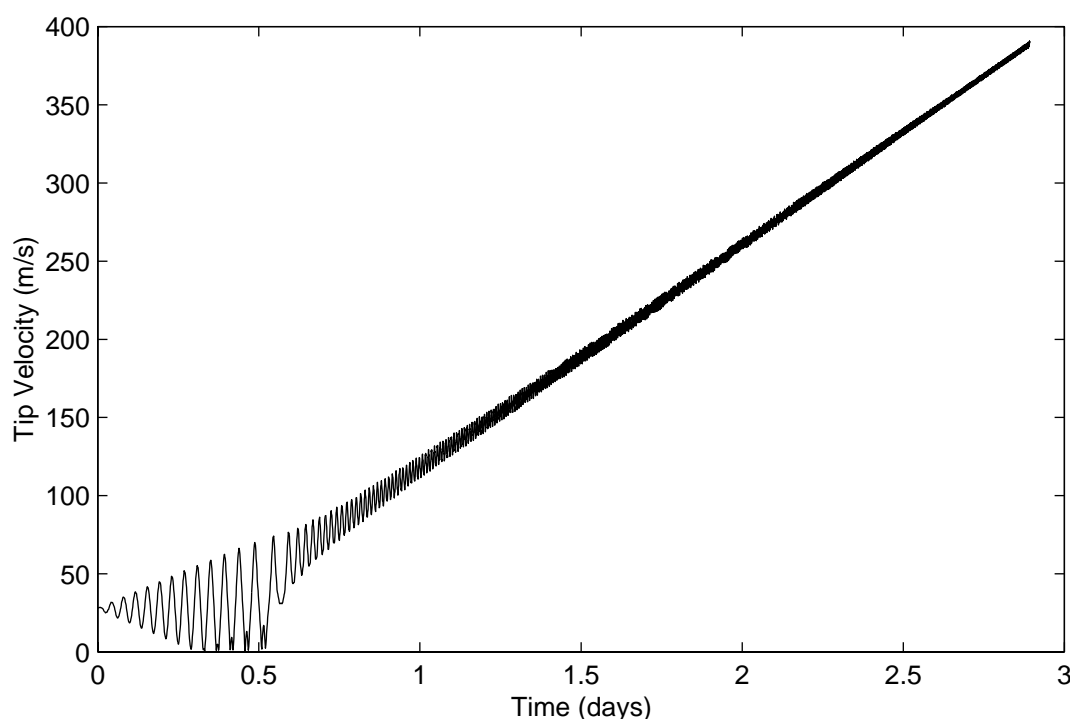


Figure 4. Tip velocity of a HEFT tether accelerating a 1000 kg payload for a transfer from 350 km to 2000 km. Tether is 25 km long, current is 6.5 A, and the facility orbit is 350 km, 0° inclination.

After tossing the payload, the tether can then reverse the current and slow the rotation rate down. Simultaneously, it would use electrodynamic tether propulsion to reboost the perigee of its own orbit, which will have dropped by approximately 40 km after throwing the payload. This reboost and de-spin operation will require about 2 days.

Thus, such a cLEO-HEFT facility could boost 1 satellite per week from a 325 km holding orbit to a transfer orbit with a 2000 km apogee, while requiring no propellant.

A potential limitation to this system concept, however, is that for deploying high-LEO constellations, a HEFT facility in circular low-LEO orbit could provide the satellite with only about $1/2$ of the ΔV needed to boost it from circular low-LEO holding orbit to a circular high-LEO operational orbit. At the apogee of

its transfer orbit, the satellite will need an additional 0.381 km/s ΔV to circularize. The satellite will thus require some onboard propulsion and propellant. Although the cLEO-HEFT facility may be able to reduce the propellant mass required to boost the satellite to its operational orbit by over half, and could simultaneously deorbit the launch vehicle, for LEO and low-MEO constellations this may not be enough of an advantage to outweigh the costs of rendezvousing with the facility and the cost of the tether system.

The cLEO-HEFT concept may be more advantageous for deploying satellites to higher altitudes. For transfer to high-MEO and GEO orbits, the apogee circularization ΔV becomes smaller relative to the ΔV needed in LEO to inject the satellite into the transfer orbit. For example, transferring a GPS satellite from a 300 km holding orbit to its 20,000 km operational orbit requires an injection ΔV of 1.02 km/s and a circularization ΔV of 0.71 km/s . From the standard rocket equation, we find that using a cLEO-HEFT facility to provide the 1.02 km/s transfer orbit injection ΔV could reduce the propellant requirements for the transfer by 66%. Transferring a payload from 300 km holding orbit to a GEO orbit requires a 2.4 km/s injection ΔV and a circularization ΔV of 1.4 km/s ; using the tether facility to provide the GTO insertion ΔV could reduce the propellant requirements by 76%. Thus for these higher target orbits the cLEO-HEFT concept may become more advantageous.

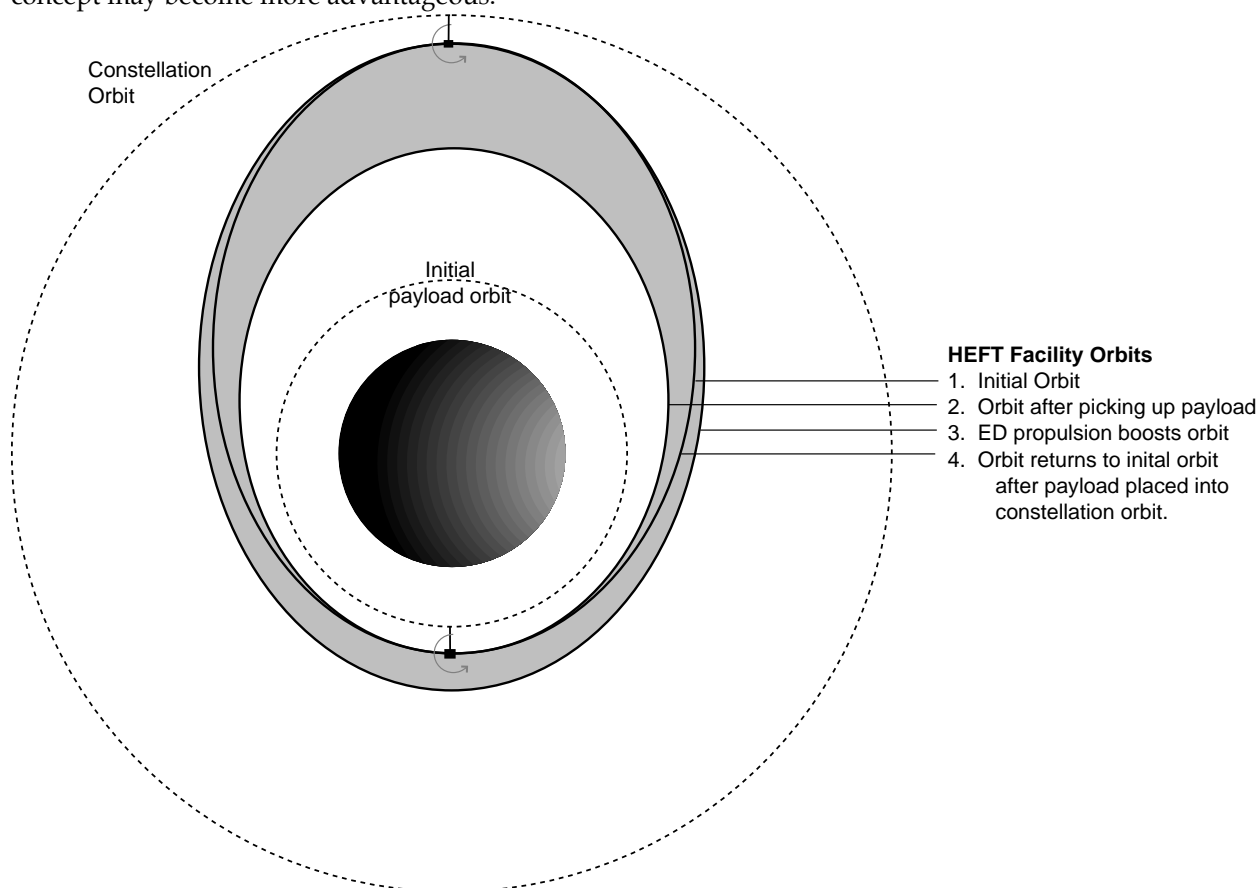


Figure 5. HEFT facility in elliptical LEO orbit designed to provide both boost and deorbit services to a constellation.

Elliptical-LEO HEFT Facility

Another possible implementation of the HEFT concept that may be more economically competitive is a tether facility placed into an elliptical LEO orbit that is used to transfer a payload from a low-LEO holding orbit to a circular operational orbit in high-LEO. This concept is illustrated in Figure 5. In this implementation, satellites would be launched into a low-LEO orbit. The eLEO-HEFT facility would initially be in an elliptical orbit with a perigee just above the satellite's holding orbit, and an apogee just below the constellation's operational orbit. The HEFT facility would have one tether rotating around a

massive central body. The initial rotation and length of the tether would be chosen such that at perigee the tether tip could rendezvous with the payload and capture it. Upon capture of the payload, the system will be in a new orbit with essentially the same perigee but a reduced apogee altitude. The system would then use electrodynamic propulsion to boost both the perigee and the apogee of its orbit, until the apogee is just below the constellation's orbit. The facility will then allow the tether to pay out to reduce the rotation rate slightly. At apogee, the tether can then release the satellite into the circular operational orbit. Upon releasing the satellite, the facility's orbit reverts back to its original values. It is then ready to boost another payload.

In addition, because the HEFT facility's apogee is just below the constellation's orbit, it can also perform de-boosting operations on satellites that need to be removed from the operational orbit, either to dispose of old satellites, or to bring malfunctioning satellites down to a low-LEO facility for repair.

As an example, we will consider the case of a HEFT facility used to boost satellites massing 1,000 kg from a holding orbit of 250 km to a 2,000 km operational orbit. For this example, we take the facility mass to be 5000 kg, the grapple mass to be 100 kg, and the tether length to be 25 km. The tether mass will be a total of 265 kg, 100 kg of which is conductor. Initially, the facility orbits with a perigee of 270 km and an apogee of 1980 km. It extends a 20 km long tether, and spins it up to rotate with a tip velocity of 418 m/s. At that length and velocity, the acceleration experienced at the tip is 0.875 gees. At perigee, it can then capture a satellite from a 250 km, circular orbit. Upon catching the satellite, the facility transfers some of its orbital momentum to the satellite, and thus the system's apogee is reduced to 1879.7 km. The HEFT facility will then use electrodynamic tether propulsion to raise the apogee by 100 km and the perigee by approximately 90 km. It will then reduce the tip velocity of the tether to 397 m/s by paying out an additional 1.03 km of tether, and when it reaches apogee it can release the satellite into the circular 2,000 km operational orbit. Upon releasing the payload, the facility's perigee will drop back to 270 km.

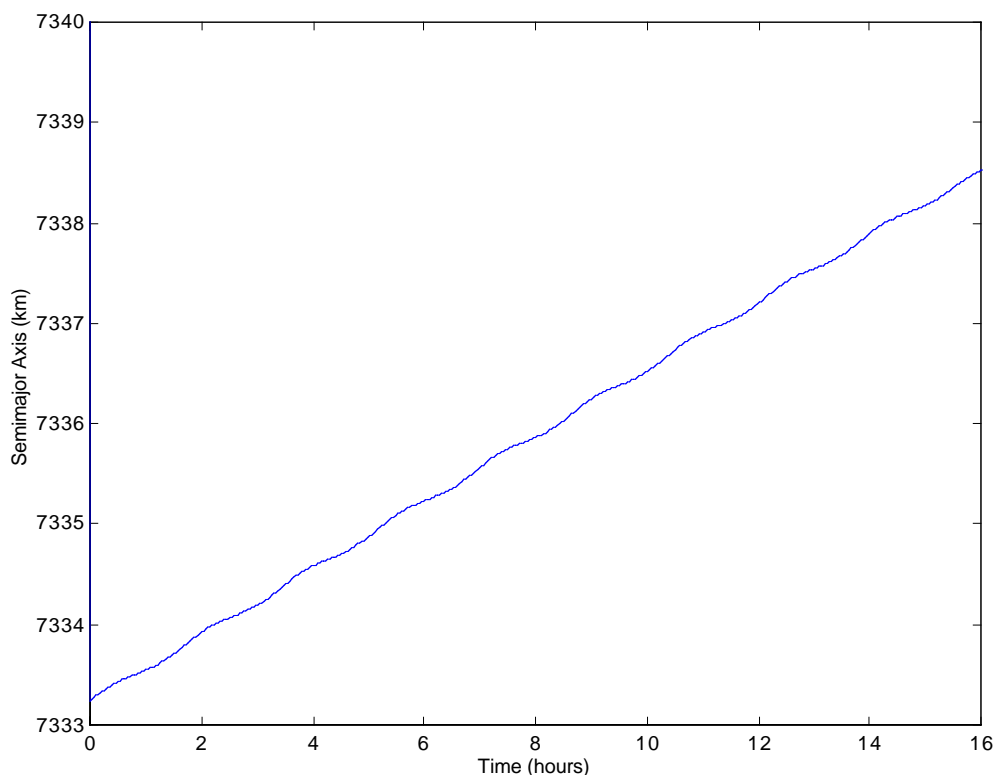


Figure 6. Increase of the semimajor axis of the HEFT tether facility's orbit due to electrodynamic reboosting.

Using the TetherSim program, we have simulated boosting of the tether orbit using electrodynamic propulsion. In this simulation, we assumed that the tether facility included a power supply able to provide a continuous 5 kW to drive current through the tether. Figure 6 shows the increase in the semimajor axis of the tether's orbit over a period of 16 hours. The semimajor axis is boosted 5200 meters in that time, a rate of 325 m/hour. At that boost rate, the tether could achieve the desired orbit within about 12 days. If we normalize by the system power and the mass of the facility plus payload, we obtain a "specific boost rate" of approximately 10 (km•mt)/(kW•day).

Reboost of Highly-Elliptical Earth-Orbit Tether Facility in the Cislunar System

This method may also have significant potential for reboosting facilities in highly elliptical orbits such as the Earth-orbit tether boost facilities in the Cislunar Tether Transport System and the MERITT system. Electrodynamic reboost of these facilities would enable them to repeatedly boost payloads to the Moon and Mars without requiring propellant expenditure. Because electrodynamic propulsion requires the presence of an ambient plasma, the electrodynamic reboosting of the orbit can only be performed while the tether is in LEO. However, since the tether needs primarily to reboost its apogee, it needs to perform its thrusting when it is near perigee, so thrusting only when in LEO is exactly what is required. Using the TetherSim program, we have modeled reboosting of the tether facility described in Appendix A. This facility masses a total of 26,250 kg. The facility uses a 11 kW solar electric power supply to generate power. While the facility is above LEO altitudes, the system stores this energy in batteries, and when the tether is below 2000 km in altitude, it expends this stored energy at a rate of 75 kW. Figure 7 shows the increase in the tether's semimajor axis over a period of one day. The semimajor axis is increased in a "stepwise" fashion because the tether is only boosting during the fraction of the elliptical orbit when its altitude is below 2000 km. The facility boosts its semimajor axis approximately 20 km in one day; at this rate, it can reboost its orbit within 85 days.

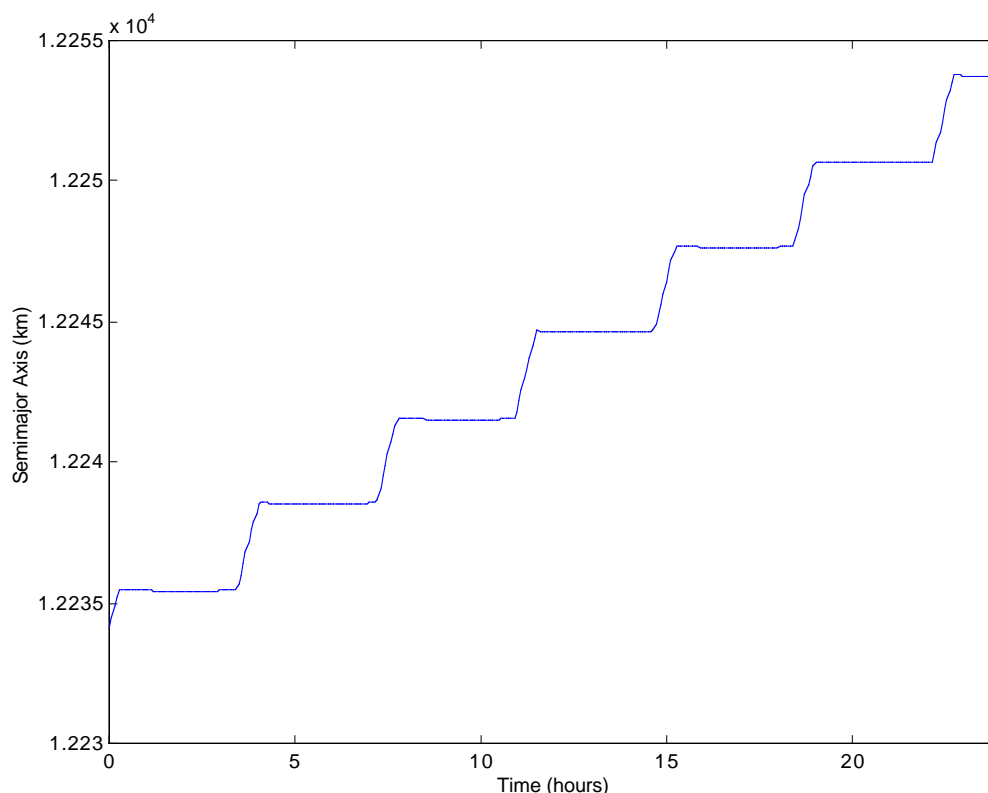


Figure 7. Reboost of the Earth-orbit tether in the Cislunar Tether Transport System.

References

1. Hoyt, R.P., Forward, R.L., *LEO-Lunar Tether Transport System Study*, Tethers Unlimited, Inc. Final Report on Smithsonian Astrophysical Observatory Purchase Order SO6-34444, subcontract on NASA Grant NAG8-1303, April 1997.
2. Carroll, J.A., *Preliminary Design of a 1 km/sec Tether Transport Facility*, March 1991, Tether Applications Final Report on NASA Contract NASW-4461 with NASA/HQ.
3. Oldson, J, Carroll, J.A., "Potential Launch Cost Savings of a Tether Transport Facility, " AIAA paper 95-2895, *31st AIAA/ASME/SAE/ASEE Joint Propulsion Conference*, July 1995.
4. Forward, R.L., "Tether Transport from LEO to the Lunar Surface," AIAA paper 91-2322, *27th AIAA/ASME/SAE/ASEE Joint Propulsion Conference*, July 1991.
5. Bekey, I., "Decreasing the Cost/Pound and Increasing the Performance of the Reusable Launch Vehicle by Using Tethered Payload Deployment, " NASA/HQ white paper, March 25, 1996.
6. Bekey, I., "Tethering: a new technique for payload deployment, " *Aerospace America*, March 1977, pp. 36-40.
7. Bangham, M.E., Lorenzini, E., Vestal, L., *Tether Transportation System Study*, NASA/TP-1998-206959, March 1998.
8. Johnson, L., "Propulsive Small Expendable Deployer System Mission (ProSEDS)", *OAST Advanced Propulsion Workshop*, JPL, Pasadena, CA, 20-22 May 1997.
9. Forward, R.L., Hoyt, R.P., *Failure Resistant Multiline Tether*, PCT/US97/05840, patent application filed 22 April 1997.

MARS-EARTH RAPID INTERPLANETARY TETHER TRANSPORT (MERITT) SYSTEM: I. Initial Feasibility Analysis

Dr. Robert L. Forward

Tethers Unlimited, Inc. 8114 Pebble Court, Clinton WA 98236
Phone/Fax: 1-360-579-1340; Email: TU@tethers.com; Web: www.tethers.com

Gerald D. Nordley

Consultant, 1238 Prescott Avenue, Sunnyvale CA 94089-2334
Phone: 1-408-739-4032; Email: gdnordley@aol.com

ABSTRACT

Routine travel to and from Mars demands an efficient, rapid, low cost means of two-way transportation. To answer this need, we have invented a system of two rotating tethers in highly elliptical orbits about each planet. At Earth, payload is picked up near periapsis and is tossed after an odd number of half-rotations later, again near periapsis, at a velocity sufficient to send the payload on a high-speed trajectory to Mars. At Mars, it is caught near periapsis and is released later at on a suborbital trajectory. The system works in both directions and is reusable; kinetic energy lost by the throwing tether can be restored by receiving payloads and/or auxiliary propulsion. Tethers with tip velocities of 2.5 km per second can send payloads to Mars in as little as 90 days if aerobraking is used at Mars. Tether-to-tether transfers without aerobraking may be accomplished in about 130 to 160 days. Tether systems using commercially available tether materials at reasonable safety factors can be as little as 15 times the mass of the payload being handled. This is a relatively new concept and tasks needing further study are listed in the final section of the paper.

BACKGROUND

The idea of using rotating tethers to pick up and toss payloads has been in the tether literature for decades [1-7]. In 1991, Forward [8] combined a number of rotating tether concepts published by others [2,6,7] to show that three rotating tethers would suffice to move payloads from a suborbital trajectory just above the Earth's atmosphere to the surface of the Moon and back again, without any use of rockets except to get out of

the Earth's atmosphere. The three tethers consisted of a "LEO" rotating tether in a nearly circular Low Earth Orbit, an "EEO" rotating tether in a highly Elliptical Earth Orbit, and a "Lunavator" rotating tether cartwheeling around the Moon in a circular orbit whose altitude is equal to the tether length, resulting in the tip of the tether touching down on the lunar surface. This concept has since been examined in detail by Hoyt and Forward [9-12], and is presently the subject of a Tethers Unlimited, Inc. Phase I Contract from the NASA Institute for Advanced Concepts, Dr. Robert A. Cassanova, Director.

In the process of thinking about ways to improve the performance of the system, Forward realized that much of the gain in the three tether system came from the EEO tether, since its center-of-mass velocity at perigee was quite high, and when the tether tip rotational velocity was added, the toss velocity was not only very high, but was taking place deep in the gravity well of Earth. It is well known in rocketry that it always pays to make your v burns deep in the gravity well of a planet, and this rule of thumb applies equally well to tether tosses. In fact, in the LEO-Lunar papers [9-12], the EEO tether throws the payload so hard toward the moon that if the Lunavator does not catch it, the payload leaves the Earth-Moon system in a hyperbolic orbit. Forward then wondered how far a single EEO tether could throw a payload if the tether were in a Highly Elliptical Orbit and rotating near the maximum tether tip velocity possible with presently available commercial tether materials. After a few back-of-the-envelope calculations, the answer was found to be: "All the way to Mars... and beyond." Not believing the answer, Forward enlisted the aid of his co-author, an experienced orbital "mechanic," who confirmed the back of the envelope calculations with more detailed calculations. The Mars-Earth Rapid

Interplanetary Tether Transport (MERITT) System is the result.

MERITT SYSTEM DESCRIPTION

The MERITT system consists of two rapidly rotating tethers in highly elliptical orbits: EarthWhip around Earth and MarsWhip around Mars. A payload capsule is launched from Earth into a low orbit or suborbital trajectory. The payload is picked up by a grapple system on the EarthWhip tether as the tether nears perigee and the tether arm nears the lowest part of its swing. It is tossed later when the tether is still near perigee and the arm is near the highest point of its swing. The payload thus gains both velocity and potential energy at the expense of the tether system, and its resulting velocity is sufficient to send it on a high-speed trajectory to Mars with no onboard propulsion needed except for midcourse guidance.

At Mars, the incoming payload is caught in the vicinity of periapsis by the grapple end of the MarsWhip tether near the highest part of its rotation and greatest velocity with respect to Mars. The payload is released later when the tether is near periapsis and the grapple end is near the lowest part of its swing at a velocity and altitude which will cause the released payload to enter the Martian atmosphere. The system works in both directions.

The MERITT system can give shorter trip times with aerobraking at Mars because the incoming payload velocity is not limited by the maximum tether tip velocity and thus payloads can use faster interplanetary trajectories.

In the following subsections we illustrate the general outlines of the system and define the terms used. This initial "feasibility" analysis has not dealt with the many problems of interplanetary phasing and trades. These issues will be addressed in future papers as time and funding allow.

Interplanetary Transfer Orbits

As shown in Figure 1, in the frame of reference of the Sun, acting as the central mass of the whole system, a payload leaves the origin planet, on a conic trajectory with a velocity v_o and flight path angle ϕ_o and crosses the orbit of the destination planet with a velocity v_d and flight path angle ϕ_d . Departure from the origin planet is timed so that the payload arrives at the orbit of the destination body when the destination body is at that point in its orbit. Many possible trajectories satisfy these conditions, creating a trade between trip time and initial velocity.

The classic Hohmann transfer ellipse (H) is a bounding condition with the least initial velocity and longest trip time. The Hohmann transfer is tangential to both the departure and destination orbits and the transfer orbits. The direction of the velocity vector is the same in both orbits at these "transfer" points and only differs in magnitude. A Δv change in payload velocity (usually supplied by onboard propulsion) is required at these points for the payload to switch from one trajectory to another.

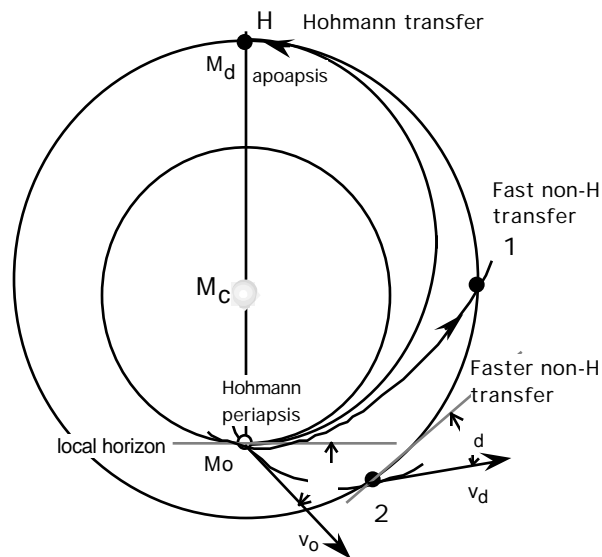


Figure 1. General Orbit Transfer Trajectories.

Faster non-Hohmann transfers may be tangential at origin, destination, or neither. They may be elliptical or hyperbolic. For a given injection velocity above the Hohmann minimum constraint, the minimum-time transfer orbit is generally non-tangential at both ends. An extensive discussion of the general orbit transfer problem may be found in Bate, Mueller and White [13]

For reasons discussed below, using tethers in an elliptical orbit with a fixed tip velocity to propel payloads results in an injection velocity constrained to the vector sum of a constant hyperbolic excess velocity of the released payload and the orbital velocity of the origin planet. When a tether only is used to receive the payload, a similar constraint exists on the destination end; the incoming trajectory is a hyperbola and the periapsis velocity of the hyperbolic orbit must not exceed what the tether can handle. This periapsis velocity is determined by the vector sum of the orbital velocity of the destination planet, that of the intersecting payload orbit at the intersection, and the fall through the gravitational field of the destination planet.

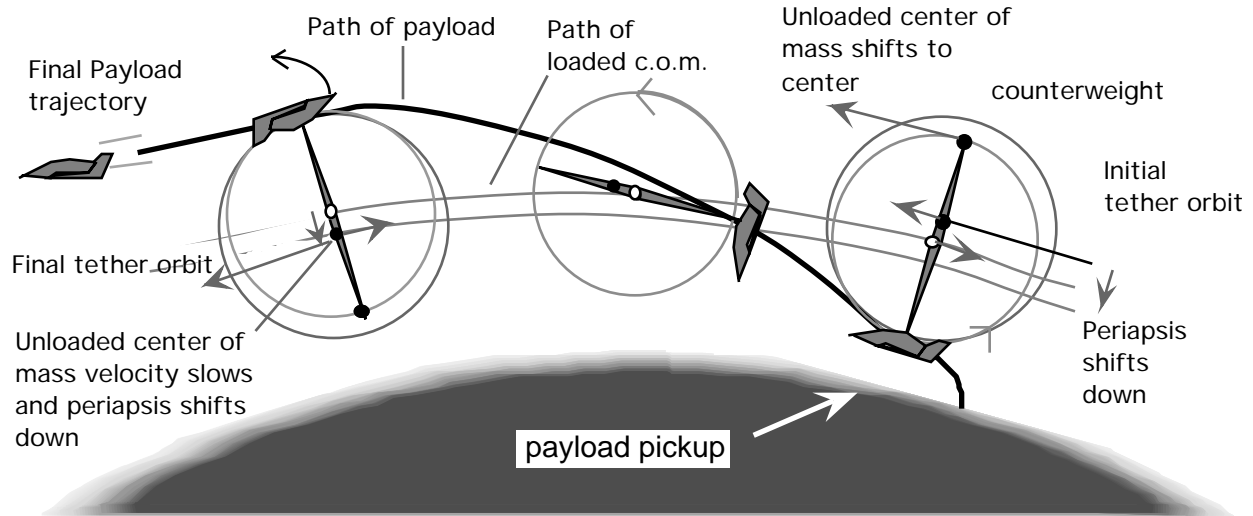


Figure 2: General geometry of tether pickup and throw orbital injection.

When passage through the atmosphere of the destination planet (aerobraking) is used to remove some of the incoming velocity, the constraint becomes an engineering issue of how much velocity can be lost in the atmospheric passage. Experience with the Apollo mission returns (circa 12 km/s) and the Mars Pathfinder landing indicates that with proper design, much more velocity can be dissipated than is required to assist tether capture.

Real passages through space take place in three dimensions. To the first order, however, transfer orbits are constrained to a plane incorporating the Sun, the origin planet at launch and the destination planet at arrival. The injection vector must occur in this plane, or close enough to it that on-board payload propulsion can compensate for any differences. This analysis considers only coplanar trajectories, but, as discussed later, this is not a great handicap.

As the payload moves out from the influence of the mass of the origin planet, its trajectory becomes more and more influenced by the mass of the Sun, until the origin planet mass can be essentially neglected. Likewise, inbound payloads become more and more influenced by the destination planet mass until the mass of the Sun may be neglected. For first order Keplerian analysis it is customary to treat the change of influence as if it occurred at a single point, called the patch point. At this point, a coordinate transformation is made.

Payload Pickup and Injection

Figure 2 shows the general geometry of a tether picking up a payload from a sub-orbital trajectory at a point just outside the atmosphere of the origin planet

and injecting it into an interplanetary transit trajectory. The payload is picked up, swung around the tether's center of mass along the circle as it moves along its orbit, and is released from the tip of the tether near the top of the circle. In the process, the tether center of mass loses both altitude and velocity, representing the loss of energy by the tether to the payload. This energy loss may be made up later by propulsion at the tether center and/or in the reverse process of catching incoming payloads.

Around the time of pick-up, the trajectory of the payload must be of equal velocity and should be very nearly tangential (no radial motion) to the circle of motion of the tether tip in the tether frame of reference. This tangential condition increases the time for a docking maneuver to be consummated. It is easy to see how this condition may be satisfied by rendezvous at the mutual apsides of the tether orbit and the payload pickup orbit, but other, more complex trajectories work as well. It is not a requirement, however, that the tether plane of rotation, the tether orbit, and the payload pickup orbit be coplanar. The mutual velocity vector at pick-up is essentially a straight line, and an infinite number of curves may be tangent to that line. The tether rendezvous acts as a kind of patch point, as the plane of the tether's rotation becomes dominant. The practical effect of this is to allow considerable leeway in rendezvous conditions. It also means that the kind of two dimensional analysis presented here has a wide range of validity.

Capturing of an incoming payload is essentially the time reversal of the outgoing scenario; the best place to add hyperbolic excess velocity is also the best place to

subtract it. If the tether orbital period is an integral multiple of the rotation period following release of a payload, the tip will be pointed at the zenith at periapsis and the capture will be the mirror image of the release.

Capturing a payload after a pass through the destination body's atmosphere is more complex than a periapsis capture, but involves the same principle: matching the flight path angle of the payload exiting trajectory to the tether flight path angle at the moment of capture and the velocity to the vector sum of the tether velocity and tip velocity. Aerodynamic lift and energy management during the passage through the atmosphere provide propellant-free opportunities to accomplish this.

There is a trade in aerobraking capture between momentum gain by the capturing tether and mission redundancy. To make up for momentum loss from outgoing payloads, the tether would like to capture incoming payloads at similar velocities. That, however, involves hyperbolic trajectories in which, if the payload is not captured, it is lost in space. Also, in the early operations before extensive ballast mass is accumulated, care must be taken that the tether itself is not accelerated to hyperbolic velocities as a result of the momentum exchange.

Payload Release

The release orbit is tangential to the tether circle in the tether frame of reference by definition, but it is not necessarily tangential to the trajectory in the frame of reference of the origin planet. The injection velocity vector is simply the vector sum of the motion of the tether tip and the tether center, displaced to the location of the tether tip. Note in the third part of Figure 2 that this does not generally lie along the radius to the tether center of mass. For maximum velocity, if one picks up the payload at tether periapsis, one must wait for the tether to swing the payload around to a point where its tip velocity vector is near parallel to the tether center of mass orbital velocity vector. By this time, the tether has moved significantly beyond periapsis, and there will be a significant flight path angle, which both orbits will share at the instant of release. Large variations from this scenario will result in significant velocity losses, but velocity management in this manner could prove useful. If, on the other hand, maximum velocity transfer and minimum tether orbit periapsis rotation is desired, the payload can be retained and the tether arm length or period adjusted to release the payload in a purely azimuthal direction at the next periapsis.

Rendezvous of Grapple with Payload

The seemingly difficult problem of achieving rendezvous of the tether tip and payload is nearly identical to a similar problem solved daily by human beings at circuses around the world. The grapple mechanism on the end of a rotating tether is typically subjected to a centrifugal acceleration of one gee by the rotation of the tether. Although the grapple velocity vector direction is changing rapidly, its speed is constant and chosen to be the same speed as the payload, which is moving at nearly constant velocity in its separate free fall suborbital trajectory. The timing of the positions of the tether tip and the payload needs to be such that they are close to the same place (within a few meters) at close to the same time (within a few seconds), so their relative spacing and velocities are such that the grapple can compensate for any differences. This situation is nearly identical to the problem of two trapeze artists timing the swings of their separate trapeze bars so that the "catcher," being supported in the 1 gee gravity field of the Earth by his bar, meets up with and grasps the "payload" after she has let go of her bar and is in a "free fall" trajectory accelerating with respect to the "catcher" at one gee. They time their swings, of course, so that they meet near the instant when both are at near zero relative velocity. The tether grapple system will have the advantages over the human grapple system of GPS guidance, radar Doppler and proximity sensors, onboard divert thrusters, electronic synapses and metallic grapples, which should insure that its catching performance is comparable to or better than the demonstrated human performance.

An essential first step in the development of the MERITT system would be the construction and flight test of a rotating tether-grapple system in LEO, having it demonstrate that it can accurately toss a dummy payload into a carefully selected orbit such that it orbits later the two meet again under conditions that will allow the grapple to catch the payload once again.

The Automated Rendezvous and Capture (AR&C) Project Office at Marshall Space Flight Center (MSFC) has been briefed on the AR&C requirements for the capture of a payload by a grapple vehicle at the end of a tether with a one-gee acceleration tip environment. MSFC has been working AR&C for over six years and has a great deal of experience in this area. It is their opinion [14] that their present Shuttle-tested [STS-87 & STS-95] Video Guidance Sensor (VGS) hardware, and Guidance, Global Positioning System (GPS) Relative Navigation, and Guidance, Navigation and Control

(GN&C) software, should, with sufficient funding, be able to be modified for this tether application.

TETHER CONSIDERATIONS

For a tether transport system to be economically advantageous, it must be capable of handling frequent traffic for many years despite degradation due to impacts by meteorites and space debris. Fortunately, a survivable tether design exists, called the Hoytether™, which can balance the requirements of low weight and long life [14,15]. As shown in Figure 3, the Hoytether™ is an open net structure where the primary load bearing lines are interlinked by redundant secondary lines. The secondary lines are designed to be slack initially, so that the structure will not collapse under load. If a primary line breaks, however, the secondary lines become engaged and take up the load.

Note that four secondary line segments replace each cut primary line segment, so that their cross-sectional area need only be 0.25 of the primary line area to carry the same load. Typically, however, the secondary lines are chosen to have a cross-sectional area of 0.4 to 0.5 of the primary line area, so as to better cope with multiple primary and secondary line cuts in the same region of the tether.

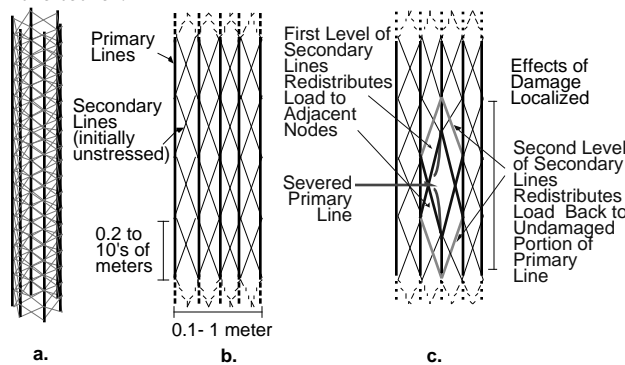


Figure 3 - The Hoytether™ design and its response to a cut line.

This redundant linkage enables the structure to redistribute loads around primary segments that fail due to meteorite strikes or material failure. Consequently, the Hoytether™ structure can be loaded at high stress levels, yet retain a high margin of safety [9].

Tether Mass Ratio

The mass of a rapidly spinning tether is determined primarily by the tip speed of the tether, not the tether length or the tether tip acceleration. In a rotating tether system, where the tether mass itself is part of the mass

being rotated, adding mass to a tether to increase its strength also increases the load, thus limiting the tip motion to a given velocity level, not acceleration level. A short, fat tether will have the same tip velocity V_T as a long, skinny tether of the same mass. The acceleration level G felt by the payload at the tip of the tether will vary as the tether length L with $G = V_T^2/L$.

The basic equation for the ratio of the mass M_T of one arm of a spinning tether to the mass M_P of the payload plus grapple on the end of the tether arm is [2,9]:

$$M_T/M_P = 1/2(V_T/V_C) \exp[(V_T/V_C)^2] \operatorname{erf}(V_T/V_C) \quad (1)$$

Where the error function $\operatorname{erf}(V_T/V_C) \approx 1$ for $V_T/V_C > 1$, V_T is the tether tip speed, and $V_C = (2U/Fd)^{1/2}$ is the maximum tip speed of an untapered tether, where U is the ultimate tensile strength of the tether material, d is its density, and $F > 1$ is an engineering safety factor derating the “ultimate” tensile strength to a safer “practical” value. The engineering safety factor F to be used in different applications is discussed in detail by Hoyt[9] and is typically between 1.75 and 3.0.

The material presently used for space tethers is a polyethylene polymer called Spectra™, which is commercially available in tonnage quantities as fishing net line. Although slightly stronger materials exist, and should be used when they become commercially available, we do not need them to make the MERITT system feasible. Spectra™ 2000 has an ultimate tensile strength of $U=4.0$ GPa, a density of 970 kg/m^3 , and an ultimate ($F=1$) characteristic velocity of $V_U = (2U/d)^{1/2} = 2.9 \text{ m/s}$. Assuming that the grapple on the end of the tether masses 20% of the payload mass, we can use Equation (1) to calculate the mass ratio of a one arm Spectra™ tether to the payload it is handling, assuming various different safety factors and various different tether tip velocities, to be:

Table 1. Ratio of Spectra™ 2000 Tether Material Mass to Payload Mass (Grapple Mass 20% of Payload Mass)

	Tether Material Safety Factor (F)			
	<u>1.75</u>	<u>2.0</u>	<u>2.4</u>	<u>3.0</u>
<u>Tip Speed V_T</u>				
1.5 km/s	2.2	2.5	3.4	4.9
2.0 km/s	3.7	4.7	6.4	10.0
2.5 km/s	8.0	11.0	17.0	30.0

From this table we can see that by using Spectra™ 2000, we can achieve tether tip velocities of 2.0 km/s with reasonable tether mass ratios (<10) and good safety factors. Higher tip velocities than 2.0 km/s are achievable using higher mass ratios, lower safety factors, and stronger materials.

Tether Survivability

There are many objects in Earth space, ranging from micrometeorites to operational spacecraft with 10 meter wide solar electric arrays. We can design interconnected multiple strand open net Hoytether™ structures that can reliably ($>99.9\%$) survive in space for decades despite impacts by objects up to 30 cm (1 foot) or so in size.

Objects larger than 30 cm will impact all the strands at one time, cutting the tether. These large objects could include operational spacecraft, which would also be damaged by the impact. Objects larger than 30 cm are all known and tracked by the U.S. Space Command. There are about 6000 such objects in low and medium Earth orbit, of which an estimated 600 will be operational spacecraft in the 2005 time frame.

Depending upon the choice of the EarthWhip orbit, calculations show that there is a small ($<1\%$) but finite chance of the EarthWhip tether striking one of the 600 operational spacecraft. It will therefore be incumbent on the tether system fabricators and operators to produce EarthWhip tether systems that maintain an accurate inventory of the known large objects and control the tether system center of mass orbital altitude and phase, the tether rotation rate and phase, and the tether libration and vibration amplitudes and phases, to insure that the tether system components do not penetrate a volume of "protected space" around these orbiting objects.

MERITT Modeling

Calculations of the MERITT system performance were performed using the mathematical modeling software package "TK Solver Plus" which allows the user to type in the relevant equations and get results without having to solve the model algebraically or structure it as a procedure, as long as the number of independent relationships equals the number of variables. This is very useful in a complex system when one may wish to constrain various variables for which it would be difficult, if not impossible, to solve and to perform numerical experiments to investigate the behavior of the system.

Two versions of a tether based interplanetary transfer system are being worked on, one for tether-only

transfers and the other incorporating an aerobraking pass at the destination body to aid in capture and rotation of the line of apsides. It should be emphasized that the results presented here are very preliminary and much remains to be done with the software. Because of the ongoing work and the growing number of variables and lines of code, we will not try to go through this line by line here. Questions concerning the code should be referred to Gerald Nordley at the above address.

The general architecture of the models is sequential. A payload is picked up from a trajectory at the origin planet, and added to a rotating tether in a highly elliptical orbit around around the origin planet. The pickup is accomplished by matching the position and velocity of the grapple end of the unloaded rotating tether to payload position and velocity.

This addition of the payload mass to one end of the tether shifts the center of mass of the tether toward the payload. The tether used in these examples is modeled as a rigid line with two arms, a grapple, a counterweight and a central mass. The tether is assumed to be designed for a payload with a given mass and a "safety factor" of two, as described in Hoyt and Forward [9] and to be dynamically symmetrical with a payload of that mass attached.

The mass distribution in the arms of the tether was determined by dividing the tether into ten segments, each massive enough to support the mass outward from its center; this was not needed for the loaded symmetric tether cases presented here, but will be useful in dealing with asymmetric counterweighted tethers. The total mass of each tether arm was determined from equation (1). The continuously tapered mass defined by equation (1) was found to differ by only a few percent from the summed segment mass of the 10 segment tether model used in the analysis, and the segment masses were adjusted accordingly until the summed mass fit the equation. The small size of this adjustment, incidentally, can be taken as independent confirmation of equation (1).

We ended up designing many candidates for the EarthWhip and MarsWhip tethers, from some with very large central station masses that were almost unaffected by the pickup or toss of a payload, to those that were so light that the toss of an outgoing payload caused their orbits to shift enough that the tether tip hit the planetary atmospheres, or the catch of an incoming payload sent the tether (and payload) into an escape trajectory from the planet. After many trials, we found some examples of tethers that were massive enough that they could toss and catch payloads without shifting into

undesirable orbits, but didn't mass too much more than the payloads they could handle. The tethers are assumed to be made of Spectra™ 2000 material braided into a Hoytube™ structure with a safety factor of 2. The tether design consists of a large central station with a solar array power supply, winches, and control systems, plus any ballast mass needed to bring the mass of the total system up to the desired final mass value. From the tether central station is extended two similar tethers, with a taper and mass determined by equation (1) according to the loaded tip velocity desired. At the end of the tethers are grapples that each mass 20% of the payloads to be handled. To simplify this initial analysis, we assumed that one grapple is holding a dummy payload with a mass equal to the active payload, so that after the grapple on the active arm captures a payload, the tether system is symmetrically balanced. Later, more complex, analyses will probably determine that a one arm tether system will do the job equally well and cost less.

Shift in Tether Center of Mass

The shift of the center of mass of the tether system when a payload was attached or released was determined by adding the moments of the unloaded tether about the loaded center of symmetry and dividing by the unloaded mass.

Figure 4. illustrates the four general circumstances of tether operations: origin pickup, origin release, destination capture and destination release. The shift of the center of mass of the tether system when a payload was attached or released was determined by adding the moments of the unloaded tether about the loaded center of symmetry and dividing by the unloaded mass. Figure 4. illustrates the four general circumstances of tether operations; origin pickup, origin release, destination capture and destination release. It turns out that the dynamics of an ideal rigid tether system with a given payload can be fairly well modeled by simply accounting for the change in the position and motion of the tether's center of mass as the payload is caught and released.

When the payload is caught, the center of mass shifts toward the payload and the tether assumes a symmetrical state. The velocity of the tip around the loaded center of mass is simply its velocity around the unloaded center of mass minus the velocity of the point which became the new center of mass about the old center of mass. The change in the tether orbital vector is fully described by the sum of the vector of the old center of mass and the vector at the time of capture or

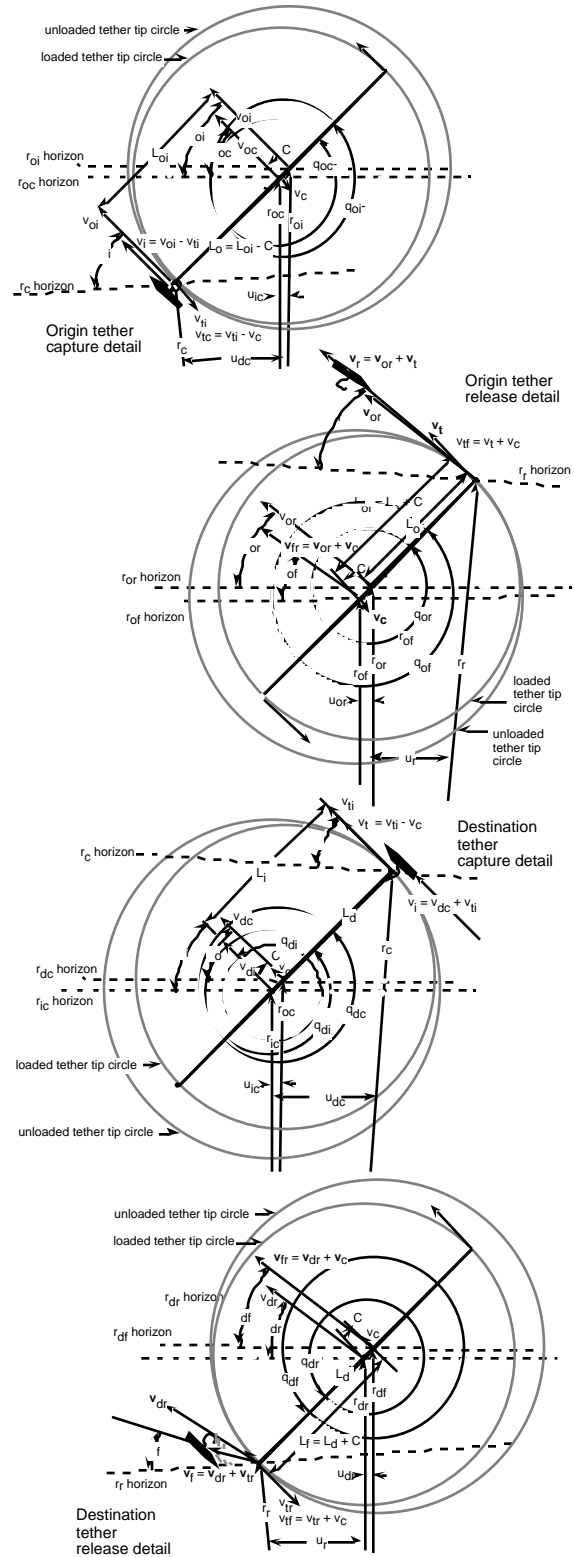


Figure 4. Tether Capture/Release Operations

release of the point that becomes the new center of mass relative to the old center of mass. Since the tether loses altitude with both the catch and the throw, its initial altitude must be high enough so that it does not enter the atmosphere after it throws the payload.

Once the payload is released, its velocity and position are converted to Keplerian orbital elements which are propagated to the outgoing patch point. At this point, they are converted back to position and velocity, and transformed to the Sun frame of reference.

The velocity of insertion into the orbit in the Sun's frame of reference is essentially the vector sum of the hyperbolic excess velocity with respect to the origin planet and the origin planet's orbital velocity about the Sun. This vector is done in polar coordinates, and the

angle portion of this vector in the origin planet frame is, at this point, a free choice. For now, an estimate or "guess" of this quantity is made. The resulting vector is then converted into Sun frame orbital elements and propagated to the patch point near the orbit of the destination planet. There, it is transformed into the destination planet coordinates.

Tether-Only Incoming Payload Capture

For the tether-only capture scenario, the velocity and radius of the tip of the tether orbiting the destination mass are calculated and iteratively matched to the velocity of the payload on an orbit approaching the destination planet, as shown in Figure 5.

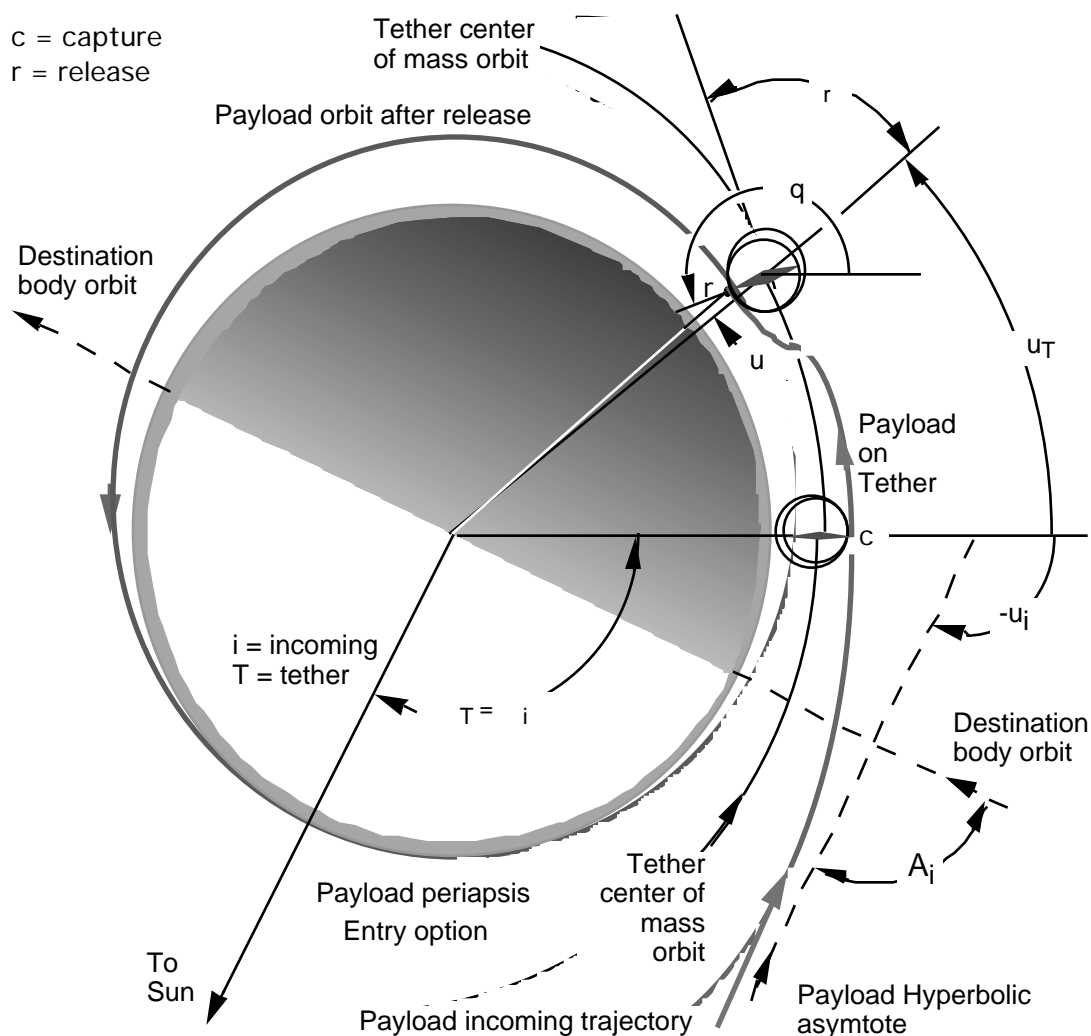


Figure 5. Tether-Only Capture Scenario

c = capture
r = release

In this work, we have generally tried to select near-resonant tether orbits that might be “tied” to geopotential features so that they precess at the local solar rate and thus maintain their apsidal orientation with respect to the planet-Sun line. The Russian Molniya communications satellites about Earth and the Mars Global Surveyor spacecraft use such orbits.

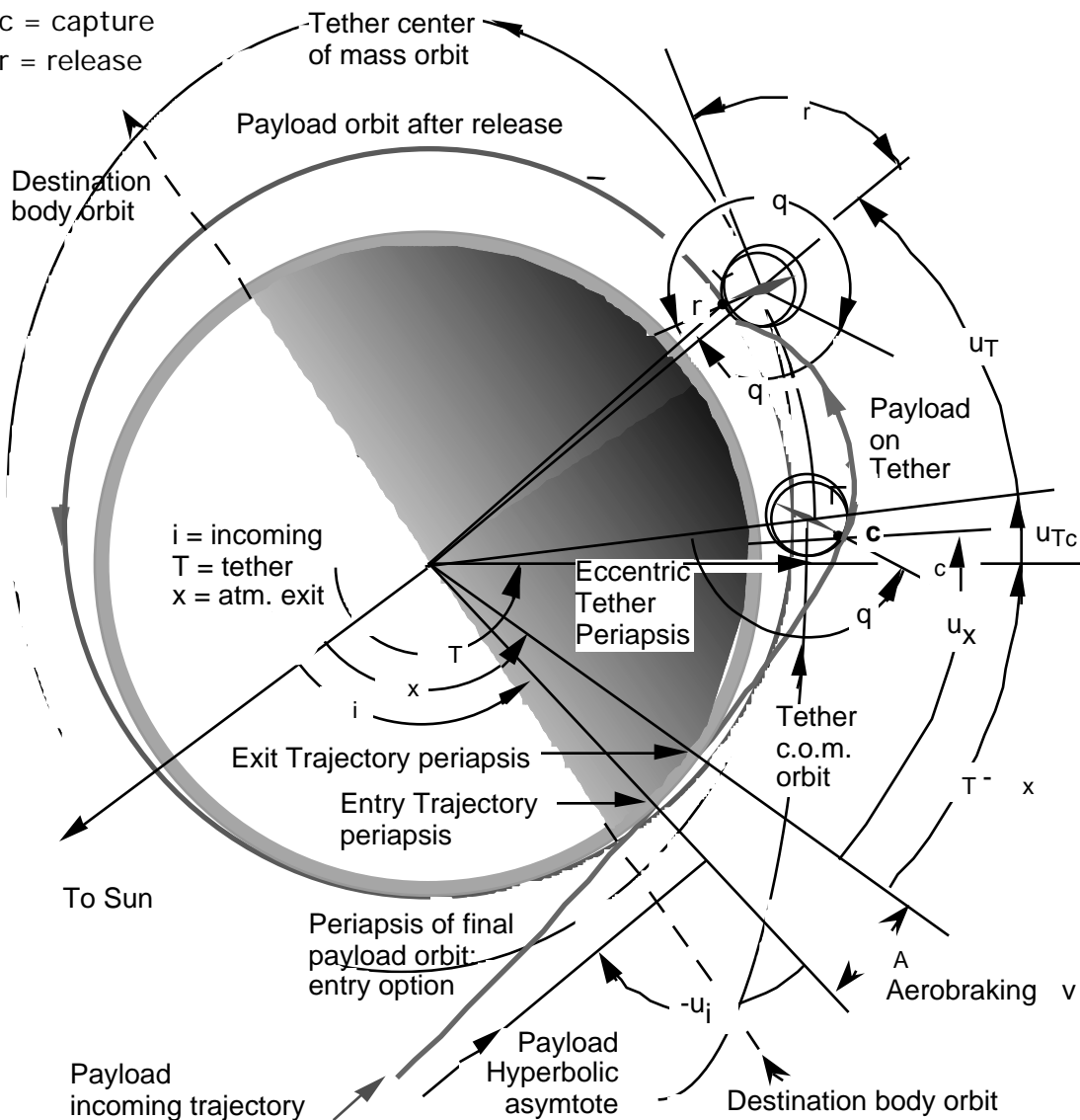


Figure 6. Aerobraking Tether Capture

precession nearly cancels the angular rate of the planet's orbit about the Sun .

The fastest transfer times are generally associated

with the fastest usable periapsis velocities. These are found when the tether is at periapsis and its tip at the zenith of its swing. In one approach to this model, these tether conditions are used to set the periapsis velocity and radius of the incoming orbit. This, in turn, defines the relative velocity at the patch point, and the origin planet injection angle can be iterated to produce a Sun frame orbit that produces that relative velocity at the destination planet patch point.

Aerobraking Payload Capture

In the case of using aerobraking in the planetary atmosphere, the injection angle can be optimized for minimum transfer time. As shown in Figure 6, the radius at which the atmosphere of the destination planet is dense enough to sustain an aerodynamic trajectory is used to define the periapsis of the approach orbit; there is no velocity limit.

In a similar manner, the tether tip at an estimated capture position and velocity, together with the radius at which the outgoing payload resumes a ballistic trajectory define an exit orbit which results in tether capture. The difference in the periapsis velocity of this orbit and the periapsis velocity of the inbound trajectory is the velocity that must be dissipated during the aerodynamic maneuver. For Mars bound trajectories, this aerobraking v is on the order of 5 km/s, as compared to direct descent v 's of 9 km to 15 km/s. Also, payloads meant to be released into suborbital trajectories already carry heat shields, though designed for lower initial velocities.

After the tether tip and the incoming payload are iteratively matched in time, position and velocity, the center of mass orbit of the loaded tether is propagated to the release point. This is another free choice, and the position of the tether arm at release determines both the resulting payload and tether orbit. In this preliminary study, care was taken to ensure that the released payload did enter the planet's atmosphere, the tether tip did not, and that the tether was not boosted into an escape orbit.

INITIAL PLANET WHIP ANALYSIS

We first carried out analyses of a number of MERITT missions using a wide range of assumptions for the tether tip speed and whether or not aerobraking was used. The trip times for the various scenarios are shown in Table 3. As can be seen from Table 3, the system has significant growth potential. If more massive tethers are used, or stronger materials become available, the tether tip speeds can be increased, cutting the transit time even further. The transit times in Table

3 give the number of days from payload pickup at one planet until payload reentry at the other planet, and include tether "hang time" and coast of the payload between the patch points and the planets. Faster transit times can be made with higher energy initial orbits for the payload and the tether. With a 2.5 km/s tip speed on the PlanetWhip tethers and using aerobraking at Mars (see Fig. 6), the Earth orbit-Mars orbit transit time can be made about 94 days.

Table 3. Potential MERITT Interplanetary Transfer Times

Tip Speed (km/s)	System Mass Ratio	Transfer direction From->To	Tether-only (days)	Aero-braking (days)
1.5	15x	Earth->Mars	188	162
		Mars->Earth	187	168
2.0	15x	Earth->Mars	155	116
		Mars->Earth	155	137
2.5	30x	Earth->Mars	133	94
		Mars->Earth	142	126

PlanetWhip Analysis

The initial mathematical model program made many simplifying assumptions, which are gradually being removed. One issue that was not addressed was the apsidal orientation of a tether expected to both catch and throw payloads.

Figure 7 is a diagram showing how a single tether toss and catch system would work on either the Earth or Mars end of the MERITT system, for a finite mass PlanetWhip tether. The incoming payload brushes the upper atmosphere of the planet, slows a little using aerobraking, and is caught by a rotating tether in a low energy elliptical orbit. After the payload is caught, the center of mass of the tether shifts and the effective length of the tether from center of mass to the payload catching tip is shortened, which is the reason for the two different radii circles for the rotating tether in the diagram. The orbit of the tether center of mass changes from a low energy elliptical orbit to a higher energy elliptical orbit with its periapsis shifted with respect to the initial orbit. The tether orbit would thus oscillate between two states: 1) a low energy state wherein it would be prepared to absorb the energy from an incoming payload without becoming hyperbolic and 2) a high energy state for tossing an outgoing payload.

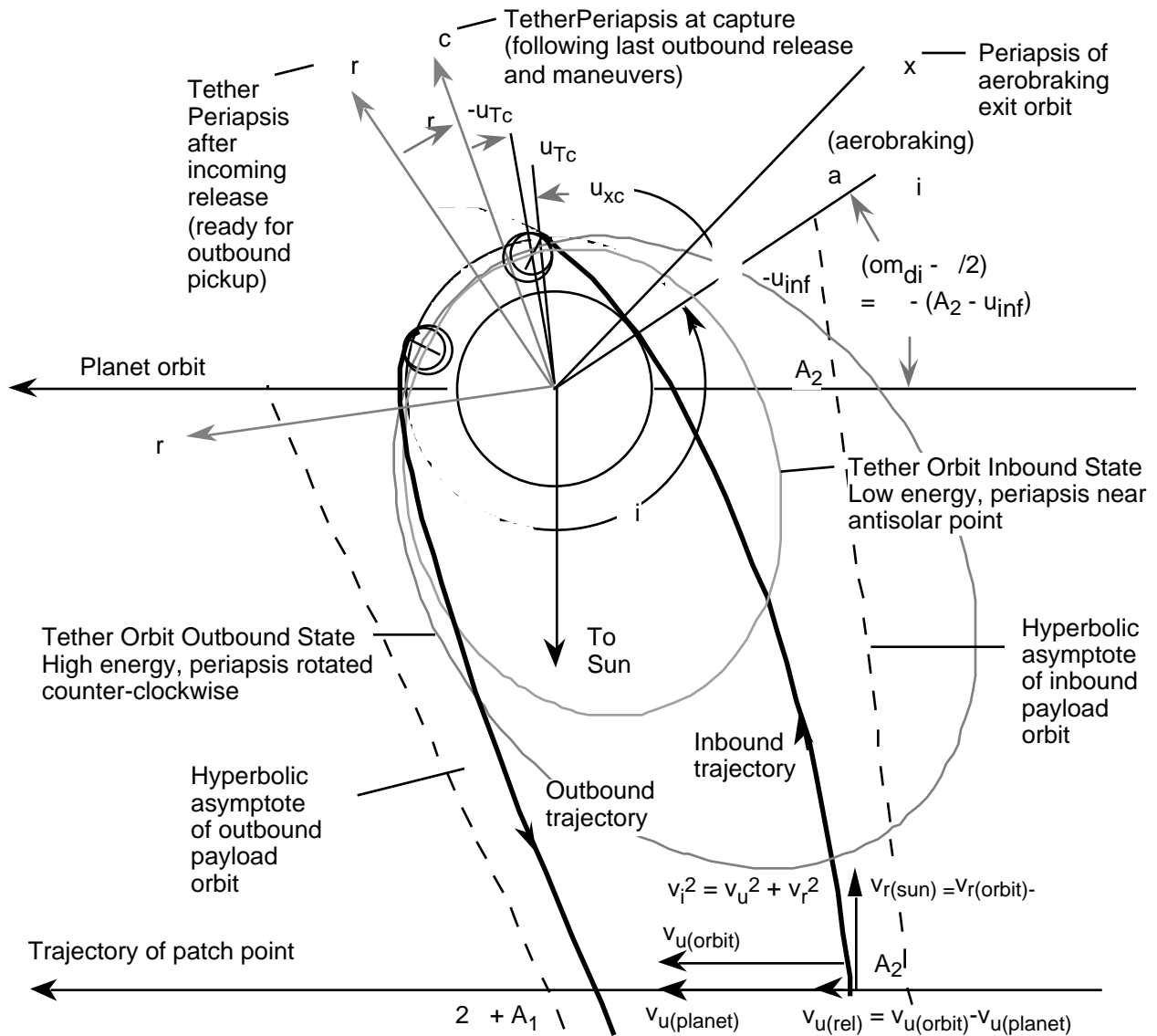


Figure 7 - "Planet"Whip showing catch and toss states using aerobraking.

The periapsis of the tether orbit is pushed counterclockwise for where a tether-only capture would occur by the angular distance needed for aerobraking and the periapsis rotations caused by capturing and releasing the payload at non-zero true anomalies. If the periapsis is shifted enough, the tether may be able to inject a payload on a return trajectory without waiting for many months, or using substantial amounts of propellant to produce the needed alignment.

DETAILED MERITT EXAMPLE

There are a large number of variables in the MERITT system concept, and many of those variables can be freely chosen at the start of the system design. We have carried out dozens of complete round-trip scenarios under various different assumptions, such as: aerobraking before tether catch versus direct tether-to-tether catch; sub-, circular, and elliptical initial and final payload orbits; 1.5, 2.0, 2.5 and higher tether tip velocities; large, small and minimum tether central

facility masses; etc. We will present here just one of the many possible MERITT scenarios using finite mass EarthWhip and MarsWhip tethers, but do it in extensive detail so the reader can understand where the broad assumptions are, while at the same time appreciating the accuracy of the simulations between the broad assumptions. In most cases, the matches between the payload trajectories and the tether tip trajectories are accurate to 3 and 4 decimal places.

The scenario we will describe uses EarthWhip and MarsWhip tethers of near minimum mass made of Spectra™ 2000 with a tip speed of 2.0 km/s. Because they have small total masses, the toss and catch operations significantly affect the tether rotation speed, center of mass, and orbital parameters, all of which are taken into account in the simulation. The payload is assumed to be initially launched from Earth into a suborbital trajectory to demonstrate to the reader that the MERITT system has the capability to supply all of the energy and momentum needed to move the payload from the upper atmosphere of the Earth to the upper atmosphere of Mars and back again. We don't have ask the payload to climb to nearly Earth escape before the MERITT system takes over.

In practice, it would probably be wise to have the payload start off in an initial low circular orbit. The energy needed to put the payload into a low circular orbit is not that much greater than the energy needed to put the payload into a suborbital trajectory with an apogee just outside the Earth's atmosphere. The circular orbit option also has the advantage that there would be plenty of time to adjust the payload orbit to remove launch errors before the arrival of the EarthWhip tether.

In the example scenario, the payload, in its suborbital trajectory, is picked up by the EarthWhip tether and tossed from Earth to Mars. At Mars it is caught by the MarsWhip tether without the use of aerobraking, and put into a trajectory that enters the Martian atmosphere at low velocity. Since this scenario does not use aerobraking, the return scenario is just the reverse of the outgoing scenario.

Payload Mass

We have chosen a canonical mass for the payload of 1000 kg. If a larger payload mass is desired, the masses of the tethers scale proportionately. The scenario assumes that the payload is passive during the catch and throw operations. In practice, it might make sense for the payload to have some divert rocket propulsion capability to assist the grapple during the catch operations. In any case, the payload will need some

divert rocket propulsion capability to be used at the midpoint of the transfer trajectory to correct for injection errors.

Tether Mass

Both the EarthWhip and MarsWhip tethers were assumed to consist of a robotic central station, two similar tethers, two grapples at the ends of the two tethers, and, to make the analysis simpler, one grapple would be holding a dummy payload so that when the active payload is caught, the tether would be symmetrically balanced.

The tether central station would consist of a solar electric power supply, tether winches, and command and control electronics. There may be no need to use center of mass rocket propulsion for ordinary tether operations. Both tethers can be adequately controlled in both their rotational parameters and center-of-mass orbital parameters by "gravity-gradient" propulsion forces and torques generated by changing the tether length at appropriate times in the tether orbit [7,16,17].

The EarthWhip tether would also have a small conductive portion of the tether that would use electrodynamic tether propulsion[9], where electrical current pumped through the tether pushes against the magnetic field of the Earth to add or subtract both energy and angular momentum from the EarthWhip orbital dynamics, thus ultimately maintaining the total energy and angular momentum of the entire MERITT system against losses without the use of propellant.

The grapple mechanisms are assumed in this scenario to mass 20% of the mass of the payload, or 200 kg for a 1000 kg payload. It is expected, however, that the grapple mass will not grow proportionately as the payload mass increases to the many tens of tons needed for crewed Mars missions.

In the scenario presented here, it is assumed that the grapples remain at the ends of the tethers during the rendezvous procedure. In practice, the grapples will contain their own tether winches powered by storage batteries, plus some form of propulsion.

As the time for capture approaches, the grapple, under centrifugal repulsion from the rotation of the tether, will release its tether winches, activate its propulsion system, and fly ahead to the rendezvous point. It will then reel in tether as needed to counteract planetary gravity forces in order to "hover" along the rendezvous trajectory, while the divert thrusters match velocities with the approaching payload. In this manner, the rendezvous interval can be stretched to many tens of seconds.

If needed, the rendezvous interval can be extended past the time when the tip of the tether passes through the rendezvous point by having the grapple let out tether again, while using the divert thrusters to complete the payload capture. The grapple batteries can be recharged between missions by the grapple winch motor/dynamos, by allowing the grapple winches to reel out while the central winches are being reeled in using the central station power supply. The grapple rocket propellant will have to be resupplied either by bringing up "refueling" payloads or extracting residual fuel from payloads about to be deorbited into a planetary atmosphere.

For this scenario, we assumed that, when loaded with a payload, the EarthWhip and MarsWhip tethers were rotating with a tether tip speed of $V_T = 2,000$ m/s. The length of each tether arm was chosen as $L=400$ km in order to keep the acceleration on the payload, $G=V_T^2/L$, near one gee. We also assumed that the total mass of the Whips are 15,000 kg for a 1000 kg payload (16,000 kg total). This mass includes the central station, both tethers, the grapples at the ends of the tethers, and the dummy payload mass. This is about the minimum tether mass needed in order for the tether center-of-mass orbits to remain stable before and after a catch of a payload with a velocity difference of 2000 m/s.

The tether material was assumed to be Spectra™ 2000 with an ultimate tensile strength of $U=4.0$ GPa, a density $d=970$ kg/m³, and an ultimate tip velocity for an untapered tether of $V_U=(2U/d)^{1/2}=2872$ m/s. The tether safety factor was initially chosen at $F=2.0$, which results in an engineering characteristic velocity for the tether of $V_C=(2U/2d)^{1/2}=2031$ m/s.

Using V_C and V_T in equation (1), we find that the mass ratio of one arm of a tapered Spectra™ 2000 tether is 3.841 times the mass at the tip of the tether. Since the mass at the end of the tether consists of the 1000 kg payload and the 200 kg grapple, the minimum total mass of one tether arm is 4609 kg, or about 4.6 times the mass of the 1000 kg payload. The amount of taper is significant, but not large. The total cross-sectional area of the tether at the tip, where it is holding onto the payload, is 6 mm² or 2.8 mm in diameter, while the area at the base, near the station, is 17.3 mm² or 4.7 mm in diameter. This total cross-sectional area will be divided up by the Hoytether™ design into a large number of finer cables.

Equation (1), however, applies to a rotating tether far from a massive body. Since the EarthWhip and MarsWhip tethers are under the most stress near

periapsis, when they are closest to their respective planets, we need to take into account the small additional stress induced by the gravity gradient forces of the planets, which raises the mass to about 4750 kg for a 1000 kg payload. We will round this up to 4800 kg for the tether material alone, corresponding to a free-space safety factor of 2.04, so that the total mass of the tether plus grapple is an even 5000 kg. With each tether arm massing 5000 kg including grapple, one arm holding a dummy payload of 1000 kg, and a total mass of 15,000 kg, the mass of the central station comes out at 4000 kg, which is a reasonable mass for its functions.

There are a large number of tether parameter variations that would work equally well, including shorter tethers with higher gee loads on the payloads, and more massive tethers with higher safety factors. All of these parameters will improve as stronger materials become commercially available, but the important thing to keep in mind is that the numbers used for the tethers assume the use of Spectra™ 2000, a commercial material sold in tonnage quantities as fishing nets, fishing line (SpiderWire), and kite line (LaserPro). We don't need to invoke magic materials to go to Mars using tethers.

Tether Rotational Parameters

When the EarthWhip or MarsWhip tethers are holding onto a payload, they are symmetrically balanced. The center-of-mass of the tether is at the center-of-mass of the tether central station. The effective arm length from the tether center-of-mass to the payload is 400,000 m, the tip speed is exactly 2000 m/s and the rotation period is $P = 1256.64$ s = 20.94 min = 0.3491 hr.

When the Whips are not holding onto a payload, then the center-of-mass of the Whip shifts 26,667 m toward the dummy mass tether arm, and the effective length of the active tether arm becomes 426,667 m, while the effective tip velocity at the end of this longer arm becomes 2,133 m/s. (Since there is no longer a payload on this arm, the higher tip velocity can easily be handled by the tether material.) The rotational period in this state is the same, 1256.64 s.

Payload Trajectory Parameters

The Earth-launched payload trajectory chosen for this example scenario is a suborbital trajectory with an apogee altitude of 203,333 m (6581.333 km radius) and an apogee velocity of 7,568 m/s. The circular orbit velocity for that radius is 7,782 m/s.

EarthWhip Before Payload Pickup

The EarthWhip starts out in an unloaded state with an effective length for its active arm of 426,667 m from the center-of-rotation, a tip velocity of 2,133 m/s and a rotational period of 1256.64 s. The center-of-mass of the EarthWhip is in a highly elliptical orbit with an apogee of 33,588 km (almost out to geosynchronous orbit), an eccentricity of 0.655, an orbital period of exactly 8 hours, a perigee radius of 7008 km (630 km altitude), and a perigee velocity of 9,701 m/s. The tether rotational phase is adjusted so that the active tether arm is pointing straight down at perigee, with the tether tip velocity opposing the center-of-mass velocity. The tip of the tether is thus at an altitude of 630 km - 426.7 km = 203.3 km and a velocity with respect to the Earth of 9,701 m/s - 2,133 m/s = 7,568 m/s, which matches the payload altitude and velocity.

EarthWhip After Payload Pickup

After picking up the payload, the loaded EarthWhip tether is now symmetrically balanced. Since the added payload had both energy and momentum appropriate to its position on the rotating tether, the EarthWhip rotation angular rate does not change and the period of rotation remains at 1257 s. The center of mass of the loaded EarthWhip, however, has shifted to the center of the tether central station, so the effective length of the loaded tether arm is now at its design length of 400,000 km and tip velocity of 2,000 m/s. With the addition of the payload, however, the orbit of the tether center-of-mass has dropped 26.7 km to a perigee of 6981.3 km, while the perigee velocity has slowed to 9,568 m/s. The apogee of the new orbit is 28,182 km and the eccentricity is 0.603, indicating that this new orbit is less eccentric than the initial orbit due to the payload mass being added near perigee. The period is 23,197 s or 6.44 hours.

Payload Toss

The catch and toss operation at the Earth could have been arranged as shown in Figure 6, so that the payload catch was on one side of the perigee and the payload toss was on the other side of the perigee, a half-rotation of the tether later (10.5 minutes). To simplify the mathematics for this initial analysis, however, we assumed that the catch occurred right at the perigee, and that the tether holds onto the payload for a full orbit. The ratio of the tether center-of-mass orbital period of 23,197 s is very close to 18.5 times the tether rotational period of 1256.64 s, and by adjusting the length of the tether during the orbit, the phase of the

tether rotation can be adjusted so that the tether arm holding the payload is passing through the zenith just as the tether center-of-mass reaches its perigee. The payload is thus tossed at an altitude of 603 km + 400 km = 1003 km (7381 km radius), at a toss velocity equal to the tether center-of-mass perigee velocity plus the tether rotational velocity or 9,568 m/s + 2,000 m/s = 11,568 m/s. In the combined catch and toss maneuver, the payload has been given a total velocity increment of twice the tether tip velocity or $v=4,000$ m/s.

EarthWhip After Payload Toss

After tossing the payload, the EarthWhip tether is back to its original mass. It has given the payload a significant fraction of its energy and momentum. At this point in the analysis, it is important to insure that no portion of the tether will intersect the upper atmosphere and cause the EarthWhip to deorbit. We have selected the minimum total mass for the EarthWhip at 15,000 kg to insure that doesn't happen. The new orbit for the EarthWhip tether has a perigee of its center of mass of 6955 km (577 km altitude), apogee of 24,170 km, eccentricity of 0.552, and a period of 5.37 hours. With the new perigee at 577 km altitude, even if the tether rotational phase is not controlled, the tip of the active arm of the tether, which is at 426.67 km from the center-of-mass of the tether, does not get below 150 km from the surface of the Earth where it might experience atmospheric drag. In practice, the phase of the tether rotation will be adjusted so that at each perigee passage, the tether arms are roughly tangent to the surface of the Earth so that all parts of the tether are well above 500 km altitude, where the air drag and traffic concerns are much reduced.

With its new orbital parameters, the EarthWhip tether is in its "low energy" state. There are two options then possible. One option is to keep the EarthWhip in its low energy elliptical orbit to await the arrival of an incoming payload from Mars. The EarthWhip will then go through the reverse of the process that it used to send the payload from Earth on its way to Mars. In the process of capturing the incoming Mars payload, slowing it down, and depositing it gently into the Earth's atmosphere, the EarthWhip will gain energy which will put it back into the "high energy" elliptical orbit it started out in. If, however, it is desired to send another payload out from Earth before there is an incoming payload from Mars, then the solar electric power supply on the tether central station can be used to generate electrical power. This electrical power can then be used to restore the

EarthWhip to its high energy elliptical orbit using either electrodynamic tether propulsion [9] or gravity-gradient propulsion [16,17].

Payload Escape Trajectory

The velocity gain of $v \approx 4,000$ m/s given the payload deep in the gravity well of Earth results in a hyperbolic excess velocity of 5,081 m/s. The payload moves rapidly away from Earth and in 3.3 days reaches the "patch point" on the boundary of the Earth's "sphere of influence," where the gravity attraction of the Earth on the payload becomes equal to the gravity attraction of the Sun on the payload. An accurate calculation of the payload trajectory would involve including the gravity field of both the Sun and the Earth (and the Moon) all along the payload trajectory. For this simplified first-order analysis, however, we have made the assumption that we can adequately model the situation by just using the Earth gravity field when the payload is near the Earth and only the Solar gravity field when we are far from the Earth, and that we can switch coordinate frames from an Earth-centered frame to a Sun-centered frame at the "patch point" on the Earth's "sphere of influence."

Payload Interplanetary Trajectory

When this transition is made at the patch point, we find that the payload is on a Solar orbit with an eccentricity of 0.25, a periapsis of 144 Gm and an apoapsis of 240 Gm. It is injected into that orbit at a radius of 151.3 Gm and a velocity of 32,600 m/s. (The velocity of Earth around the Sun is 29,784 m/s.) It then coasts from the Earth sphere-of-influence patch point to the Mars sphere-of-influence patch point, arriving at the Mars patch point at a radius of 226.6 Gm from the Sun and a velocity with respect to the Sun of 22,100 m/s. (The velocity of Mars in its orbit is 24,129 m/s.) The elapsed time from the Earth patch point to the Mars patch point is 148.9 days.

Payload Infall Toward Mars

At the patch point, the analysis switches to a Mars frame of reference. The payload starts its infall toward Mars at a distance of 1.297 Gm from Mars and a velocity of 4,643 m/s. It is on a hyperbolic trajectory with a periapsis radius of 4451 km (altitude above Mars of 1053 km) and a periapsis velocity of 6,370 m/s. The radius of Mars is 3398 km and because of the lower gravity, the atmosphere extends out 200 km to 3598 km. The infall time is 3.02 days.

MarsWhip Before Payload Catch

The MarsWhip tether is waiting for the arrival of the incoming high velocity payload in its "low energy" orbital state. The active tether arm is 426,667 m long and the tip speed is 2,133 m/s. The center-of-mass of the unbalanced tether is in an orbit with a periapsis radius of 4025 km (627 km altitude), periapsis velocity of 4,236 m/s, apoapsis of 21,707 km, eccentricity of 0.687, and a period close to 0.5 sol. (A "sol" is a Martian day of 88,775 s, about 39.6 minutes longer than an Earth day of 86,400 s. The sidereal sol is 88,643 s.) The orbit and rotation rate of the MarsWhip tether is adjusted so that the active arm of the MarsWhip is passing through the zenith just as the center-of-mass is passing through the perigee point. The grapple at the end of the active arm is thus at $4024.67 + 426.67 = 4,451.3$ km, moving at $4,236 \text{ m/s} + 2,133 \text{ m/s} = 6,370 \text{ m/s}$, the same radius and velocity as that of the payload, ready for the catch.

MarsWhip After Payload Catch

After catching the payload, the MarsWhip tether is now in a balanced configuration. The effective arm length is 400,000 m and the tether tip speed is 2,000 m/s. In the process of catching the incoming payload, the periapsis of the center-of-mass of the tether has shifted upward 26,667 m to 4,051 km and the periapsis velocity has increased to 4,370 m/s, while the apoapsis has risen to 37,920 km, and the eccentricity to 0.807. The period is 1.04 sol.

Payload Release and Deorbit

The payload is kept for one orbit, while the phase of the tether rotation is adjusted so that when the tether center-of-mass reaches periapsis, the active tether arm holding the payload is approaching the nadir orientation. If it were kept all the way to nadir, the payload would reach a minimum altitude of about 250 km (3648 km radius) at a velocity with respect to the Martian surface of $4370 \text{ m/s} - 2000 \text{ m/s} = 2370 \text{ m/s}$. At 359.5 degrees (almost straight down), this condition is achieved to four significant figures. The payload is then moving at a flight path angle with respect to the local horizon of 0.048 radians and enters the atmosphere at a velocity of 2,442 km/s.

MarsWhip after Deorbit of Payload

After tossing the payload, the MarsWhip tether is back to its original mass. The process of catching the high energy incoming payload, and slowing it down for a gentle reentry into the Martian atmosphere, has given

the MarsWhip a significant increase in its energy and momentum. At this point in the analysis, it is important to check that the MarsWhip started out with enough total mass so that it will not be driven into an escape orbit from Mars.

The final orbit for the tether is found to have a periapsis radius of 4078 km (676 km altitude so that the tether tip never goes below 253 km altitude), a periapsis velocity of 4,503 m/s, an apoapsis radius of 115,036 km, an eccentricity of 0.931, and a period of 6.65 sol. The tether remains within the gravity influence of Mars and is in its high energy state, ready to pick up a payload launched in a suborbital trajectory out of the Martian atmosphere, and toss it back to Earth.

Elapsed Time

The total elapsed transit time, from capture of the payload at Earth to release of the payload at Mars, is 157.9 days. This minimal mass PlanetWhip scenario is almost as fast as more massive PlanetWhip tethers since, although the smaller mass tethers cannot use extremely high or low eccentricity orbits without hitting the atmosphere or being thrown to escape, the time spent hanging on the tether during those longer orbit counts as well and the longer unbalanced grapple arm of the lightweight tether lets it grab a payload from a higher energy tether orbit.

FUTURE MERITT STUDIES

As emphasized before, this paper is only the first of a series of papers that will continue to demonstrate the engineering and economic feasibility of the MERITT concept by finding optimum solutions to the various technical challenges, and illustrating ways to augment and expand the concept. The follow-on papers, numbered II to VII, will cover the following topics:

II - Finite PlanetWhip Mass Analysis

This paper will document the detailed effects of the finite mass of the EarthWhip and MarsWhip tethers on the operation of the MERITT system, especially the capture and toss phases. Special attention will be given to scenarios where the payload "helps" in the transfer by starting out in circular or elliptical orbits with significant energy and angular momentum in them, so the PlanetWhip does not shoulder the whole transport burden. Then, for various values of interplanetary travel time and transit velocity, this paper will determine the minimum mass needed by the PlanetWhip to prevent it from being deorbited during a toss or recoiling to escape during a catch.

III - Full Trajectory Analysis

This paper will remove the simplifying assumptions made during the initial feasibility analysis concerning the gravity fields of the planets, the orbits of the planets, the tilt of the planet axes, the interplanetary trajectory, and the actual positions of the planets in the coming two decades. It is not expected that including these corrections will affect the feasibility of the concept. It will, however, result in an accurate estimate of the width of the launch windows, optimum launch times for different toss velocities and resultant transit times, and, hopefully, some attractive case studies.

IV - Tether Dynamics Analysis

This paper has assumed ideal rigid tethers. Real tether materials have both elasticity and damping. The Hoytether™ structure then adds its own damping and a non-linear elasticity and strength response as the secondary strands come into play after sufficient elongation. Then, depending upon the placement of intermediate masses along the tether, the long tether structure has libration, pendulum, and skip-rope modes, plus longitudinal, transverse, and torsional vibrational modes. The analysis would study the effect of the catch and throw operations on the excitation of those modes, ways to minimize the excitation, and how the existence of high amplitude oscillations of those modes could affect the accuracy of the catch and throw operations.

V - Energy/Momentum Management

One of the major advantages of the MERITT system over rocket methods for getting to Mars is that once two-way traffic is established, the system can, in principle, be self-powered, with incoming payload capsules restoring energy and angular momentum lost by the tethers when throwing outgoing payloads. A payload thrown to Mars from a tether on Earth typically arrives with much more velocity than the tether can handle at feasible tip velocities, and trajectories have to use aerobraking or be deliberately deoptimized to allow capture. Energy will be needed to make up drag losses, for tether damping, for periapsis rotation, and for phasing maneuvers, so we need to study methods for restoring that energy and momentum. The EarthWhip tether can supply both of these by including a Hoyt Electrodynamic Force Tether (HEFT™) system[9] in its structure. MarsWhip tether energy management can be accomplished by including a solar electric power supply on the central facility and using the electrical energy to power a tether winch to periodically change the tether length at the proper point in the MarsWhip elliptical

trajectory [15,16], making the orbit more or less elliptical for the same angular momentum.

VI - Incremental Construction

The objective of this paper would be to show how the EarthWhip tether can be built up incrementally, first serving to send small science payloads to Mars, while at the same time accumulating central facility mass by keeping upper stages and other unwanted masses. The Hoytether™ design also lends itself to incremental construction, not only in length but in thickness and taper, so that a 10, 20 or even 100 ton tether can be built out of a large number of 1 to 5 ton deploy-only canisters each containing a 10-20 km long section of tether.

Preliminary analysis also shows that a minimal mass MarsWhip can be tossed to Mars by a similar mass EarthWhip tether, arriving at Mars 180 days after toss. The MarsWhip could halt itself by use of an aerobraking module. Alternatively, it could employ the Landis [18] tether assisted planetary orbital capture procedure, where prior to close approach to Mars, the tether is deployed so that one end is ahead of and much closer to Mars than the other, pulling that end of the tether into a different trajectory than the other end. If properly done, the tether system gains rotational energy and angular momentum from the non-linear gravity-whip interaction, at the expense of its center-of-mass orbital energy and angular momentum, and thus ends up rotating around its center-of-mass, with the center-of-mass in a highly elliptical capture orbit around Mars. Once in the capture orbit, the MarsWhip tether can use tether pumping [15,16] to change the rotation rate of the tether and the ellipticity of its orbit to the desired values. After the MarsWhip is ready to receive incoming payloads, its tether and central facility can then be built up by additional incremental payloads.

VII - Spinning Tether Payload

Once the MERITT system has proved its reliability in handling science probes, sample return missions, and cargo missions, to robotically build up a Mars orbital station and surface base camp, then it could be considered for delivery of crewed interplanetary transit capsules. For these missions, the short trip times available using the MERITT system will minimize the radiation exposure to the crew. In addition, the MERITT system could also provide a method of completely eliminating the biological effects of long periods in zero gee. The payload tossed by the EarthWhip and caught by the MarsWhip would consist

of two capsules connected by a tether and put into slow rotation during the toss operation. After the toss, a solar electric powered winch on one of the payload capsules would change the length of the tether to attain any desired artificial gravity level during the transit time interval. Since the payload can be caught by the tether grapple at either capsule end, and the capsule velocity can add or subtract from the MarsWhip tether tip velocity, the existence of a spinning payload opens up a whole new series of system optimizations to be explored.

VIII - Transport to Other Planets

Although Mars is the obvious first target for a Rapid Interplanetary Tether Transport (RITT) system, there is no reason why the RITT concept couldn't be used for rapid transport among other planets and moons in the Solar System, as well as between planets and moons. The objective of this study would be to define "Planet"Whip tether systems for each planet that could provide two-way transport not only between that planet and Earth, but between that planet and other planets, ultimately resulting in a solar-system-wide tether transportation network.

CONCLUSIONS

We have shown that two rapidly spinning tethers in highly elliptical orbits about Earth and Mars, can be combined into a system that provides rapid interplanetary transport from a suborbital trajectory above the Earth's atmosphere to a suborbital trajectory above the Martian atmosphere and back.

ACKNOWLEDGMENTS

This research has been supported in part by Contract 07600-011 from the NASA Institute for Advanced Concepts, Dr. Robert A. Cassanova, Director; and in part by the Tethers Unlimited, Inc. IR&D program.

REFERENCES

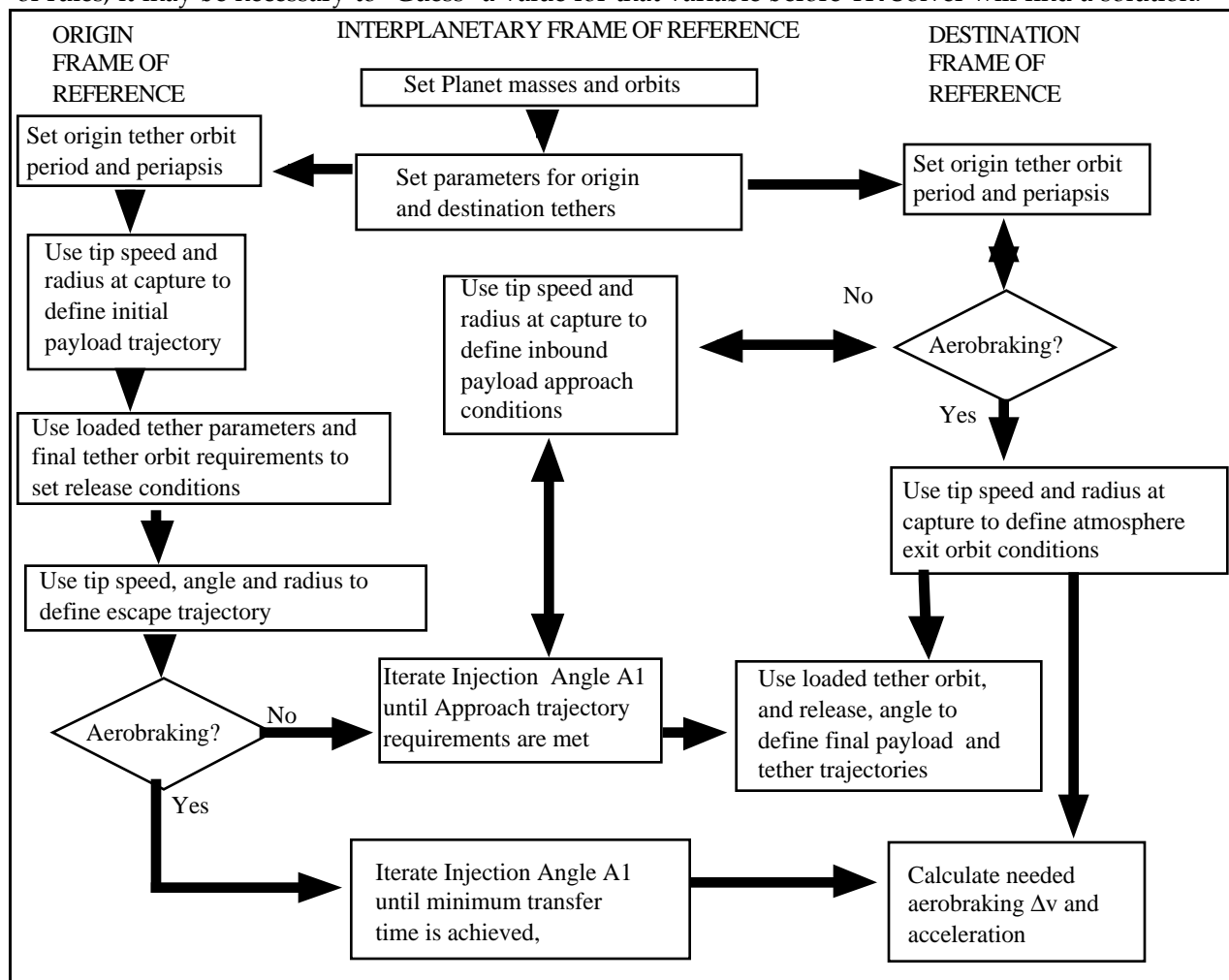
- [1] M.L. Cosmo and E.C. Lorenzini, Tethers In Space Handbook - Third Edition, prepared for NASA/MSFC by Smithsonian Astrophysical Observatory, Cambridge, MA, Dec 1997.
- [2] Hans Moravec, "A Non-Synchronous Orbital Skyhook," J. Astronautical Sci., Vol. 25, No. 4, pp. 307-322, Oct-Dec 1977.
- [3] Hans Moravec, "Free Space Skyhooks," November 1978. Unpublished notes obtained from the author.
- [4] Paul A. Penzo, "Tethers for Mars Space Operations," The Case For Mars II, Ed. C.P. McKay, AAS Vol. 62, Science and Technology Series, pp. 445-465, July 1984.
- [5] Paul A. Penzo, "Prospective Lunar, Planetary, and Deep Space Applications of Tethers," Paper AAS 86-367, AAS 33rd Annual Meeting, Boulder, CO, Oct 1986.
- [6] Martin O. Stern, Advanced Propulsion for LEO-Moon Transport, NASA CR-17084. Technical Report, California Space Institute, UCSD, La Jolla, CA 92093, June 1988. Progress Report on NASA Grant NAG 9-186, NASA/JSC, Houston, TX.
- [7] Joseph Carroll, "Preliminary Design for a 1 km/s Tether Transport Facility," NASA OAST Third Annual Advanced Propulsion Workshop JPL, Pasadena, CA, 30-31 Jan 1992.
- [8] Robert L. Forward. "Tether Transport from LEO to the Lunar Surface," AIAA Paper 91-2322, AIAA/SAE/ASME/ASEE 27th Joint Propulsion Conference, Sacramento, CA, June 1991.
- [9] R.P. Hoyt and R.L. Forward, LEO-Lunar Tether Transport System Study, Tethers Unlimited Final Report on Subcontract S06-34444 on NASA Grant P3776-5-96 to the Smithsonian Astrophysical Observatory, April 1997.
- [10] R.P. Hoyt and R.L. Forward, "Tether Transport from Sub-Earth-Orbit to the Moon... And Back!," 1997 International Space Development Conference, Orlando FL, May 1997.
- [11] R.P. Hoyt, "LEO-Lunar Tether Transport System," AIAA Paper 97-2794, 33rd Joint Propulsion Conference, 1997.
- [12] R.P. Hoyt, "Tether System for Exchanging Payloads Between the International Space Station and the Lunar Surface," 1997 Tether Technical Interchange Meeting, Huntsville AL, Sept 10-11, 1997.
- [13] Bate, R.R., D.D. Mueller and J.E. White, Fundamentals of Astrodynamics, Dover, 1971
- [14] Robert L. Forward, "Failsafe Multistrand Tether Structures for Space Propulsion," In AIAA/SAE/ASME/ASEE 28th Joint Propulsion Conference, Nashville, TN, July 1992. AIAA Paper 92-3214.
- [15] R.L. Forward and R.P. Hoyt, "Failsafe Multiline Hoytether Lifetimes," AIAA paper 95-28903 1st AIAA/SAE/ASME/ASEE Joint Propulsion Conference, San Diego, CA, July 1995.
- [16] M. Martinez-Sanchez and S. Gavit, "Orbital Modifications Using Forced Tether Length Variations." Journal of Guidance, Control and Dynamics, Vol. 10, pp. 233-241, May-June 1987.
- [17] G. Landis, "Reactionless Orbital Propulsion Using Tether Deployment," Acta Astronautica, Vol. 26, No. 5, pp. 307-312, 1992; paper IAF-90-254.
- [18] G. Landis, "Reactionless Orbital Capture Using Tethers," paper AIAA-98-3406, 24th AIAA/ASME/SAE/ASEE Joint Propulsion Conference, July 13-15, 1998, Cleveland OH. In slightly earlier form: 8th Advanced Space Propulsion Workshop, JPL 13-15 May, 1997.

NOTES ON THE MARS-EARTH RAPID INTERPLANETARY TETHER TRANSPORT MODEL

Gerald Nordley
Consultant, Tethers Unlimited, Inc.

The attached model is being developed using the TK Solver Plus mathematical modeling program and was used to generate the data provided in the appendices.

TK Solver Plus for Macintosh was distributed by Universal Technical Systems, 1220 Rock Street, Rockford IL 61101. A model consists of a list of equations, or "rules," listed in a rule window which relate a set of variables. The rules are non procedural and the program will generally solve for a variable provided sufficient other variables are defined regardless of where the variable appears in a rule (as long as it appears only once) or what order the rules are listed in. Thus it is possible to "tie down" certain variables (such as a periapsis of a final orbit) and ask the program to iterate the other inputs of the model to produce value desired, if possible. If a variable is listed more than once in the set of rules, it may be necessary to "Guess" a value for that variable before TK Solver will find a solution.



The following model is generally presented in chronological order, but this is not necessary in TK Solver, nor is it necessary to solve an equation for the variable needed; it only needs to appear in a rule where all other variables are defined. TK Solver allows for the creation of user-defined subroutines called "functions" which may be either rule driven or procedural in nature. What happens with variables in functions does not appear in the variables window, however, and initial development was thus done entirely in the rules window. Now that the model is working, much of what is listed here

will be moved into such subroutines. This will make it easier to expand the model.

The current model is two dimensional and treats planetary orbits as circles. These and other simplifications will be removed as development continues. The documentation of this model is in rough engineering form and consist mainly of comments and notes in the listing itself. It is not the intent of the engineer to offer this to other parties for use as a finished piece of software at this time. The listing attached is for the case of an Earth to Mars payload with aerodeceleration before tether capture at the Mars end. Except for the inclusion of an atmospheric exit orbit and the calculation of the aerobraking Δv , the equations are the same as used for the tether-only cases. Outbound Earth to Mars and inbound Mars to Earth listings differ only in the calculation of the patch points. The flowchart below sketches the approach of the model to interplanetary transport by rotating tethers.

"Equations for orbital transfer program, using aerodeceleration at the destination."

"Astronomical conditions for the outbound problem"

```

mus = Ms*G           "transfer central mass"
muo = Mo*G           "departure mass"
wo^2 = (Mo+Ms)*G/aMo^3 "average angular velocity of origin mass"
vciro^2 = (Mo+Ms)*G/aMo "circular orbital velocity at origin"
PdMd = PsMd/(1 - PsMd*wd/(#2pi)) "diurnal rotation period of Md"
mus/(aMo+ropat)^2 + muo/ropat^2 = (aMo+ropat)*wo^2 "origin patch condition"
mud = Md*G           "destination mass"
wd^2 = (Md+Ms)*G/aMd^3 "average angular velocity of destination mass"
mus/(aMd-rdpat)^2 = mud/rdpat^2 + (aMd-rdpat)*wd^2 "dest. patch condition"
vcird^2 = (Md+Ms)*G/aMd "circular orbital velocity at destination"
PdMo = PsMo/(1 - PsMo*wo/(#2pi)) "diurnal rotation period of Mo"

```

"original tether loaded design"

```

Lo = vtipo^2/cgo      "used centrifugal gravity to constrain length"
qdoto = vtipo/Lo      "loaded tether angular rate"
Proto = #2pi/qdoto    "loaded tether rotation period"

```

"original tether initial conditions, subscript 'oi' "

```

Loi = Lo+Cshifto      "unloaded tether grapple arm length"
vco = Cshifto*qdoto    "rotational speed of unloaded c.o.m."
vtipoi = vtipo + vco   "rotational speed of unloaded tether tip"
qdotoi = vtipoi/Loi    "unloaded tether angular rate"
Protoi = #2pi/qdotoi   "period of rotation of unloaded tether"
Poi = #2pi*sqrt(aoi^3/muo)/PdMo "period of initial tether orbit"
noi = PdMo*Poi/Protoi "number of rotations per orbit of unloaded teth"
rpoi = Ro + Alto + Loi + 2*Cshifto "set initial tether periapsis"
poi = ((rpoi*vpoi)^2)/muo "set one of the following to get orbit"
eoi = poi/rpoi - 1     "eccentricity"
aoi = poi/(1-eoi^2)    "semimajor axis"
Esoi = -muo/(2*aoi)    "specific energy"
Esoi = vpoi^2/2 - muo/rpoi "vpoi is periapsis velocity"
roic = rpoi            "tether c.o.m. radius at capture = periapsis"
cos(uoic)=(poi/roic-1)/eoi "uoic = true anom. set to zero for the time bein
duoc = atan2(Cshifto*sin(qoic),(roic-Cshifto*cos(qoic))) "c.o.m. offset"
qoc = qoic + duoc      "tether arm angle from straight down = 0 for now

```

"payload pickup conditions, subscript 0"

```

      "These have not yet been modified for nonperiapsis pickup."
r0c = rpoi - Loi      "radius of capture assumes pickup at periapsis"
vcoc^2 = muo/r0c      "circular orbital velocity at roc, for comparis
v0c = vpoi-vtipoi     "capture velocity assumes pickup at periapsis"
Es0 = v0c^2/2 - muo/r0c "specific energy"
v0R = sqrt(2*(Es0 + muo/Ro))*1.2 "estimated payload Δv from surface"
h0 = v0c*r0c          "specific angular momentum (assumes apsidal pic
a0 = -muo/(2*Es0)     "semimajor axis"
p0 = h0^2/muo         "semilatus rectum"
e0^2 = 1-p0/a0        "eccentricity"
rp0 = a0*(1-e0)       "periapsis"
ra0 = a0*(1+e0)       "apoapsis"
P0 = #2pi*sqrt(a0^3/muo)/60 "period of initial payload orbit"
tmax = 2*asin(vymax/vtipoi)/qdotoi "unloaded origin tether pickup time"

```

"post pickup original tether orbit, subscript 'o' "

```

vpo = vpoi -qdotoi*Cshifto "periapsis velocity of c.o.m. assumes q = 0"
rpo = rpoi-Cshifto        "new tether c.o.m. periapsis and altitude"
po = ((rpo*vpo)^2)/muo    "semilatus rectum"
eo = po/rpo - 1           "eccentricity"
ao = po/(1-eo^2)          "semimajor axis"
Eso = -muo/(2*ao)        "specific energy from ao"
Eso = vpo^2/2 - muo/rpo   "defines periapsis velocity"
Po = #2pi*sqrt(ao^3/muo)/PdMo "period of loaded tether orbit"

```

```

cos(uoc) = (po/rpo-1)/eo      "cosine of true anomaly"
      "tether conditions at payload release, subscript 'or' "
Duo = uor-uoc
nrev = int(Duo/((#2pi))      "number of orbits while on tether"
      "eccentric and Mean anomalies of tether orbit at release"
Eor/2 = atan2(sqrt(1-eo)*Tan(uor/2),sqrt(1+eo)) + nrev*pi()
      IF Eor<0 THEN MAor = #2pi+Eor-eo*sin(Eor) Else MAor = Eor - eo*sin(Eor)
cgor = cos(qor)
uordeg=uor*raddeg           "uor in degrees--for user convenience"
tor = (Po*86400)*MAor/((#2pi) "time since tether periapsis of release"
Dgor = qdoto*tor            "number of rotations since tether periapsis"
duor = atan2(Lo*sin(qor),(ror-Lo*cos(qor))) "offset for tether angle"
qor = qoc + Dgor - Duo       "tether tip angle from straight down at release"
qordeg = Mod(qor*raddeg,360) "qor in degrees"
no = Po*86400/Proto          "number of rotations per tether revolution"
nor = tor/Proto              "number of revolutions since periapsis pick up"
cos(phor) = sqrt(po*muo)/(ror*vor) "phi of loaded tether c.o.m. at release"
phor = atan2(eo*sin(uor),(1+eo*cos(uor))) "above as a function of uor"
vor^2 = muo*(2/(ror - 1/ao)    "velocity of loaded tether c.o.m. at release"
ror = po/(1+eo*cos(uor))      "radius of loaded tether c.o.m. at release"
ruqor = -Lo*sin(qor)          "tether arm length projected on tether c.o.m. h
zqor = ror-Lo*cos(qor)        "radius plus tether arm length projected on rad
rlr^2 = ruqor^2 + zqor^2      "radius of tether tip at release"
vuqor = -vtipo*cos(qor)      "tip velocity projected on c.o.m. horizon"
vzqor = vtipo*sin(qor)       "tip velocity projected on tether c.o.m. radius
Test2^2 = vuqor^2 + vzqor^2  "test variable to check on above"
vuor = vor*cos(phor)         "tether c.o.m. horizontal velocity"
vzor = vor*sin(phor)         "tether c.o.m. vertical velocity"

      "final origin tether orbit, subscript of"
ruof = -Cshifto*sin(qor)      "unloaded c.o.m. horizontal position"
rzof = ror + Cshifto*cos(qor) "unloaded c.o.m. vertical position"
rof^2 = ruof^2 + rzof^2       "unloaded c.o.m. radius from Mo after release"
vquf = qdoto*Cshifto*cos(qor) "unloaded c.o.m. horizontal vel, in or frame"
vqzf = -qdoto*Cshifto*sin(qor) "unloaded c.o.m. vertical vel, in or frame"
vuof = vor*cos(phor) + qdoto*Cshifto*cos(qor) "unloaded c.o.m. horizontal
vzof = vor*sin(phor) - qdoto*Cshifto*sin(qor) "unloaded c.o.m. vertical ve
vof^2 = vuof^2 + vzof^2      "Mo frame velocity of tether center of mass
duf = atan2(-ruof,rzof)      "horizontal angular offset due to c.o.m. shift"
phof = atan2(vzof,vuof) - duf "final origin tether c.o.m. flight path angle"
Esof = vof^2/2 - muo/rof      "specific energy"
hof = vof*rof*cos(phof)      "specific angular momentum"
aof = -muo/(2*Esof)          "semimajor axis"
pof = hof^2/muo              "semilatus rectum"
eof^2 = 1-pof/aof            "eccentricity"
Pof = #2pi*sqrt(aof^3/muo)/PdMo "period of final tether orbit"
rpof = aof*(1-eof)           "periapsis"
vpof^2/2 = Esof + muo/rpof    "periapsis velocity of final origin tether"
altof = rpof-Loi-Ro          "Constraint: min. tip altitude"
uof = atan2(tan(phof),(1-rof/pof)) "true anomaly of tether orbit after rel"

      "payload release orbit in Mo frame of reference, subscript 1r"
vuolr = vuor + vuqor         "payload horizontal velocity at release"
vzolr = vzor + vzqor         "payload vertical velocity at release"
v1lr^2 = vuolr^2 + vzolr^2    "payload velocity at release"
pholr = atan2(vzolr,vuolr)    "payload flight path angle wrt tether horizon"
phlr = pholr + duor           "duor adjusts phi for tip displacement"
rlr^2 = (ror - Lo*cos(qor))^2 + (-Lo*sin(qor))^2 "payload radius at releas
pl = (rlr*v1lr*cos(phlr))^2/muo "semilatus rectum of payload injection"
Esl = v1lr^2/2 - muo/rlr      "specific energy"
el^2 = 1+2*Esl*pl/muo        "eccentricity"
rpl = al*(1-el)              "periapsis radius"
IF Abs(el-1)<1E-9 THEN al = ropat/2 ELSE al = -muo/(2*Esl)
      "payload release orbit may be either elliptical or hyperbolic (al<0)

```



```

ulr = Mod(#2pi+atan2( tan(phlr),(1-rlr/pl) ), #2pi)      "true anomaly"

"Time of flight"
sulr = sgn(sin(ulr))      "sign of u used to resolve ambiguities"
coElr = (e1+cos(ulr))/(1+e1*cos(ulr))
  IF e1>=1 THEN Elr=sulr*acosh(coElr) Else Elr=sulr*acos(coElr)
  IF e1>=1 THEN siElr = sinh(Elr) Else siElr = sin(Elr)

"payload orbit in Mo frame of reference at patch point, subscript 1i"
culinf = -1/e1      "cosine of u at infinity ≈ cos(ul) at ropat"
  IF Abs(culinf)>1 THEN cos(ulinf)=1-1E-19 Else cos(ulinf) = culinf
vhl^2 = -sgn(culinf)*abs(vlr^2 - 2*muo/rlr)      "hyperbolic excess velocity"
  "a negative number means non escape"
culi = (pl/ropat-1)/e1      "cosine ul at patch: can't use ulinf:t=inf"
  IF Abs(culi)>1 THEN cos(uli)=1-1E-19 Else cos(uli) = culi
coEli = (e1+culi)/(1+e1*culi)      "cosine of Eccentric anom at patch"
  IF e1>=1 THEN Eli= acosh(coEli) Else Eli = acos(coEli)
  IF e1>=1 THEN siEli = sinh(Eli) Else siEli = sin(Eli)
  "logic statements interpret coElr as either cos or cosh, depending on e"
tlr = sulr*sgn(al)*sqrt(abs(al^3/muo))*(Elr-e1*siElr)
tli = sgn(al)*sqrt(abs(al^3/muo))*(Eli-e1*siEli)
delti = (tli-tlr)/86400      "time of flight from release to patch"
Aldeg = Al*raddeg      "Mo frame departure angle, clockwise from orbit path"
  "Set Al, or GUESS to match proper destination constraints"

"Orientation of periapses,"
oml = mod(1.5*pi() + (uli + Al),#2pi)      "payload arg of periapsis at patch"
omof = mod(#2pi*nrev + oml + ulr -(uof-duor),#2pi)      "tether om at release"
omofD = omof*raddeg      "in degrees"
omoi = mod(omof + (uor-uof) + uoic,#2pi)      "initial argument of periapsis"
omoiD = omoi*raddeg

"Energetics: If DETo is less than DEpo, periapsis has rotated"
DETo = -mTo*(Esof-Esoi)      "mass times change of specific energy of tether"
DEpo = mpo*(Es1-Es0)      "mass times change of payload specific energy"

"Ms transfer orbit, subscript 2"
ro = aMo + ropat*sin(Al)      "distance from center of Ms"
v2iz = vhl*sin(Al)      "radial component of transfer orbit injection"
v2iu = vcir + vhl*cos(Al)      "azimuthal component of transfer orbit inj."
v2i^2 = (v2iu^2 + v2iz^2)      "velocity of transfer orbit injection"
ph2i = atan2(v2iz,v2iu)      "flight path angle"
sph2 = sgn(ph2i)      "sign of flight path angle, +,outgoing"
Es2 = 0.5*v2i^2 - mus/ro      "specific energy"
a2 = -mus/(2*Es2)      "semimajor axis"
h2 = v2iu*ro      "specific angular momentum"
p2 = h2^2/mus      "semilatus rectum"
e2^2 = 1 + 2*Es2*(h2/mus)^2      "eccentricity"
cos(sph2*u2i) = (p2/ro-1)/e2      "cosine of true anomaly"
rp2 = p2/(1+e2)      "periapsis radius"
ra2 = a2*(1+e2)      "apoapsis radius, negative if hyperbolic"
coE2i = (e2+cos(u2i))/(1+e2*cos(u2i))      "Eccentric anomaly"
  IF e2>=1 THEN siE2i^2=1+coE2i^2 Else siE2i^2=1-coE2i^2
  IF e2>=1 THEN E2i=acosh(coE2i) Else E2i=acos(coE2i)
to = sgn(Al)*sqrt(abs(a2^3/mus))*sgn(a2)*(E2i - e2*siE2i)/86400
  "imputed time of flight from periapsis to injection"
rd = aMd - sph2*rdpat      "radius at destination patch, assumes
  " ph2d≈π/2 to avoid an additional iterative solving loop"
v2d^2 = 2*(Es2+mus/rd)      "Ms frame velocity at patch point"
cosph2d = h2/(rd*v2d)      "cosine of flight path angle"
IF cosph2d > 1 THEN ph2d = 0 ELSE cos(ph2d) = cosph2d

```

```

cosu2d = (p2/rd-1)/e2          "cosine of true anomaly"
IF cosu2d <= -1 THEN u2d = pi() Else u2d = acos(cosu2d)
cosE2d = (e2+cos(u2d))/(1+e2*cos(u2d))  "Eccentric anomaly"
IF e2>=1 THEN sinE2d^2=1+cosE2d^2 Else sinE2d^2=1-cosE2d^2
IF e2>=1 THEN E2d=acosh(cosE2d) Else E2d=acos(cosE2d)
td = sgn(A1)*sqrt(abs(a2^3/mus))*sgn(a2)*(E2d - e2*sinE2d)/86400
delt = td - to                 "time of flight in Ms trajectory"
delu = u2d - u2i               "Ms-centered angle traversed"

"destination patch conditions; assumes circular destination orbit"
v2du = v2d*cosph2d             "azimuthal velocity"
v2dz = v2d*sin(ph2d)           "radial velocity"
Dv2du = v2du-vcird             "positive if payload"
v3i^2 = Dv2du^2 + v2dz^2       "velocity with respect to planet"
A2 = atan2(v2dz,Dv2du)         "approach angle in Md frame of reference"
A2deg = A2*raddeg              "approach angle in degrees for convenience"

"hyperbolic approach orbit, Md frame, subscript 3"
r3i^2 = rdpat^2 + (rdpat/tan(A2))^2 "initial radius from Md"
rp3 = rpx                      "sets target periapsis of approach orbit"
Es3 = 0.5*v3i^2 - Md*G/r3i     "specific energy"
vp3^2 = 2*(Es3 + Md*G/rp3)     "periapsis velocity"
e3 = 1 + 2*rp3*Es3/(Md*G)      "eccentricity"
a3 = -mud/(2*Es3)              "semimajor axis"
p3 = rp3*(1+e3)                "semilatus rectum"
ph3i = -acos(sqrt(p3*mud)/(rdpat*v3i)) "incoming flight path angle"
ph3deg = ph3i*raddeg           "ph3i in degrees"
cos(u3inf) = -1/e3             "cosine of hyperbolic asymptote"
om3 = (u3inf-A2)+pi()/2        "angle to inbound periapsis from Ms->Md radius"
cos(u3i) = (p3/r3i - 1)/e3     "cosine of true anomaly at incoming patch"
cosh(E3i) = (e3+cos(u3i))/(1+e3*cos(u3i)) "Eccentric anomaly"
t3i = -sqrt(-(a3^3/mud))*(E3i - e3*sinh(E3i))/86400 "flight time to rp3"

"destination tether, subscript d"
Ld = vtipd^2/cgd              "used centrifugal gravity to constrain length"
qdotd = vtipd/Ld              "loaded design angular rate"
Prd = #2pi/qdotd              "loaded tether period"
Ldi = Ld + Cshiftd            "unloaded grapple arm radius from c.o.m."
vtipdi = vtipd + Cshiftd*qdotd "unloaded tip velocity"
qdotdi = vtipdi/Ldi           "unloaded rotation rate"
Prdi = #2pi/qdotdi            "period of rotation of unloaded tether"

"destination tether initial orbit parameters, subscript di"
Pdi = Prdi*ndi/PdMd           "tether orbit period, GUESS or set by ndi"
rpdi = Rd + Altd + Ldi        "tether orbit periapsis set by min tip altitude"
Pdi=#2pi*sqrt(adi^3/mud)/PdMd "defines tether orbit semimajor axis"
Esdi = -mud/(2*adi)           "specific energy"
Esdi = vpd^2/2 - mud/rpdi     "defines periapsis velocity"
edi^2 = 1 + 2*Esdi*pd/mud     "eccentricity"
pdi = rpdi*(1+edi)            "semilatus rectum"
rpdi = adi*(1-edi)            "periapsis"
radi = adi*(1+edi)            "apoapsis radius"
edi^2 = 1 + 2*Esdi*pd/mud     "eccentricity"
radi = adi*(1+edi)            "apoapsis radius"
rpdi = adi*(1-edi)            "periapsis"
pdi = rpdi*(1+edi)            "semilatus rectum"

"aerodeceleration approximated by constant deceleration from rp3 to rpx"
dva = vp3-vpx                 "delta v needed set by capture requirements"
dta = dva/adec                 "deceleration time"
dua = (.5*adec*dta^2)/rpx     "arc of deceleration, set by orientation needs"
dsa = .5*adec*dta^2           "distance covered during deceleration"
Duc = uxc + dua                "arc covered from rp3 to capture"

```

```

"atmospheric exit orbit"
hx = rxc*vdxc*cos(phxc)      "specific angular momentum required for phxc"
phxcD = phxc*raddeg          "phxc in degrees for convenience"
vpx = hx/rpx                 "periapsis velocity of exit orbit"
Esx = vpx^2/2 - mud/rpx      "specific energies"
Esx = vdxc^2/2 - mud/rxc     "secific energy by another method as check"
px = hx^2/mud                "semilatus rectum of exit orbit"
ex = px/rpx - 1              "eccentricity"
ax = -mud/(2*Esx)            "semimajor axis"
uxc = acos( (px/rxc-1)/ex )   "true anomaly of capture on payload orbit"
cExc = (ex+cos(uxc))/(1+ex*cos(uxc)) "cos of eccentric anomaly"
  IF ex>=1 THEN siExc^2=1+cExc^2 Else siExc^2=1-cExc^2
  IF ex>=1 THEN Exc=acosh(cExc) Else Exc=acos(cExc)
txc = sqrt(abs(ax^3/mud))*sgn(ax)*(Exc - ex*siExc)/86400 "time from exit to capture"
  "capture phasing Capture conditions obtained by iterating, subscript dic"
  "The general strategy is to set udic at a desirable value and find a tether
  "arm angle that generates an orbit that has the desired periapsis. A number
  "of solutions are possible, so one needs to guess close to the one desired"
udicdeg = udic*raddeg        "GUESS true anomaly of tether at capture event"
cos(Edic) = (edi+cos(udic))/(1+edi*cos(udic)) "eccentric anomaly"
Mdic = Edic*edi*sin(Edic)     "mean anomaly"
tdic = Mdic*Pdi*86400         "time since periapsis"
qdic = qdotdi*tdic + Dqdc     "assumes q= 0 at periapsis; relax if needed"
qdideg = qdic*raddeg          "q in degrees"
rdic = pdi/(1+edi*cos(udic))  "radius of tether c.o.m. at capture"
  zdic = rdic - Ldi*cos(qdic)  "z component of tether tip position"
  rudic = -Ldi*sin(qdic)       "horizontal component of tether tip position"
rxc^2 = zdic^2 + rudic^2       "radius of tether tip from center of Md"
vdic^2 = 2*(Esdi + mud/rdic)   "velocity of c.o.m. at capture"
tan(phdic) = edi*sin(udic)/(1+edi*cos(udic)) "flight path angle"
  vtDicz = vtipdi*sin(qdic)    "radial projection of tip velocity"
  vtDicu = -vtipdi*cos(qdic)   "horizontal projection of tip velocity"
  vtdcz = vdic*sin(phdic)+vtipdi*sin(qdic) "z velocity of tip"
  vtdcu = vdic*cos(phdic)-vtipdi*cos(qdic) "u velocity of tip"
vdxc^2 = vtdcz^2 + vtdcu^2     "velocity of tether tip and payload at capture"
sin(udic) = Atan2(Ldi*sin(qdic),rdic) "Md-centered offset angle of tip"

"destination tether after capture"
qdc = qdic-dudc               "loaded tether arm orientation"
zdc = -Cshiftd*cos(qdic)+rdic "loaded tether center of mass z position"
rudc = -Cshiftd*sin(qdic)     "loaded tether c.o.m. horizontal position"
dudc = atan2(rudc,rdic)        "angular shift due to c.o.m. offset"
rdc^2 = zdc^2 + rudc^2         "loaded tether c.o.m. radius"
vuqdic = -Cshiftd*qdotdi*cos(qdic) "new com u velocity, com ref"
vzqdic = Cshiftd*qdotdi*sin(qdic)  "new com z velocity, com ref"
vudc = vdic*cos(phdic) + vuqdic   "new com horizontal velocity, Md ref"
vzdc = vdic*sin(phdic) - vzqdic    "new com z velocity, Md ref"
vdc^2 = vudc^2 + vzdc^2         "loaded tether c.o.m. velocity"
Esd = vdc^2/2 - mud/rdc         "specific energy"
Esd = -mud/(2*ad)              "defines semimajor axis, ad"
phdc = atan2(vzdc,vudc)+dudc     "flight path angle"
hd = rdc*vdc*cos(phdc)          "specific angular momentum"
pd = (hd^2)/mud                "semilatus rectum"
ad = pd/(1-ed^2)               "defines eccentricity"
rpd = ad*(1-ed)                "periapsis"
rad = ad*(1+ed)                "apoapsis"
vpd = hd/rpd                   "periapsis velocity"
Pd = #2pi*sqrt(ad^3/mud)/PdMd "Period"
nd = PdMd*Pd/Prd               "loaded tether rotations per tether period"
udc = atan2( tan(phdc),(1-rdc/pd) ) "true anomaly at capture"
cos(Edc) = (ed+cos(udc))/(1+ed*cos(udc)) "eccentric anomaly"
Mdc = Edc - ed*sin(Edc)        "Mean aomaly at capture"
omdc = om3 + dudc - udc        "tether argument of periapsis at capture"

```

"tether orbit at release; GUESS one of the following parameters"

```

udr = mod(udc + Dud,#2pi)      "angle traversed while on tether"
udrdeg = udr*raddeg
urrev = int((Dud+udc)/(#2pi)) "number of revs while on tether"
sudir = sgn(sin(udr))
Edx = sudr*acos((ed+cos(udr))/(1+ed*cos(udr))) "cosine of eccentric anomaly"
Edr = mod(#2pi+Edx,#2pi)
Mdr=Edr-ed*sin(Edr)+urrev*#2pi      "Mean anomaly at release"
tldr = (Mdr-Mdc)*Pd*86400/#2pi      "time spent on tether"
cos(phdr) = sqrt(pd*mud)/(rdr*vdr)   "cosine of flight path angle"
udx = atan2( tan(phdr),(1-rdr/pd) )  "true anomaly from phdr"
udr = mod(#2pi+udx,#2pi)            " mod 2π "
vdr^2 = mud*(2/rdr - 1/ad)          "velocity of tether c.o.m at release"
rdr = pd/(1+ed*cos(udr))            "radius at release"

```

"tether payload release conditions"

```

Dqdr = qdotd*tldr              "tether rotation over Dqdr"
qdr = mod(qdc+Dqdr-Dud,#2pi)   "tether arm angle from down"
qdrdeg = qdr*raddeg             " " in degrees"
qdrrev = qdr/(#2pi)            " " in rotations"
phdr = atan2(ed*sin(udr),(1+ed*cos(udr))) "flight path angle of c.o.m at rel."
vdr^2 = mud*(2/rdr - 1/ad)      "velocity of c.o.m. at release"
rdr = pd/(1+ed*cos(udr))        "radius of c.o.m. at release"
cos(phdr) = sqrt(pd*mud)/(rdr*vdr) "cosine of tether flight path angle"
dudr = atan2(Ld*sin(qdr),(rdr-Ld*cos(qdr))) "Md-frame offset angle due to Ld"
rugdr = -Ld*sin(qdr)            "horizontal distance of tip"
zqdr = rdr-Ld*cos(qdr)          "radial distance of tip"
r4r^2 = rugdr^2 + zqdr^2        "radial distance of tip&payload"
vuqdr = -vtipd*cos(qdr)        "c.o.m. horizontal velocity of tip"
vzqdr = vtipd*sin(qdr)         "c.o.m. radial velocity of tip"
Test3^2 = vuqdr^2 + vzqdr^2    "test to verify tip speed right"
vudr = vdr*cos(phdr)           "horizontal velocity of c.o.m."
vzdr = vdr*sin(phdr)           "radial velocity of c.o.m."
vur = vudr + vuqdr             "horizontal vel. of tip"
vzr = vzdr + vzqdr             "radial velocity of tip"
v4r^2 = vur^2 + vzr^2          "Md frame velocity of tip"
ph4dr = atan2(vzr,vur)          "c.o.m. flight path angle of tip"
ph4r = ph4dr + dudr            "flight path angle of tip/payload"

```

"final tether orbit, subscript df"

```

rudf = Cshiftd*sin(qdr)        "relative hor. dist. of final c.o.m."
rzdf = rdr + Cshiftd*cos(qdr)  "relative radius of final c.o.m."
rdf^2 = rudf^2 + rzdf^2        "Md radius of final c.o.m."
vqudf = qdotd*Cshiftd*cos(qdr) "rel. hor. velocity of final c.o.m."
vqudf = -qdotd*Cshiftd*sin(qdr) "rel. radial velocity of final c.o.m."
vudf = vdr*cos(phdr) + qdotd*Cshiftd*cos(qdr) "Md horizontal velocity"
vzdf = vdr*sin(phdr) - qdotd*Cshiftd*sin(qdr) "Md radial velocity"
vdf^2 = vudf^2 + vzdf^2        "velocity of final c.o.m."
dudf = atan2(-rudf,rzdf)       "azimuthal offset angle due to Cshiftd"
phdf = atan2(vzdf,vudf) - dudf "flight path angle of tether c.o.m."
Esdf = vdf^2/2 - mud/rdf       "specific energy at rdf"
Esdf = vpdf^2/2 - mud/rpdf     "specific energy at periapsis"
adf = -mud/(2*Esdf)            "semimajor axis"
hdf = vdf*rdf*cos(phdf)        "specific angular momentum"
pdf = hdf^2/mud                "semilatus rectum"
edf^2 = 1-pdf/adf              "eccentricity"
Pdf = #2pi*sqrt(abs(adf)^3/mud)/PdMd "period"
rpdf = adf*(1-edf)             "periapsis"
altpdf = rpdf - Rd - Loi       "periapsis altitude"
radf = adf*(1+edf)             "apoapsis"
udf = atan2( tan(phdf),(1-rdf/pd) ) "true anomaly"

```

```

"final payload orbit, subscript 4"
h4 = r4r*v4r*cos(ph4dr)      "flight path angle"
p4 = h4^2/mud                "semilatus rectum"
Es4 = v4r^2/2 - mud/r4r      "specific energy"
v4e^2 = 2*Es4 + 2*mud/(Rd+Altd) "payload velocity on entry of Md atm"
e4^2 = 1 + 2*Es4*h4^2/mud^2  "eccentricity"
cos(u4r) = (p4/r4r-1)/e4     "true anomaly"
rp4 = p4/(1+e4)              "radius of periapsis"
a4 = rp4/(1-e4)              "semimajor axis"
ra4 = a4*(1+e4)              "apapsis radius"
vp4 = h4/rp4                 "periapsis velocity (may reenter first!)"
P4 = #2pi*sqrt(abs(a4^3)/mud)/3600 "period"
vcp^2 = mud/r4r              "circular orbit velocity at r4r"

"total time from Mo tether capture to release by the Md tether"
Total = tor/86400 + delti + delt + t3i + (txc + tdr)/86400

"tether periapsis orientation"
omdi = #2pi + om3 - udi + udi     "initial argument of periapsis"
omdiD = mod(omdi*raddeg,360)      "omdi in degrees"
omdf = omc + udr - udf            "final argument of periapsis"
omdfD = mod(omdf*raddeg,360)      "omdf in degrees"

"diagnostics"
DEsd = -mTd*(Esdf-Esdi)          "change in tether energy"
DEsdp = mdp*(Es4-Es3)            "change in payload energy"

Flag1 = int((Rd+150000)/r4r)      "flag for payload reentry"
Flag2 = int((Rd+150000)/rp4)      "flag for tether reentry"

raoi = aoi*(1+eoi)               "initial origin tether apoapsis"
rao = ao*(1+eo)                  "loaded origin tether apoapsis"
raof = aof*(1+eof)               "final origin tether apoapsis"

radi = adi*(1+edi)               "initial destination tether apoapsis"
rad = ad*(1+ed)                  "loaded destination tether apoapsis"
radf = adf*(1+edf)               "final destination tether apoapsis"

rpdf - rpdi = Drpd               "tether c.o.m. periapsis shift"
vpdf - vpdi = Dv                 "tether c.o.m. velocity change"

```

(Earth-Mars) aerobraking transfer mode

6.2831853	#2pi		2*pi() (used to reduce operations)
57.29578	raddeg		deg/rad degrees per radian conversion const.
6.672E-11	G		Nm ² /kg ² universal gravitational constant
1.496E11	AU		m astronomical unit
1.9891E30	Ms		kg mass of central body of orbit 1
	mus	1.3271E20	Nm ² /kg grav. parameter
6378000	Ro		m Radius of origin body
5.974E24	Mo		kg Mass of origin body
	muo	3.9859E14	Nm ² /kg grav. parameter of origin body
86165.045	PsMo		s sidereal rotation period of Md
	PdMo	86400	s diurnal rot. period defines "day"
1.5E11	aMo		semimajor axis of Mo orbit
	wo	1.983E-7	rad/s average orbit angular velocity of Mo
1.50555E9	ropat		m patch radius from origin body
L 1000	mpo		kg mass of payload
L 100000	mTo		kg mass of tether system
L 4000	Cshifto		m tether C.O.M. shift with payload
3398000	Rd		m Radius of destination body
6.4163E23	Md		kg Mass of destination body
	mud	4.2809E13	Nm ² /kg grav. parameter
88642.66	PsMd		s sidereal rotation period of Md
	PdMd	88775.724	s diurnal rot. period defines "day"
2.2739E11	aMd		semimajor axis of Md orbit
	wd	1.0624E-7	rad/s average orbit angular velocity of Md
1.07957E9	rdpat		m patch radius from destination body
1000	mpd		kg mass of payload
L 100000	mTd		kg mass of tether system
L 4000	Cshiftd		m tether C.O.M. shift with payload
"origin tether"			
150000	Alto		m minimum altitude of tether tip
L 2500	vtipo		m/s tip velocity of departure tether
	cgo	15.625	m/s ² acceleration at loaded tether tip
L 400000	Lo		m arm length of loaded tether
	Proto	1005.3096	s design rotation period,loaded tether
	qdoto	.00625	rad/s rotation rate of Tether
L	Loi	404000	m arm length of unloaded origin tether
	vco	83.333313	m/s velocity of unloaded c.o.m.
L	vtipoi	2525	m/s unloaded tip velocity
	noi	129.52619	number of rotations per orbit
	Protoi	1005.3096	s Period of rotation before pickup
	qdotoi	.00625	rad/s rotation rate before pickup
L 2.5003579	Poi		PdMo Period of unloaded tether
L	rpoi	6940000	m periapsis before pickup
	raoi	104082913	m initial origin tether apoapsis
L	vpoi	10475.871	m/s periapsis vel. before pickup
	aoi	55525456	m semimajor axis before pickup
	poi	13061572	m semilatus rectum before pickup
	eoi	.87450801	eccentricity before pickup
	Esoi	-3589212	J/kg tether specific energy
"payload pickup conditions"			
L	r0c	6536000	m radius of pick-up
L	v0c	7950.8709	m/s pickup velocity of payload
	v0R	9582.5547	estimated launch velocity
	vcoc	7798.0376	m/s circular orbit velocity at r0c
	Es0	-30609969	J/kg Initial specific energy of payload
	a0	6510710.3	m semimajor axis
	h0	5.0941E10	m ² /s specific angular momentum
	p0	6510413.5	semilatus rectum
L	e0	.03662146	eccentricity
L	rp0	6536000	m periapsis

L	ra0	7032913.4	m	apoapsis
L	P0	92.691648	min	Period
0	qoic		rad	tether arm angle at pickup
	uoic	0	rad	true anomaly at capture
	roic	6968000	m	radius of initial c.o.m. at pickup
2	vymax		m/s	max tolerable tether vertical motion
	tmax	.24774196	s	origin tether pickup time $v_y < v_{\max}$
"post pickup tether and tether orbit"				
	vpo	10271.669	m/s	periapsis velocity of c.o.m. orbit
L	rpo	6936000	m	periapsis radius of tether orbit
	rao	80486536	m	loaded origin tether apoapsis
	Eso	-4558323	J/kg	specific energy of tether orbit
	po	12803050	m	semilatus rectum of tether orbit
	eo	.8409293		eccentricity of tether orbit
	ao	43720601	m	semimajor axis of dep.tether orbit
L	Po	2.1234839	PdMo	period of loaded departure tether orbi
	no	90.5	rot/rev	number of rotations/rev
	uoc	0	rad	true anomaly after capture
	duoc	0	rad	Md centered c.o.m. offset at pickup
	qoc	0	rad	tether arm angle after pickup
"tether payload release conditions"				
L 360	uordeg		deg	Tether true anomaly of release
L	uor	6.2831853	rad	u at release NOTE:GUESS
	nrev	1	number	of revolutions before release
	Eor	6.2831853	rad	eccentric anomaly of release
	MAor	6.2831853	rad	mean anomaly of release
	tor	90980.523	s	time of release since periapsis
	Duo	6.2831853	rad	Δu while on tether
	Dqor	568.62827	rad	Δq while on tether
	duor	1.48E-14	rad	tether arm true anomaly offset
	nor	90.5	number	of rotations while on tether
	cqor	-1	cosine	of quor
L	qor	1140.3981	rad	release rotation from suborbital point
L 180	qordeg		deg	release angle in degrees, mod 2π
L	ror	6936000	m	radius of tether c.o.m. at release
L	vor	10450.871	m/s	velocity of tether com at release
L	phor	-2.06E-19	rad	flight path angle of tether at release
	vugor	2500	m/s	tip horizontal velocity in c.o.m. fram
	vzqor	6.803E-10	m/s	tip radial velocity in c.o.m. frame
	vuor	10271.669	m/s	tether c.o.m. horiz vel at release
	vzor	-2.03E-15	m/s	tether c.o.m. radial vel at release
	Test2	2500		
"tether final orbit"				
	ruof	-3.628E-9	m	azimuthal dist to final c.o.m.
	rzof	6941333.3	m	radial dist to final c.o.m.
	rof	6941333.3	m	radius of final c.o.m.
	vquf	-83.33331	m/s	old u vel of new c.o.m.
	vqzf	-2.27E-11	m/s	old r vel of new c.o.m.
	vuof	10188.336	m/s	azimuthal vel. of final c.o.m.
	vzof	-2.27E-11	m/s	radial vel. of final c.o.m.
	vof	10188.336	m/s	final c.o.m. velocity
	duf	5.227E-16	rad	Δu between centers of mass
	phof	-2.75E-15	rad	final tether orbit phi
	Esof	-5520912	J/kg	specific energy
L	Pof	1.8331234	PdMo	period
	aof	36097778	m	semimajor axis
	hof	7.0721E10	m ² /s	specific angular momentum
	eof	.80770746		eccentricity
	pof	12547900	m	semilatus rectum
L	rpof	6932000	m	periapsis
	raof	65254223	m	final origin tether apoapsis
L	altof	150000	m	tip altitude

	vpof	10188.336	m/s	periapsis velocity
	uof	-6.15E-15		tether u after release
				"payload release orbit, Mo ref"
	rugor	-1.088E-7	m	tip horiz. position in tether frame
	zgor	7354666.7	m	tip radial projection on tether axis
	rlr	7354666.7	m	tip/payload radius from Mo
	vuolr	12771.669	m/s	tip horiz vel in tether-Mo frame
	vzolr	6.803E-10	m/s	tip radial vel in tether-Mo frame
L	vlr	12950.871	m/s	tip/payload release velocity-Mo frame
	pholr	5.327E-14	rad	payload phi in tether Mo frame
L	phlr	6.2E-15	rad	flight path angle of departure
	p1	22136046	m	semilatus rectum of dep. orbit
	Es1	27362886	J/kg	specific energy of dep. orbit
	e1	2.009796		eccentricity of dep. orbit
	a1	-7283319		semimajor axis of dep. orbit
	rp1	7354666.7		periapsis radius
	ulr	1.019E-13	rad	true anomaly of dep. orbit at release
L	vhl	7685.0176	m/s	hyperbolic excess velocity of departur
	culinf	-.4975629		cosine of true anomaly at infinity
L	ulinf	2.0704911	rad	hyperbolic departure asymptote
	sulr	1		sign of sine of ulr
	coElr	1		cos of Eccentric anomaly at release
	siElr	0		sine of Eccentric anomaly at release
	Elr	0	rad	Eccentric anomaly at release
	t1r	0	s	time from periapsis at release
	uli	2.0831698	rad	true anomaly at patch point
	culi	-.4902473		cosine of true anomaly
	coEli	103.3497		cosine of eccentric anomaly
	Eli	5.3312422	rad	eccentric anomaly "
	siEli	103.34486		sine of eccentric anomaly
	t1i	199242.21	s	time from periapsis at patch
	delti	2.3060441	day	Δt from release to patch point
				"diagnostics"
	DETo	5.7951E10	J	Change in Tether system energy
	DEpo	5.7973E10	J	Change in payload energy
	A1	.20943951	rad	departure asymptote wrt Mo orbit
L	33	Aldeg	deg	departure asymptote wrt Mo orbit
L	om1	1.0674603	rad	argument of periapsis of departure
	omoi	.72181296	rad	solar arg of peri, init tether orbit
L	omoiD	61.160968	deg	" in degrees
	omof	.72181296	rad	solar arg of peri, final tether orbit
L	omofD	61.160968	deg	' in degrees
				"Ms transfer orbit injection condition
L	ro	1.5082E11	m	departure radius from Ms
	vciro	29744.82	m/s	circular velocity at ro
	v2iz	1538.0656	m/s	radial velocity of injection
	v2iu	36980.85	ms	tangential velocity of injection
	v2i	37012.821	m/s	velocity at ro
L	ph2i	.11514354	rad	flight path angle at ro
	sph2	1		sign of ph2 at ro: + for outgoing
	h2	5.5587E15	m2/s	specific angular momentum wrt Ms
	Es2	-1.9793E8	J/kg	specific energy of orbit wrt Ms
L	e2	.51786257		eccentricity for orbit wrt Ms
	p2	2.3283E11	m	semilatus rectum for orbit wrt Ms
L	u2i	.33885778	rad	true anomaly at ro
L	a2	3.0675E11	m	semimajor axis (<0 for hyperbolic)
	rp2	1.4995E11	m	radius of perigee of orbit wrt Ms
L	ra2	4.656E11	m	radius of apoapsis (<0, if hyperbola)
	coE2i	.99803184		cos or cosh of eccentric anomaly at ro
	siE2i	.06270915		sin or sinh of eccentric anomaly at ro
	E2i	.06275032	rad	eccentric anomaly at ro

	to	5.4779722	day	time since periapsis at departure
				"destination conditions"
L	rd	2.1499E11	m	radius to Md patch point from Ms
L	v2d	28318.31	m/s	payload velocity at rd
L	ph2d	.45897111	rad	flight path angle at rd
	cosph2d	.88118636		cos(ph2d)
	u2d	1.5186615	rad	true anomaly at rd
	cosu2d	.05211116		cosine of true anomaly at rd
	cosE2d	.58789649		cos or cosh of eccentric anomaly at rd
	sinE2d	.80893616		sin or sinh of eccentric anomaly at rd
	E2d	.94234029	rad	eccentric anomaly at rd
	td	96.577199	day	time from periapsis at arrival
L	delu	1.1466085	rad	angle traversed between ro and rd
L	delt	74.927428	day	time from ro to rd
				"patch conditions"
	v2dz	13178.094	m/s	radial velocity at rd wrt Ms
	v2du	24562.428	m/s	azimuthal velocity at rd wrt Ms
	vcird	24158.607	m/s	circular orbit velocity of Md
	Dv2du	403.82071	m/s	relative azimuthal velocity (v2u-vc)
L	v3i	12560.152	m/s	relative vel. at rd wrt Md
L	A2	1.522896	rad	approach angle in Md frame
L	A2deg	87.255513	deg	
				"hyperbolic approach orbit, Md frame"
L	r3i	1.02677E9	m	initial payload radius from Md
L	rp3	3498000	m	periapsis of approach orbit
L	vp3	13496.318	m/s	periapsis velocity of approach orbit
	Es3	86872978	J/kg	specific energy of approach orbit
	a3	-246391.3	m	semimajor axis
	e3	15.19693		eccentricity of incoming orbit
	p3	56656860	m	semilatus rectum of approach orbit
	ph3i	-1.567336	rad	flight path angle at r2d
L	ph3deg	-89.79001	deg	" in degrees
	u3inf	1.6366467	rad	true anomaly at infinity
	om3	1.6672804	rad	arg of periapsis of inbound orbit
	u3i	1.6331878	rad	true anomaly at rp3
	E3i	6.3579009	rad	eccentric anomaly
L	t3i	.9450767	day	time to periapsis/entry
				"destination tether initial orbit"
L 150000	AltD		m	min altitude of tether tip
L 2500	vtipd		m/s	unloaded tether tip velocity
	cgd	15.625	m/s2	acceleration at tip
	qdotd	.00625	rad/s	angular rate
	Prd	1005.3096	s	period of rotation, loaded
400000	Ld		m	new length of tether arm
	nd	111.61619		number of rotations per orbit
L	vtipdi	2525		unloaded tether tip vel
L	Ldi	404000	m	unloaded radius of capture/release arm
	ndi	67.757443		number of rotations per orbit, initial
	qdotdi	.00625	rad/s	tether rotation rate
	Prdi	1005.3096	s	initial period of rotation
L .5	Pdi		PdMd	initial period of tether orbit
	adi	17135549	m	
	Esdi	-1249143	J/kg	specific energy of tether orbit
L	vpdi	4282.6594	m/s	periapsis velocity of tether orbit
L	rpdi	3952000	m	periapsis radius of tether orbit
	radi	30309764	m	initial destination tether apoapsis
	edi	.76882367		eccentricity of tether orbit
	pdi	7006900.2	m	semilatus rectum of tether orbit
				"capture phasing"

L	0	udic	rad	tether true anomaly at capture GUESS
L		udicdeg 0	deg	" in degrees NOTE: SET
		qdic 2.6339362	rad	angle from Md center to tether tip
L	151.1105	qdideg	deg	" in degrees NOTE: GUESS
		Dqdc 2.6339362	rad	offset from 0 at periapsis
		dudc -.0016363		u offset due to tether arm length
L		rdic 3952000	m	tether center radius from Md at captur
L		phdic 0	rad	flight path angle of tether center
		zdic 4322539.6	m	tether frame radial coordinate of tip
		rudic -200934	m	tether frame azimuthal coord. of tip
L		rx 4310145	m	radius of capture
L		vdic 4282.6594	m/s	velocity of tether center at capture
		vtcdz 1255.8373		tether frame radial tip velocity
		vtcdcu 6629.6547		tether frame azimuthal tip velocity
		dudic .05070211	rad	capture true anomaly offset due to tip
		vtcdicz 1255.8373	m/s	tether frame z component of tip vel.
		vtcdicu 2257.5393	m/s	tether frame u component of tip vel.
		vdxc 6747.5513	m/s	tip velocity at capture
		Edic 0	rad	tether eccentric anomaly at capture
		Mdic 0	rad	mean anomaly of capture
		tdic 0	s	time since periapsis of capture
				"atmosphere exit orbit"
	3498000	rp 6947.2864	m	periapsis of exit orbit NOTE: SET
L		vp 12871625	m/s	velocity at periapsis of exit orbit
		Es 12871625	J	specific energy of exit orbit
		hx 2.4789E10	m2/s	specific angular momentum of exit orbi
		px 14354053	m	semilatus rectum of exit orbit
		ex 3.1035028		eccentricity of exit orbit
		ax -1662940	m	semimajor axis
		phxc .55672229	rad	capture flight path angle
L		phxcd 31.419898	deg	" in deg
L		uxc .72640827		true anomaly
		cExc 1.1606698		cosine of eccentric anomaly
		siExc 1.5320426		sine of eccentric anomaly
		Exc .55954031	rad	eccentric anomaly
		txc .01591401	day	time of flight
				"aerodeceleration"
L	3	adec	m/s2	average payload aerodeceleration
		dta 2330.8528	s	time of aerodeceleration
L		dva 6549.0313	m/s	aerodeceleration delta v
		dsa 8149312.2	m	arc distance of deceleration
L		dua 2.0435397	rad	angle traversed during deceleration
		Duc 3.0575232	rad	angle from entry to capture
				"tether orbit after capture
		udc -.0236429	rad	u of new tether orbit at capture
		Edc .00711817	rad	eccentric anomaly "
		Mdc .00118314	rad	mean anomaly "
		qdc 2.6355725	rad	capture arm angle
		zdc 3972985.1	m	radial comp of new c.o.m. position
		rudc -6481.739	m	azimuthal comp of new c.o.m. posit.
L		rdc 3955502.7	m	new c.o.m. radius
		vugdic 72.823831		new c.o.m. u vel, old c.o.m. ref
		vzqdic 40.510871		new c.o.m. z vel, old c.o.m. ref
		vudc 4444.9392	m/s	new c.o.m. u vel, Md ref
		vzdc -40.51087	m/s	new c.o.m. z vel, Md ref
L		vd 4304.5652	m/s	new c.o.m. velocity
		Esd -895568.1	J/kg	specific energy
L		Pd .55071093	PdMd	period of tether orbit
		ad 23900746	m	semimajor axis
		phdc -.0107499	rad	flight path angle
		hd 1.7659E10	m2/s	specific angular momentum

	ed	.83379241	eccentricity
	pd	7284713.9	m semilatus rectum
	vpd	4445.4319	m/s periapsis velocity of tether
L	rpd	3955451.1	m periapsis
	rad	43829007	m loaded destination tether apoapsis
			"release phasing"
L.75	Dud		rad Δu while on tether NOTE: GUESS
	udx	-.0236429	tether true anomaly at release
L	udr	.74207796	rad " mod 2π
L	udrdeg	42.517935	" in degrees
	sudr	-1	sign of $\sin(udr)$ + for outbound
	urrev	0	rev revolutions while on tether
	Dqdr	682.28053	rad tether rotation over Dud
	dudr	.00547167	rad tether arm true anomaly offset
	qdr	.04890204	rad angle tether radius to tip at release
L	qdrdeg	2.8540318	deg " in degrees. GUESS:Multiple solution
	qdrrev	.007783	rev " in revolutions
	Edx	-.0071182	tether eccentric anomaly at release
	Edr	6.2760671	rad " mod 2π
	Mdr	6.2820022	rad mean anomaly of release
L	tdr	711.22125	s time between capture and release
L	rdr	4441119.8	m radius to tether c.o.m. at release
L	vdr	4020.2532	m/s velocity of c.o.m. at release
	phdr	-.0107499	rad tether flight path angle at release
	ph4dr	.03818884	rad tip phi at release (tether radius)
L	ph4r	.78937605	rad tip phi at release (tip radius)
			"payload final orbit insertion"
	ruqdr	-19553.02	m azimuthal tether tip position
	zqdr	3573468.6	m radial tether tip position
L	r4r	4041665	m radius of payload release
	vuqdr	-2497.011	m/s x velocity of tether tip
	vzqdr	122.20638	m/s y velocity of tether tip
	Test3	2500	Test to make sure components add up
	vudr	4444.867	m/s u velocity of tether at release
	vzdr	-47.78385	m/s z velocity of tether release
	vur	1947.8556	m/s azimuthal velocity of release
	vzr	74.422537	m/s radial velocity of release
L	v4r	1888.9531	m/s final orbit injection velocity
	vcp	3461.161	m/s circular orbit velocity at inject radi
	h4	6.96071E9	m2/s final orbit specific angular momentum
	p4	1131791.4	m semilatus rectum
	e4	.6833912	eccentricity
	Es4	-10079795	J specific energy
L	P4	1.0105518	hr period
	a4	2123529.8	m semimajor axis
L	v4e	2552.5822	m/s velocity at atmospheric entry
	u4r	3.1238846	rad true anomaly of payload at release
	vp4	10353.135	m/s periapsis velocity
	rp4	672328.24	m periapsis NOTE:SET FOR ITERATION
	ra4	3574731.4	m apoapsis
			"tether final orbit"
	rudf	651.76722	m azimuthal dist to final c.o.m.
	rzdf	3986307.8	m radial dist to final c.o.m.
	rdf	3986307.9	m radius of final c.o.m.
	vqudf	83.23369	m/s old u vel of new c.o.m.
	vqzdf	-4.073545	m/s old r vel of new c.o.m.
	vudf	4528.1007	m/s azimuthal vel. of final c.o.m.
	vzdf	-51.85739	m/s radial vel. of final c.o.m.
	vdf	4528.3976	m/s final c.o.m. velocity
	dudf	-.0001635	rad Δu between centers of mass
	phdf	-.0112883	rad final tether orbit, phi
	Esdf	-485941	J specific energy

L	PdF	.61033029	PdMd	period (in Md days)
	adf	44048032	m	semimajor axis
	hdf	1.805E10	m ² /s	specific angular momentum
	edf	.90951299		eccentricity
	pdf	7610888.5	m	semilatus rectum
L	rpdf	3972712.3	m	periapsis
	radf	84110290	m	final destination tether apoapsis
L	vpdf	4317.6276	m/s	periapsis velocity
	udf	-.0249268		u of final tether orbit at release
L	altpdf	170712.28	m	altitude
"Orientation"				
	omdi	8.0011678	rad	argument of periapsis at capture
L	omdiD	99.703482	deg	" in degrees, mod 2π
	omdc	1.689287	rad	argument of periapsis after capture
	omdf	7.9737563	rad	argument of periapsis from ES line
L	omdfD	97.46196	deg	" in degrees, mod 2π
L	Total	80.227152	day	Total time between Mo tether capture and Md tether release
"Diagnostics"				
	DEsdp	-9.695E10	J	Change in payload system energy
	DEsd	-2.29E10	J	Change in tether system energy
	Drpd	24441.336		Change in tether periapsis radius
	Dv	156.59946		Change in tether periapsis velocity
	Flag1	0		Flag1 = 1 if release alt < 150 km
	Flag2	5		Flag2 > 1 if payload deorbit

DESIGN OF A TETHER BOOST FACILITY FOR THE HUMAN MARS MISSION

Robert P. Hoyt
Tethers Unlimited, Inc.
1917 NE 143rd St., Seattle WA 98125-3236
www.tethers.com

Abstract

We have developed a preliminary architecture for a tether boost facility designed to handle cargo payloads for the Human Mars Mission. This facility will impart a total ΔV of 2.5 km/s to the payloads, boosting them from LEO holding orbits to high-energy elliptical orbits in preparation for Trans-Mars-Injection rocket burns. Our analyses indicate that the total system mass required, using currently available tether materials and reasonable safety factors, would be approximately 4.6 times the payload mass, or 391 mt of facility mass for a 85 mt payload. Economically, this system would compare very favorably to a SEP boost stage if it is used for repeated missions. The system would provide rapid transfer times, comparable to chemical rocket transfer times, yet require no propellant resupply. The system could also provide direct Mars transfer insertion for 15 mt payloads, and handle significant traffic to GEO and the Moon.

Introduction

NASA is currently developing preliminary designs for the first Human Mars Mission, targeted for flights during the 2011 and 2013/2014 Mars transfer opportunities.¹ For mankind to be able to afford a sustained human presence on Mars beyond this first visit, the cost of frequent transportation to and from Mars must be reduced by an order of magnitude. A significant portion of the cost reduction must come from minimization of expendables and the amount of propellant that must be launched into Earth orbit.

Tether systems can provide the fully-reusable propellantless in-space propulsion capability needed to achieve the cost reductions for frequent travel to Mars. In this paper, we will develop and analyze a design for a tether system capable of providing 2.5 km/s of the 3.8 km/s total ΔV needed to inject payloads in LEO into a 178 day Mars transfer. We will then compare this system to solar-electric propulsion (SEP) upper stages currently being considered for this part of the mission.

The Mission:

In order to facilitate an apples-to-apples comparison, we will design the tether system to accomplish the same mission as the SEP stage in the baseline Human Mars Mission design. The SEP stage would boost cargo payloads massing approximately 85 mt from low-LEO orbits to an 800 x 67,000 km High Elliptical Orbit (HEO). From this orbit, the cargo vehicles would perform a ~1.3 km/s Trans-Mars Injection (TMI) maneuver.

Elliptical-Orbit HEFT System

For this system, we will use the High-Strength Electrodynamic Force Tether (HEFT) Facility concept, which combines rotating momentum-exchange tether principles with electrodynamic tether propulsion to provide a means for repeatedly boosting payloads from LEO to higher orbits or interplanetary trajectories without requiring propellant expenditure.² The HEFT facility would consist of a central station with a power supply, a long, tapered, high-strength tether, and a grapple vehicle at the end of the tether. The tether would have a conducting core so that current can be driven along the tether by the station's power supply. The HEFT facility would be placed in an elliptical orbit with a perigee in LEO, and its rotation would be chosen so that the grapple vehicle at the tether tip could rendezvous with payloads in low-LEO orbits when the tether is at the bottom of its rotation. After picking up a payload, the tether facility would carry the payload for one orbit and, on its return to perigee, release the payload at the top of its rotation,

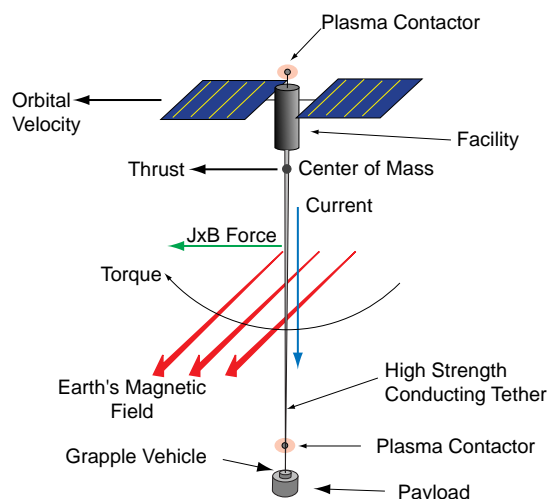


Figure 1. Schematic of the HEFT Facility concept.

injecting the payload into a high elliptical orbit. In this design, the total ΔV the HEFT facility imparts to the payload is 2.5 km/s. In boosting the payload's orbit, the facility will have imparted some of its orbital momentum and energy to the payload, reducing its own apogee. The HEFT facility will then use its power supply to drive current through the tether when it is near perigee, reboosting its apogee. This will enable it to restore its orbital momentum and energy so that it can boost additional payloads. This combination of momentum-exchange and electrodynamic tether propulsion enables the HEFT system to reliably boost payloads out of LEO without requiring propellant expenditure.

In order to determine the feasibility and required mass of this system, we must determine the tether length, rotation rate, and orbit characteristics that will permit the tether to rendezvous with the payload and throw it into the desired high energy orbit.

In this analysis, the payload of mass M_p begins in a circular orbit with radius r_{IPO} . The payload orbits with a velocity of

$$V_{p,0} = \sqrt{\frac{\mu_e}{r_{IPO}}}. \quad (1)$$

The facility is placed into an elliptical orbit with a perigee above the payload's orbit, with the difference between the facility's initial perigee and the payload orbital radius equal to the distance from the tether tip to the center of mass of the facility and tether:

$$r_{p,0} = r_{IPO} + (L - l_{cm,unloaded}), \quad (2)$$

where L is the tether's total length, and $l_{cm,unloaded}$ is the distance from the facility to the center of mass of the system before the payload arrives (this distance must be calculated numerically for a tapered tether).

The initial tether tip velocity is equal to the difference between the payload velocity and the perigee velocity of the tether facility's center-of-mass:

$$V_{t,0} = V_{p,0} + V_{IPO}. \quad (3)$$

In order to ensure that a payload will not be "lost" if it is not caught by the tether on its first opportunity, we choose the semimajor axis of the facility's orbit such that its orbital period will be some rational multiple N of the payload's orbital period:

$$P_{f,0} = NP_{IPO} \Rightarrow a_{f,0} = N^{2/3} r_{IPO} \quad (4)$$

For example, if $N=8/3$, this condition means that every three orbits the facility will have an opportunity to rendezvous with the payload, because in the time the facility completes three orbits, the payload will have completed exactly eight orbits.

An additional consideration in the design of the system are the masses M_f and M_t of the facility and tether, respectively. A significant facility mass is required to provide "ballast mass." This ballast mass serves as a "battery" for storing the orbital momentum and energy that the tether transfers to and from payloads. If all catch and throw operations are performed at perigee, the momentum exchange results primarily in a drop in the facility's apogee. A certain minimum facility mass is necessary to keep the post catch and throw apogees of the facility orbit above the Earth's upper atmosphere. Most of this "ballast mass" will be provided by the mass of the tether deployer and winch, the facility power supply and power processing hardware, and the mass of the tether itself. If additional mass is required, it could be provided by waste material in LEO, such as spent upper stage rockets and shuttle external tanks.

The tether mass will depend upon the maximum tip velocity and the choices of tether material and design safety factor, as described in Reference 3. For a tapered tether, the tether's center-of-mass will be closer to the facility end of the tether. This can be an important factor when the tether mass is significant compared to the payload and facility masses. In the calculations below, we have used a model of a tether tapered in a stepwise manner to calculate tether masses and the tether center-of-mass numerically.

By conservation of momentum, the perigee velocity of the center of mass of the tether and payload after rendezvous is:

$$V_{p,1} = \frac{V_{p,0}(M_f + M_t) + V_{IPO}M_p}{(M_f + M_t) + M_p}. \quad (5)$$

When the tether catches the payload, the center-of-mass of the tether system shifts downward slightly as the payload mass is added at the bottom of the tether:

$$r_{p,1} = \frac{r_{p,0}(M_f + M_t) + V_{IPO}M_p}{(M_f + M_t) + M_p} \quad (6)$$

In addition, when the tether catches the payload, the angular velocity of the tether does not change, but because the center-of-mass shifts closer to the tip of the tether when the tether catches the payload, the tether tip velocity decreases. The new tether tip velocity can be calculated as

$$V_t' = V_t \frac{(L - l_{cm,loaded})}{(L - l_{cm,unloaded})} \quad (7)$$

At this point, it is possible to specify the initial payload orbit r_{IPO} , the payload/facility mass ratio χ , the facility/payload period ratio N , and the desired final orbit, and derive a system of equations from which one particular tether length and one tether tip velocity can be calculated that determine an “exact” system where the tether tip velocity need not be adjusted to provide the desired C_3 of the payload lunar trajectory. However, the resulting system design is rather restrictive, working optimally for only one particular value of the facility and tether masses, and results in rather short tether lengths that will require very high tip acceleration levels. Fortunately, we can provide an additional flexibility to the system design by allowing the tether facility to adjust the tip velocity slightly by reeling the tether in or out a few percent. If, after catching the payload, the facility reels the tether out by an amount ΔL , the tip velocity will increase due to conservation of angular momentum:

$$V_t'' = \frac{V_t'(L - l_{cm,loaded})}{(L - l_{cm,loaded}) + \Delta L} \quad (8)$$

When the facility returns to perigee, it can throw the payload into higher energy orbit with perigee characteristics:

$$r_{p,LTO} = r_{p,1} + (L - l_{cm,loaded}) \quad V_{p,LTO} = V_{p,1} + V_t'' \quad (9)$$

System Design:

Using the equations above, standard Keplerian orbital equations, and equations describing the shift in the system's center-of-mass as the payload is caught and released, we have calculated a design for a “MarsHEFT” system capable of transferring picking up payloads from a circular LEO orbit and throwing them to a 800 x 67,000 km pre-TMI orbit. The payload and tether orbits are shown to scale in Figure 2.

Payload:

- mass $M_p = 85$ mt
- altitude $h_{IPO} = 545$ km
- velocity $V_{IPO} = 7.59$ km/s

Tether Facility:

- tether length $L = 210$ km
- tether mass $M_t = 221$ mt $(2.6 \times M_p)$ (Spectra 2000 fiber, safety factor of 3)
- station mass $M_f = 170$ mt $\approx 2 \times$ payload mass
- **total system mass** $M = 391$ mt $\approx 4.6 \times$ payload mass
- initial tether tip velocity: $V_{t,0} = 1507$ m/s
- High Energy [Pre-Catch] Orbit:
 - perigee altitude $h_{p,0} = 699$ km,
 - apogee altitude $h_{a,0} = 13170$ km
 - eccentricity $e_0 = 0.468$
 - period $P_0 = 8/3 P_{IPO}$ (rendezvous opportunity every 12.7 hrs)

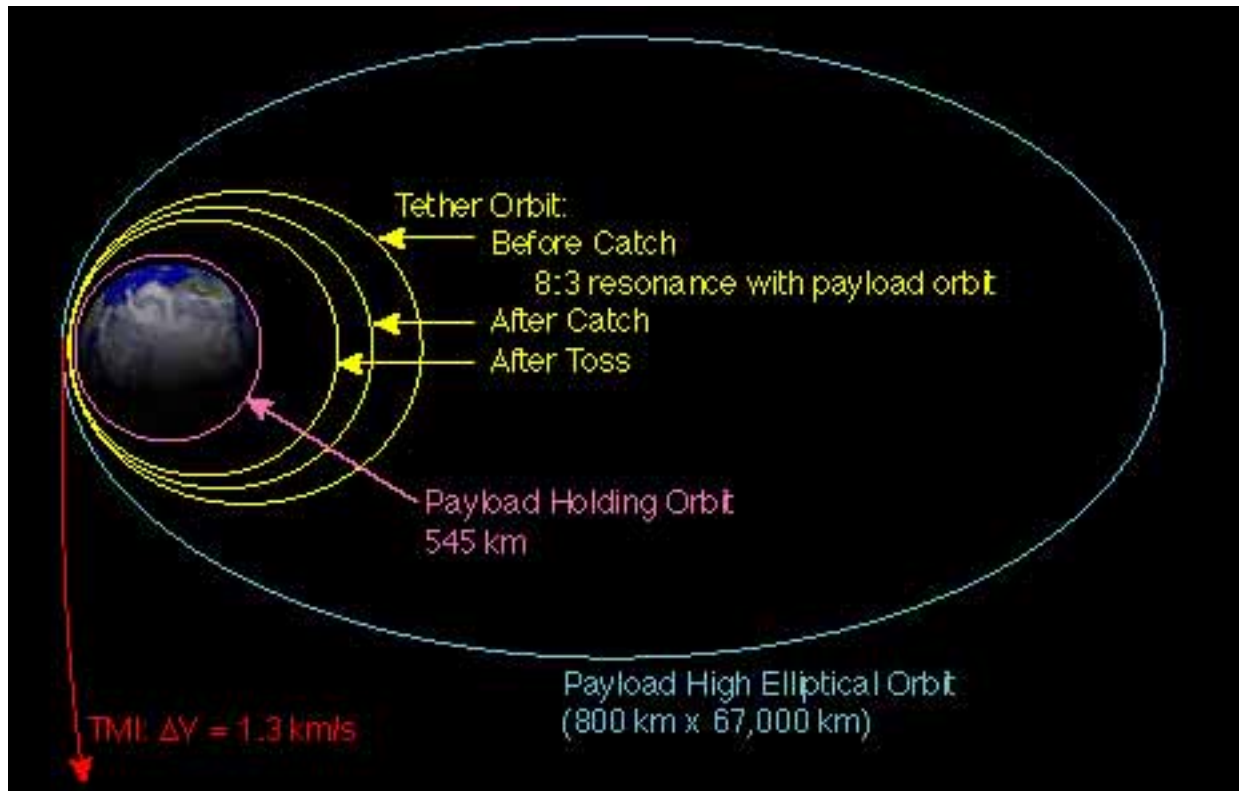


Figure 2. Orbital architecture of the MarsHEFT system.

- rendezvous acceleration $g_{tip} = 1.5 \text{ gees}$
- post-catch orbit (COM):
 - perigee altitude $h_{p,0} = 671 \text{ km}$,
 - apogee altitude $h_{a,0} = 9219 \text{ km}$
 - eccentricity $e_0 = 0.377$
- After catching payload, facility unreels 0.9 km of tether to absorb capture shock and adjust tip velocity
- Adjusted Post-Catch tip velocity: $V_t'' = 1236 \text{ m/s}$
- Post Catch tip acceleration: $g'_{tip} = 1.2 \text{ gees}$
- Low Energy [post-throw] orbit:
 - perigee altitude $h_{p,0} = 643 \text{ km}$,
 - apogee altitude $h_{a,0} = 6375 \text{ km}$
 - eccentricity $e_0 = 0.29$

High-Energy Payload Orbit:

- perigee altitude $h_{p,lto} = 800 \text{ km}$
- apogee altitude $h_{a,lto} = 67,000 \text{ km}$
- perigee velocity $V_{p,lto} = 10.058 \text{ km/s}$
- orbit energy parameter $C_3 = -9.9 \text{ km}^2/\text{s}^2$

HEFT System Reboost

After boosting the payload, the HEFT facility will be left in a lower energy elliptical orbit with a semimajor axis that is approximately 3400 km less than its original orbit. It can then use electrodynamic propulsion to reboost its apogee by driving current through the tether when the tether is near perigee. Because the tether is rotating, the direction of the current must be alternated as the tether rotates to produce a net thrust on the facility. Modeling of reboost of HEFT tether systems indicate that the system could reboost its semimajor axis at a rate of 50 km·mt / day·kW. Thus if the 391 mt facility has a 100 kWe power supply, it can reboost its orbit within about 270 days. If, instead, it has the 800 kWe power supply baselined for the Mars mission SEP stage, it could reboost its orbit in about 1 month.

Comparison to SEP Stage

In the SEP/Human Mars Mission scenario, a SEP stage massing 22 mt would use roughly 48 mt of Xenon fuel to boost the 85 mt payload into the 800 x 67,000 km HEO, with a transfer time on the order of one year.⁴ Typically, a SEP stage would have a lifetime of two mission, limited by thruster and solar panel degradation. Thus, for this comparison, we will take the required on-orbit mass of a SEP stage to be $(2 \times 48 + 22)/2 = 59$ mt. If Earth-to-Orbit launch costs are the primary cost driver, the 391 mt tether facility would gain a cost advantage within 7 boost missions. Additional factors, such as the limited world supply and high cost of Xenon, may reduce the number of missions needed for break-even. Use of the tether facility for other missions, as described below, would further improve its economic competitiveness.

Because much of the tether facility mass is simply ballast mass, used as a “battery” to store orbital energy and momentum, the system can utilize spent upper stages, shuttle external tanks, and other on-orbit mass to provide this ballast. Thus it may be possible to significantly reduce the total launch costs for deploying the HEFT tether system.

An additional advantage of the HEFT system is that it provides transfer times comparable to high-thrust chemical rocket systems, without requiring propellant expenditure. This can help to significantly reduce degradation of the Mars cargo systems due to the extended radiation exposure they would experience in a SEP slow-spiral boost scenario, and could reduce the radiation health risks to astronauts when it is eventually used for transporting personnel.

System Use for Direct Mars Injection, GTO Injection, and Lunar Transport

In addition to boosting large Mars-bound payloads into high elliptical orbits in preparation for TMI burns, this HEFT system could also perform numerous other important propulsion missions. With a tether sized to provide the 2.5 km/s ΔV to Mars-bound payloads massing 85 tons, could, by boosting its orbit and increasing its rotation rate, be used to inject 15 mt payloads directly into rapid Mars transfer trajectories. It could also boost 100 mt payloads from 300 km circular holding orbits into GTO trajectories, providing a reusable system for deploying ambitious space solar power stations and other GEO satellites. It could also be used to throw 40 mt payloads into minimum-energy lunar transfer trajectories. Thus, such a system could defray its development and launch costs by handling multiple propulsion missions. Furthermore, these other missions would provide opportunities to validate the HEFT system before it is used for a high-value Mars mission.

Summary

We have developed a preliminary architecture for a HEFT tether facility designed for the Human Mars Mission. This facility would impart a total ΔV of 2.5 km/s to the payloads, boosting them from LEO holding orbits to high-energy elliptical orbits in preparation for TMI rocket burns. Our analyses indicate that the total system mass required, using currently available tether materials and reasonable safety factors, would be approximately 4.6 times the payload mass, or 391 mt of facility mass for a 85 mt payload. Economically, this system would compare very favorably to a SEP boost stage if it is used for repeated missions. The system would provide rapid transfer times, comparable to chemical rocket transfer times, yet require no propellant resupply. The system could also provide direct Mars transfer insertion for 15 mt payloads, and handle significant traffic to GEO and the Moon.

References

1. Kos, L., “Human Mars Mission: Transportation Assessment”, AIAA Paper 98-5118.
2. Forward, R.L., Hoyt, R.P., *Failure Resistant Multiline Tether*, Robert L. Forward and Robert P. Hoyt, Patent Application PCT/US97/05840, filed 22 April 1997.
3. Hoyt, R.P., Uphoff, C.W., “Cislunar Tether Transport System”, AIAA Paper 99-2690.
4. Kos, L., personal communication, email dated 4/15/99.

THE HOYTETHER™: A FAILSAFE MULTILINE SPACE TETHER STRUCTURE

Robert P. Hoyt, Robert L. Forward
Tethers Unlimited

8114 Pebble Court, Clinton, WA 98236 USA

Phone/Fax: (206) 306-0400; Email: TU@tethers.com; Web: www.tethers.com

Abstract

The Hoytether is a failsafe, multiline space tether structure for long-duration and high-value tether missions. The Hoytether structure is an open net which provides redundant linkage in such a way as to maintain spatial separation between the individual lines composing the structure as the structure is degraded by orbital debris and meteoroid impactors. The spatial separation between lines prevents one small impactor from severing the entire Hoytether as can happen with a single-line tether. The Hoytether structure thus can suffer many cuts by small impactors while maintaining the design load. Analytical modeling of tether lifetimes in the space debris environment indicate that the redundancy of the Hoytether enables it to provide >99% survival probabilities for periods of months to years. Using numerical modeling, we have developed designs for Hoytether structures capable of operating at very high stress levels while maintaining high reliability for long lifetimes.

Introduction

The successes of the two Small Expendable-tether Deployment System (SEDS) and the Plasma Motor Generator (PMG) experiment have demonstrated the feasibility and reliability of small, inexpensive self-deploying tether systems.¹ Tethers are now being considered for a variety of applications such as studies of the upper atmosphere,² facilities for orbital transfer of payloads,^{3,4} electrodynamic power and propulsion systems for the International Space Station and other satellites,⁵ synthetic aperture radar systems, and rapid de-orbiting of post-operational LEO satellites.⁶ For tethers to be viable candidates for many of these applications, they must be designed to survive the flux of orbital debris and meteoroids for periods of many years and/or operate with high safety factors.

The need for a space tether structure with redundant linkage was illustrated by the results of the SEDS-2 mission flown in 1994¹ and the recent TSS-1R mission.⁷ The second SEDS mission used a tether consisting of a single cylindrical braided line with a diameter of 0.8 mm and a length of 20 km. This tether was cut by a debris or meteoroid impactor roughly 4 days after deployment. In the TSS-2 mission, the conducting-core tether was severed by a high-voltage arc caused by a defect in the electrical insulation, resulting in loss of the tether and the Italian satellite. Clearly, for a tethered system to complete a many-year mission, or for a crewed tether experiment to operate with an acceptable safety factor, a tether structure with built-in redundancy is required.

A tether structure capable of achieving the multi-year lifetimes and high safety factors required for many applications is illustrated in Figure 1.⁸ This design was invented in 1991 by Robert Hoyt and subsequently named the "Hoytether" by Dr. Robert L. Forward. The "Hoytether" is a tri-axial net consisting of a number of primary load-bearing lines running the length of the structure. These "primary" lines are periodically interconnected by diagonal secondary lines. A section of a tubular Hoytether is illustrated in Figure 1a. Where the secondary lines intersect the primary lines they are firmly connected so that one line does not slip relative to the other. The secondary lines are only put

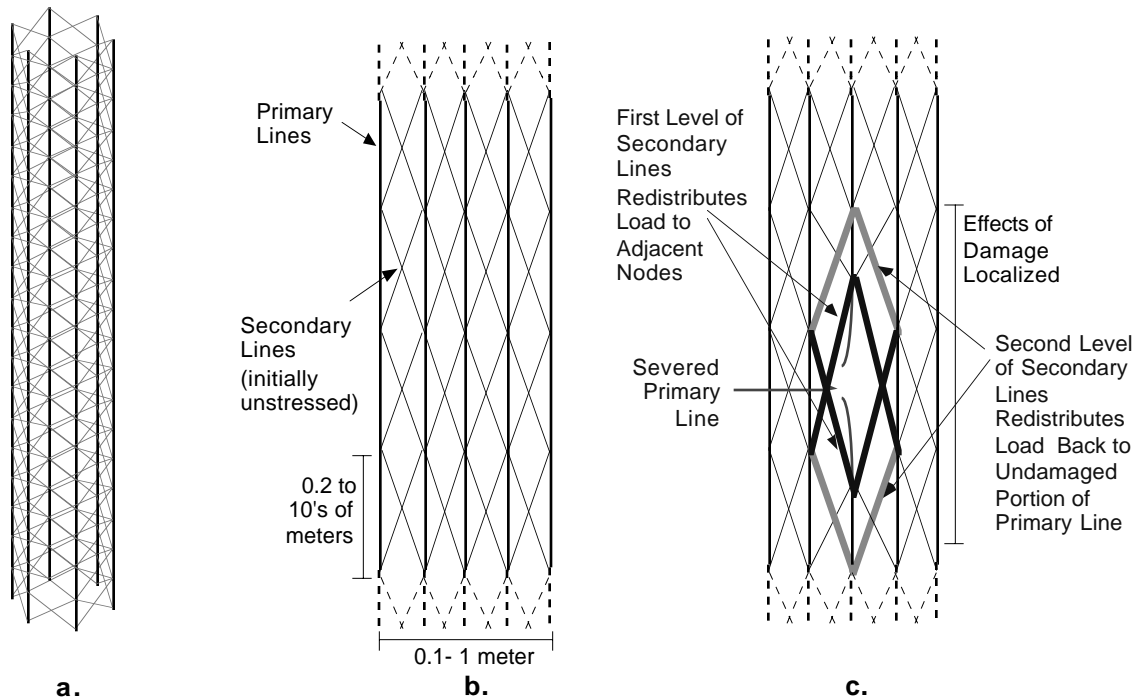


Figure 1. a) Section of a tubular Hoytether ("Hoytube"). b) Schematic of undisturbed tape Hoytether ("Hoytape"). c) Secondary lines redistribute load around a failed primary line without collapsing structure. *Note:* the horizontal scale is expanded relative to the vertical scale; in reality the secondary lines are nearly parallel to the primary lines.

under load if a section of primary line is cut by space debris. At either end of the structure, a support ring enforces the cylindrical spacing between the primary lines. Because the secondary lines are not initially loaded, they do not cause the structure to neck down, and thus no solid spacers are required along the length of the tether. In this cylindrical configuration, the Hoytether structure is also called a "Hoytube."

The principle of the Hoytether is illustrated in Figures 1b and 1c. The secondary lines are almost parallel to the primary lines and thus are ready to pick up the load if a primary line fails. When a section of primary line is cut by debris, the secondary lines assume the load and redistribute the stresses in such a way that the effects of the damage are localized to a region near the failure. Because the secondary lines are nearly parallel to the primary lines, and because they are initially slack, the structure necks down only slightly. Thus such a tether can suffer many cuts without catastrophic failure. Moreover, the structure degrades gracefully, maintaining separation between the individual lines to minimize the chances that a single object could cut more than one line.

In experimental and numerical investigations during early 1992 under SBIR contract NAS8-39318,⁹ this design was found to:

1. Withstand multiple cuts of individual line segments while retaining structural integrity and degrading gracefully.
2. Have lifetimes several orders of magnitude longer than comparable-mass single-line tethers.

3. Redistribute loads around a cut primary line in such a way that the distortion due to a line segment failure was localized to within a few sections on either side of the cut. This localization of the effects of a cut keeps the structure from “pinching” severely, even after many cuts.
4. Maintain separation between the strands without the need for solid bracing structures.

The availability of a multistrand tether system with such failsafe attributes will enable NASA and other organizations to pursue the many advanced propulsion applications of tethers, particularly the many missions in which long-life and safety are important considerations.

Tether Survival Probabilities

A tether deployed in space will be subjected to impacts by both meteorites and man-made orbital debris. For a conventional single-line tether, one strike by an impactor with sufficient energy will cut the tether and cause failure of the mission. A Hoytether, however, has many redundant links, and thus can suffer many cuts to individual lines while continuing to support its design load.

Survival Probability of a Single Line

Currently, experimental data on the rate of failure of a tether line is limited to the results of the SEDS-2 experiment and the ongoing TiPS experiment.^{1,10} Consequently, the most appropriate method of estimating the lifetimes of space tether lines currently available is to utilize models of the flux of debris and meteoroid particles such as that given by Kessler.^{11,12} This data is typically given as the cumulative flux of particles larger than a specified diameter. This particle flux, $F(d_{particle})$, is converted to a flux of lethal impactors by assuming that a tether line will be cut by particles with diameters equal to or greater than a specified fraction k_L of the tether diameter, $f(d_{line}) = F(k_L d_{line})$. This fraction is called the “lethality coefficient.” For the analyses in this work, a lethality coefficient of 0.3 is assumed based upon the results of the SEDS-2 experiment.¹³ This value of k_L is in the middle of the range of values (0.2-0.5) that are commonly used.¹⁴

For a single line tether of diameter d and length L , the probability of survival of the tether for a duration T is obtained by first multiplying the flux of lethal impactors by the surface area of the tether line to obtain a rate of cuts c ,

$$c = \pi d L F(k_L d), \quad (1)$$

and multiplying this rate by the lifetime to obtain the expected number of cuts in the time T ,

$$N = c T. \quad (2)$$

The survival probability is then obtained using Poisson statistics to determine the probability that the line suffers no cuts during the period T ,

$$P(T) = P_N(0) = \frac{N^0}{0!} e^{-N} = e^{-cT}. \quad (3)$$

The $1/e$ lifetime of a single line tether is thus $\tau_1 = 1/c$.

Survival Probability of Hoytethers

Hoytether Parameters and Cut Ratesⁱ

In a generic Hoytether, there are n primary lines and m secondary lines. The lines are divided up into h segments or tether “levels,” determined by the interconnection points of the primary lines with the secondary lines. For a Hoytether of length L , the length of the individual primary line segments is $l_p = L/h$, and there are a total of nh of these segments.

The primary lines are separated by a distance a , and are connected by diagonal secondary lines which are deliberately made slightly longer than the distance between interconnection points by a “slack coefficient,” k_s , which is typically around 1.005.

The survival probability of the individual primary and secondary line segments is found in a manner equivalent to the estimation of single line tether survival in Eqns. (1)-(3):

$$P_p(T) = P_{N_p}(0) = e^{-N_p} \quad (4)$$

$$P_s(T) = P_{N_s}(0) = e^{-N_s} \quad (5)$$

The probability of at least one cut on a given line segment is given by

$$P_N(>0) = 1 - P_N(0) = 1 - e^{-N}. \quad (6)$$

Because the Hoytether design provides redundant paths for bearing the tether load, the loss of a single primary or secondary line segment does not lead to failure of the tether as a whole. In a low load case, where any one of the primary or secondary lines can carry the full load, then *all* of the primary and *all* of the secondary lines would have to be cut *at the same level* before the Hoytether as a whole will be severed. When the tether is under more substantial loading, the tether will survive until the number of uncut primary lines plus the number of uncut secondary lines is insufficient to bear the tether load. Predicting the survival probabilities in this case can be done using Monte Carlo simulation of tether structures subjected to random fluxes of impactors. Alternatively, this problem can be approached analytically by calculating the survival probabilities of the primary lines and the secondary lines separately and then combining them.

The number of primary line segments that must be cut before failure, x , is approximately proportional to the ratio of the applied load W_a compared to the maximum load capacity W_p of all the n primary lines in the uncut Hoytether. Similarly, the number of secondary line segments that must be cut before failure, y , is approximately proportional to the ratio of the applied load W_a compared to the maximum load capacity W_s of all the m secondary lines in the uncut Hoytether:

$$x \geq (1 - \frac{W_a}{W_p}) n \quad y \geq (1 - \frac{W_a}{W_s}) m \quad (7)$$

Survival Probability of Hoytether Structure

If N_p and N_s are small, then the probability of survival of any one level is very high. However, there are many levels, and failure of any one of them causes failure of the whole tether. The

ⁱ For a detailed derivation of the survival probability analysis, please refer to AIAA paper 95-2890, “Failsafe Multiline Hoytether Lifetimes,” R.L. Forward and R.P. Hoyt, or to Appendix E, “Small Impactor Survival Probabilities of Hoytethers,” in *Failsafe Multistrand Tether SEDS Technology Demonstration*, Tethers Unlimited Final Report on NASA contract NAS8-40545, June 1995.

probability of survival of the entire Hoytether is thus the product of the survival of all the h primary line levels in the tether:

$$S_T = S_L^h = [1 - (1 - e^{-Np})^x (1 - e^{-Ns})^y]^h$$

$$S_T(t) = [1 - (1 - e^{-c_p t})^x (1 - e^{-c_s t})^y]^h. \quad (8)$$

Tether Lifetime

When the Hoytether structure is fabricated with many levels (large h), analysis of Eqn. (8) results in an effective tether “lifetime” of

$$\tau = \frac{1}{c_p \left(\frac{x}{x+y}\right) c_s \left(\frac{y}{x+y}\right) h \left(\frac{1}{x+y}\right)} \quad (9)$$

Although the lifetime given by Eqn. (9) is a “1/e lifetime,” in that the probability of survival of the Hoytether at time $t = \tau$ is $S_T = 1/e = 0.368$, the probability of survival with mission duration does *not* have the standard “1/e curve” decay with time. Although the individual line segments will have lifetimes described by the traditional exponential decay, the probability of failure of *all* of the line segments on a particular level is the *product* of those lifetimes. This “sharpens” the drop time, so that the Hoytether maintains a high probability of survival for periods shorter than the tether lifetime, and has a low probability of surviving after that lifetime is exceeded.

Lifetimes: Hoytethers vs. Single-Line Tethers

A single-line tether with diameter D_1 and length L_1 has a 1/e lifetime of $\tau_1 = 1/c_1 = 1/S_1 F_1$, where $S_1 = \pi D_1 L_1$ is the total surface area of the tether and F_1 is the flux of space impactors capable of severing the tether. The single-line tether’s probability of survival decays exponentially as described by Eqn. (3).

The effectiveness of the Hoytether design can be examined by replacing this single-line tether with a failsafe multiline tether having the same mass as the single-line tether. The Hoytether has n primary lines and $m=2n$ secondary lines. The secondary lines will have half the cross-sectional area of the primary lines, so the secondary lines will have the same total mass as the primary lines. Using these rates in Eqn. (12), the Hoytether lifetime is found to be proportional to the single-line tether lifetime $\tau_1 = 1/C_1$ by the factors:

$$t = \frac{1}{c_p \left(\frac{x}{x+y}\right) c_s \left(\frac{y}{x+y}\right) h \left(\frac{1}{x+y}\right)} \approx \frac{h \left(1 - \frac{1}{x+y}\right)}{2 \left(\frac{0.75y}{x+y}\right) (2n)^{0.75}}. \quad (10)$$

Inspection of this equation reveals that the lifetime of a Hoytether is greater than the lifetime of an equal mass single-line tether roughly by a factor of the number of interconnection levels h divided by the number of primary lines n .

The analytical relationship for the survival probability as a function of time, expressed in Eqn. (8), shows that the survival probability of the tether does not drop as a simple 1/e decay but rather

maintains a high survival probability until the Hoytether lifetime is reached. This means that the Hoytether can achieve very high (>99%) survival probabilities for long periods of time.

Figure 2 shows a comparison between the survival probabilities of a single-line tether and an equal-mass Hoytether designed for a low-load mission such as a gravity-gradient stabilized synthetic aperture radar satellite system.¹⁵ Both tethers are 10 km long and mass 25.5 kg. The Hoytether would be a tubular structure with 6 primary lines connected by secondary lines every 0.2 m. While the single-line tether has a survival probability that drops exponentially with time, the Hoytether can have a >99% survival probability for many decades. [Note: decade-long lifetimes will likely require system capability to avoid large (>1 m) objects, such as derelict satellites.]

High-Strength Survivable Tethers

Motivation

Tethers have the potential to significantly reduce the cost of in-space transportation by providing a means of transferring payloads from one orbit to another without the use of fuel.^{3,4} Systems composed of rotating tethers attached to orbiting facilities could be used to boost payloads from low Earth orbit or even suborbital trajectories to higher orbits by transferring orbital momentum and energy from the tether facility to the payload; the orbit of the facility could be restored by “recycling” orbital momentum from return traffic.

Such tether transport systems will require tethers capable of operating at very high stress levels. In addition, for a tether transport system to be economically advantageous, it must be capable of handling frequent traffic for a periods of at least several years. Consequently, a tether transport system will require the use of tethers designed to remain fully functional at high stress levels for many years despite degradation due to impacts by meteorites and space debris. An additional requirement for this system is that the tether mass be minimized to reduce the cost of fabricating and launching the

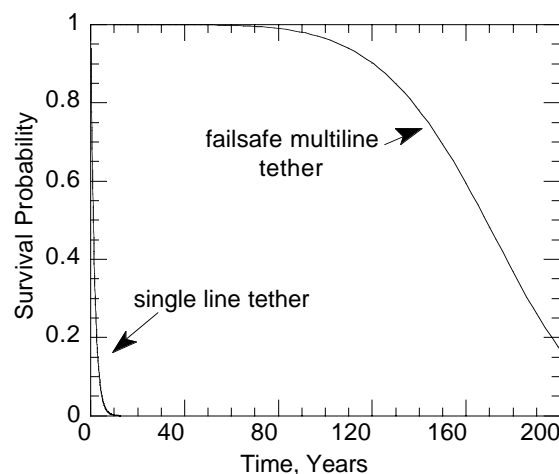


Figure 2. Small-impactor survival probabilities of equal-weight single-line and failsafe multiline tethers for a low-load mission.

tethers. These two requirements present conflicting demands upon the tether design that make conventional single-line tethers impractical for this application. For a single-line tether to achieve a high probability of survival for many years, it would have to be very thick and massive. Fortunately

the Hoytether design can balance the requirements of low weight and long life, enabling tether transport facilities to become feasible. The redundant linkage in the Hoytether enables the structure to redistribute loads around primary links that fail due to meteorite strikes or material failure. Consequently, the Hoytether structure can be loaded at high stress levels yet still achieve a high margin of safety for long periods of time.

Because minimizing the tether mass is critical to the viability of tether systems for in-space propulsion, we have sought to optimize the design of the Hoytether structure so as to maximize the strength-to-weight ratio of the tether while achieving high probability of survival for periods of years.

Minimizing the “Safety Factor” While Maintaining Reliability

When a tension member is developed, it is normally designed to operate at a load level somewhat lower than the maximum it could support without breaking. This derating provides margin of error in case of imperfections in the material or the construction. Typically, a tether is designed to carry a maximum load that is 50% of its breaking limit; this tether would have a “design safety factor” of $F = 1/50\% = 2.0$.

Because a high-speed rotating tether must support its own weight in addition to the weight of its payload, the required mass for a rotating tether increases exponentially as the design safety factor is increased.¹⁶ For rotating tether systems, therefore, it is necessary to operate at the minimum acceptable safety factor so as keep the required tether mass within economically feasible levels. For conventional single-line tethers, however, reducing the safety factor causes a corresponding increase in the likelihood of failure.

For the Hoytether, we define the safety factor as the ratio of the maximum load capacity of *both primary and secondary lines* to the design load. The safety factor thus provides the same measure of the strength-to-weight ratio of the Hoytether structure as it does for a single-line tether. However, this definition of the safety factor does not accurately represent the true margin of safety for the Hoytether. Because the Hoytether has redundant links that can reroute loads around parts of the tether that have failed, it is possible to load the Hoytether at a large fraction of the capacity of the primary lines (ie.- small “safety factor”) and still have a large margin of safety. Consequently, using the Hoytether structure allows us to design the tether with a low “safety factor” to minimize the tether mass and yet still have a very reliable structure. In this effort we have sought to optimize the Hoytether by finding a design that minimizes the safety factor and thus minimizes the required mass while still providing the ability to withstand many cuts due to meteorite strikes.

It should be noted that the manner in which we calculate the Hoytether safety factor below is not obvious. Typically, we refer to Hoytether designs by the level of stress on the *primary lines*. Thus, if each secondary can support 1/2 as much tension as a single primary line can support (i.e.- each secondary has half the cross-sectional area of a primary line), and if it is loaded at 50% of the capacity of the primary lines, it will be loaded at a design safety factor of

$$F = \frac{[(\# \text{ of primaries})(\text{primary line area}) + (\# \text{ of secondaries})(\text{secondary line area})]}{(\# \text{ of primaries})(\text{primary line area})} \frac{1}{(\text{primary stress level})} \quad (11)$$

$$F = [1 + 2(1/2)]/50\% = 4.$$

Method: Simulation with the SpaceNet Program

To study the optimization of the Hoytether structure for high-load applications, we performed a series of simulations of variations of the structure using the SpaceNet program.¹⁷ The SpaceNet program uses a combination of finite-element methods with a structural relaxation scheme to calculate the effects of damage to complex 3-D net structures such as the Hoytether.

Results

We began by studying multi-line Hoytethers with secondary lines having 1/4 the cross-sectional area of the primary lines; the secondary lines thus have a total mass of 1/2 of the mass of the primary lines (there are two secondary lines per primary line). In addition, the secondary line length was chosen so that they would be slack under design load. We found that if this tether is loaded at 90% of capacity of the primary lines, giving it a design safety factor of $F=1.67$, it can survive a cut to one of the primary lines. Moreover, the tether can survive an additional cut on the same level. However, if the second cut is on a primary line immediately adjacent to the first cut, the structure will fail. While the probability of two adjacent primary lines being cut by two separate meteoroid impacts is very small, it is possible that two lines could be cut by one impactor if it is large enough. Consequently, it is necessary to design the tether to withstand several localized cuts. Therefore, a larger safety factor is required.

The results of our subsequent analyses indicate that the design of an optimal Hoytether depends upon how much of its mission duration will be spent under high load. Consequently, there are two classes of Hoytether designs, one for tethers that are always under high load, and one for tethers that are heavily loaded for brief periods only.

Continuous-High Load Tether

If the tether will be under high load for most of its mission, then it should be designed with secondary lines slack at the expected load level. This will enable the tether lines to remain spread apart at all times, minimizing the chances of a single impactor cutting several lines. For this case, a near-optimal tether design would be a cylindrical Hoytether with a large number of primary lines (~20) stressed at 75% of their maximum load and with initially-slack secondary lines that each have a cross-sectional area 0.4 times that of a primary line. Splitting the tether up into a large number of primary lines is necessary. From Eqn. (11), such a tether will have a design safety factor of $F=2.4$. However, the redundant nature of the structure will make the Hoytether far more reliable than a single line tether with the same safety factor. Simulations with the SpaceNet program have shown that this tether design can withstand multiple cuts on a single level. In fact, even if all of the primary lines on one level are cut, the secondary lines will support the load.

Intermittent High-Load Tether

A tether on a transfer facility, however, would likely be loaded at high levels for only a few hours every month. Therefore, it is possible to reduce the tether weight by designing it to have slack secondaries at the load level experienced during its long “off-duty” periods, but to have the secondaries bear a significant portion of the load during a brief high-stress operation such as a payload catch-and-throw operation. During the high-stress period, the loading of the secondaries will cause the structure to collapse to a cylindrical tube. Once a payload is released and the stress is reduced, however, the tether lines will drift back apart. If this high-load period is brief, it will only slightly increase the chances of tether failure due to impact by a large object. Because the secondaries bear a significant fraction of the stress at high load levels, the tether can safely be loaded to higher levels. Simulations

indicate that a 20-primary line Hoytether with secondary lines having cross-sectional area $1/4$ of that of the primary line area can be loaded to more than 100% of the primary line capacity and still survive cuts to two adjacent primary line segments. A reliable design for this class of tether would be a cylindrical Hoytether with primary lines sized so that they will be loaded at 85% of their capacity during peak stress operations, and secondary lines with cross-sectional areas $1/4$ of the primary lines. The secondary line lengths would be chosen so that they would be slightly slack during off-duty periods. Eqn. (11) above gives the design safety factor of this tether as $F=1.75$.

Conclusions

Tether applications in which the tether must survive the space environment for long periods will require tether structures capable of surviving multiple impacts by debris and micrometeorites. The Hoytether open-net structure provides multiply redundant linkage, enabling the tether to withstand many cuts yet still provide reliable load-bearing capabilities. While the survival probability of a standard single-line tether decreases exponentially with time, the Hoytether structure has a survival probability that remains very high for a long period of time, dropping only when its "lifetime" is reached. We have investigated the design of Hoytethers for demanding applications such as tether transport systems and found that the redundant linkage of the structure enables the Hoytether to operate reliably at very high stress levels; this design thus can minimize the tether mass required for tether transport systems.

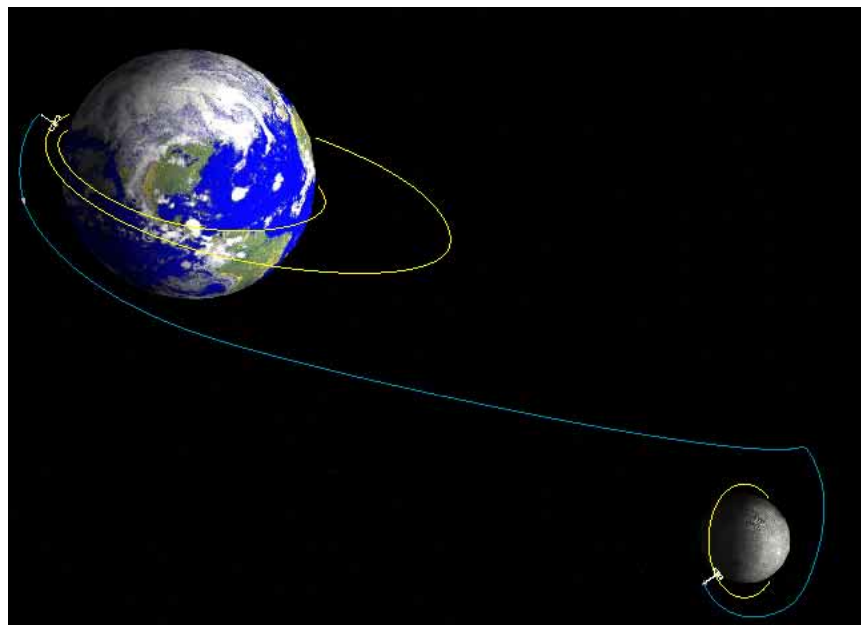
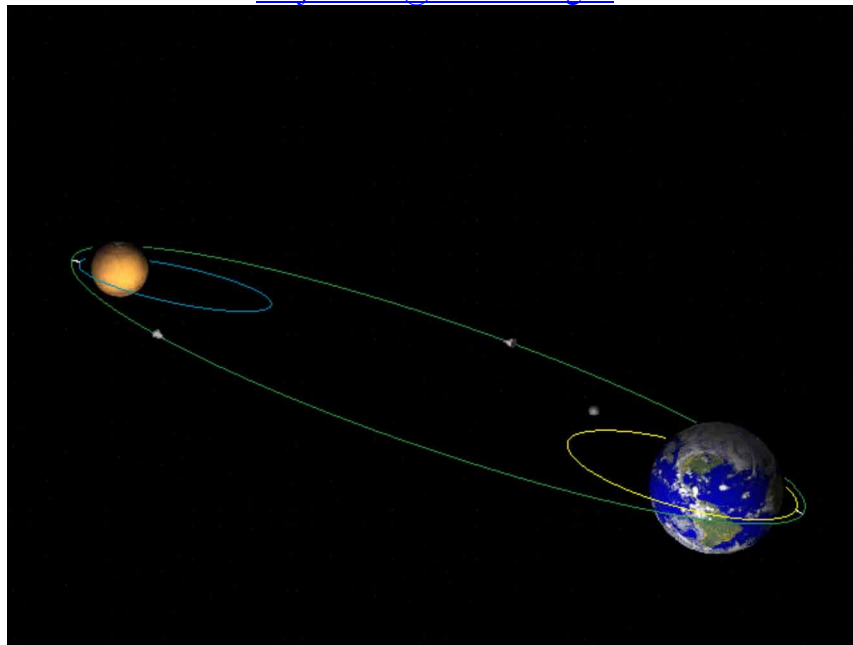
References

1. Carroll, J.A., "SEDS Deployer Design and Flight Performance," paper WSEDSA-1 at 4th International Conference on Tethers in Space, Washington DC, 1995.
2. Heelis, R.A., "Tethered Satellite Investigation of The Ionosphere and Lower Thermosphere", *Proceedings of the Tether Technology Interchange Meeting*, Huntsville, AL (9-10 September 1997).
3. Carroll, J.A., *Preliminary Design of a 1 km/sec Tether Transport Facility*, Tether Applications Final report on NASA contract NASW-4461, March 1991.
4. Hoyt, R.P., Forward, R.L., "Tether System for Exchanging Payloads between Low Earth Orbit and the Lunar Surface," AIAA Paper 97-2794, 33rd Joint Propulsion Conference, Seattle WA, July 1997.
5. Johnson, L., "Propulsive Small Expendable Deployer System Mission (ProSEDS)", *OAST Advanced Propulsion Workshop*, JPL, Pasadena, CA, 20-22 May 1997.
6. Forward, R.L., Hoyt, R.P., Uphoff, C., "The 'Terminator Tether'TM : A Near-Term Commercial Application of the NASA/MSFC ProSEDS Experiment", *Proceedings of the Tether Technology Interchange Meeting*, Huntsville, AL (9-10 September 1997).
7. NASA Press Release, Tuesday June 4, 1996, email from NASAnews@hq.nasa.gov 14:32:22
8. Forward, R.L., "Failsafe Multistrand Tether Structures for Space Propulsion," Final Report on NAS8-39318 with NASA/MSFC, AL 35812. See also AIAA preprint 92-3214, 28th Joint Propulsion Conference, Nashville, TN, 1992.
9. Forward, R.L., *Failsafe Multistrand Tethers for Space Propulsion*, Forward Unlimited Final Report on NASA SBIR Phase I Research Contract NAS8-39318, 1992.
10. Barnes, J., "TiPS: Results Of A Tethered Satellite Experiment", *Proceedings of the Tether Technology Interchange Meeting*, Huntsville, AL (9-10 September 1997).
11. B. Jeffrey Anderson, Editor and Robert E. Smith, Compiler, *Natural Orbital Environment Guidelines for Use in Aerospace Vehicle Development*, Section 7 "Meteoroids and Orbital Debris", Don Kessler and Herb Zook, NASA TM-4527, June 1994.

12. D.J. Kessler, et al., *A Computer Based Orbital Debris Environment Model for Spacecraft Design and Observations in Low Earth Orbit*, NASA TM (to be released). [Beta test copies of the program can be obtained by contacting Christine O'Neill at JSC at 713-483-5057 or oneill@snmail.jsc.nasa.gov]
13. Hoyt, R.P., Forward, R.L., "Failsafe Multi-strand Tether SEDS Technology," *Fourth International Conference on Tethers in Space*, Washington, DC, April 1995.
14. private communication from F. Jarrossy of Martin Marietta Corp.
15. R.L. Forward and R.P. Hoyt, "Failsafe Multiline Hoytether Lifetimes," AIAA paper 95-2890, presented at the *31st Joint Propulsion Conference*, July 1995.
16. Hoyt, R.P., "Tether System for Exchanging Payloads between the International Space Station and the Lunar Surface," *Proceedings of the Tether Technology Interchange Meeting*, Huntsville, AL (9-10 September 1997).
17. "Simulation of Complex Multiline Space Tether Structures: SpaceNet" Appendix N in *Failsafe Multistrand Hoytether SEDS Technology Demonstration*, Tethers Unlimited Final Report on NASA SBIR contract NAS8-40545, June 1995.

Momentum-Exchange Tether Propulsion Technology

Prepared for
Les Johnson
National Aeronautics and Space Administration
George C. Marshall Space Flight Center
Mail Code RA10
Huntsville, Alabama 35812
256-544-0614
les.johnson@msfc.nasa.gov



Momentum-Exchange Tether Propulsion Technology

INTRODUCTION

If the US space program is to develop a sustained and prosperous human presence on Mars, the Moon, and elsewhere in the solar system, the cost of transporting supplies and personnel to and from these destinations must be reduced by orders of magnitude. The first step in achieving this goal is the reduction of the cost of Earth-to-orbit launch. The Reusable Launch Vehicle (RLV) Program is addressing these cost reductions. However, for missions beyond low-Earth-orbit, the propellant and propulsion systems for orbit raising and interplanetary transfer are major cost drivers. Thus it will be necessary to reduce by orders of magnitude the cost of in-space transportation as well. These cost reductions must be achieved by greatly reducing the amount of propellant and other expendables required to provide transportation beyond LEO.

Momentum-Exchange Tether systems are capable of providing frequent round-trip travel between LEO and numerous important destinations, including GEO, the surface of the Moon, Mar orbit, and other planetary bodies, with little or no propellant expenditure required. Systems of several rotating tethers can create a fully-reusable “public transit system” in space that will provide both rapid transit times *and* minimal propellant usage.

Momentum-Exchange Tether propulsion has the potential to directly support Goal 10 by providing a fully-reusable system technology capable of reducing the cost of interorbital transfer by an order of magnitude in the near term.

ABSTRACT

Momentum-exchange tethers are rotating high-strength cables that can be used to throw payloads back and forth between LEO and GEO, the Moon, and Mars. A tether facility serves as a “battery” to store orbital momentum and energy, and transfers this momentum and energy to payloads by catching and releasing them with the rotating tether. By balancing the flow of mass to and from the destination, the total orbital energy and momentum of the system can be conserved, eliminating the need for large quantities of propellant for the transfer maneuvers. Combining the principles of rotating momentum-exchange tethers with propellantless electrodynamic tether propulsion can create facilities that can repeatedly boost payloads from LEO to higher orbits or interplanetary trajectories without requiring propellant or return traffic.

1.0 TECHNOLOGY DESCRIPTION

A momentum-exchange tether system will typically consist of a central facility, a long, tapered, high-strength tether, and a grapple vehicle at the end of the tether. The tether will be deployed from the facility, and the system will be induced to spin using tether reeling maneuvers or electrodynamic forces. The direction of tether spin is chosen so that the tether tip is moving behind the tether facility’s center-of-mass on its downswing, and moving ahead of it on its upswing, as illustrated in Figure 1. With proper choice of tether orbit and rotation, the tether tip can then rendezvous with a payload when the tether is at the bottom of its swing and later release the payload at the top of its swing, tossing the payload into a higher orbit. The orbital energy and momentum given to the payload comes out of the energy and momentum of the tether

facility. The tether's orbit can be restored by reboosting with propellantless electrodynamic tether propulsion or with high-Isp electric propulsion; alternatively, the tether's orbit can also be restored by using it to de-boost return traffic payloads.

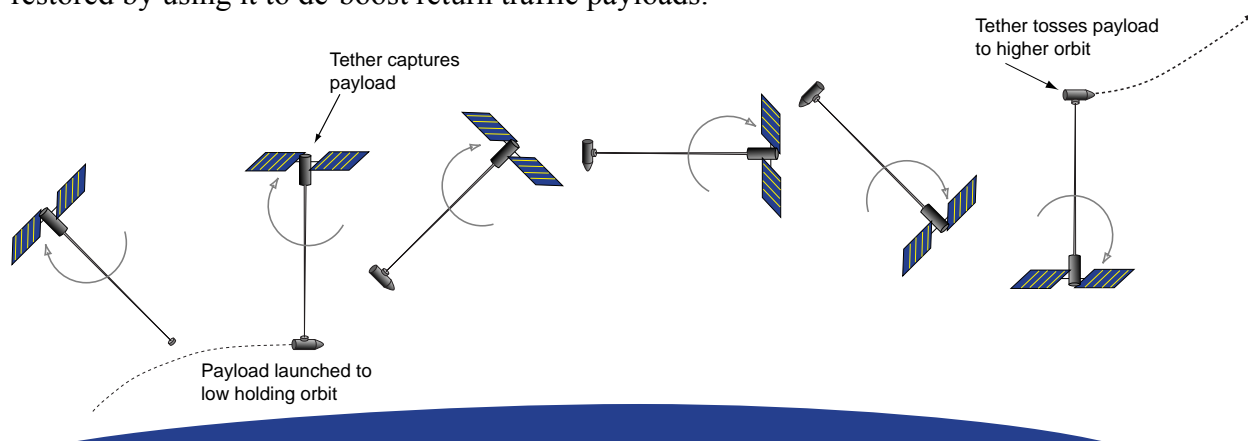


Figure 1. Illustration of a momentum-exchange tether boosting a payload.

2.0 POTENTIAL IN-SPACE APPLICATIONS

Momentum-exchange tethers can reduce costs for a wide range of in-space propulsion missions, including LEO, MEO, and GEO satellite constellation deployment, travel from LEO to the surface of the moon, and rapid interplanetary travel from Earth to Mars and back:

2.1 Geostationary Satellite Deployment

A recent study by Boeing, SAO, and NASA/MSFC concluded that a two-stage tether system for boosting communications satellites to geostationary orbit could significantly reduce the costs of launching payloads compared to the use of chemical upper stages. This study proposed the use of two rotating tethers in elliptical orbit around the Earth to transfer satellites from 300 km holding orbit to geostationary orbit, and assumed that the tether facilities would use high-Isp electric propulsion to reboost the tethers' orbits after each boost operation. This approach provides the high fuel-economy of electric propulsion but with the rapid transit times of chemical rockets. The study concluded that a system sized for 4000 kg satellites could be constructed with a total system mass of under 25,000 kg, and could reduce the costs of boosting the satellites to their operational orbits by more than 50%.

2.2 LEO/MEO Satellite Constellation Deployment

A momentum-exchange tether in elliptical LEO orbit may provide a cost-effective method to transfer constellation satellites from low-LEO holding orbits to circular high-LEO or MEO operational orbits. This concept is illustrated in Figure 2. The tether facility would initially be deployed in an elliptical orbit with a perigee just above the satellite's holding orbit, and an apogee just below the constellation's operational orbit. The initial rotation and length of the tether would be chosen such that at perigee the tether tip could rendezvous with the payload and capture it. Upon capture of the payload, the system will be in a new orbit with essentially the same perigee but a reduced apogee altitude. The system would then use electrodynamic tether propulsion to boost both the perigee and the apogee of its orbit, until the apogee is just below the constellation's orbit. The facility will then let the tether pay out to reduce the rotation rate slightly. At apogee, the tether can then release the satellite into the circular operational orbit. Upon releasing the satellite, the facility's orbit reverts back to its original values. It is then ready to boost another payload.

The advantage of such a system is that it provides a fully reusable capability for deploying satellites with short transit times and without propellant expenditure. Moreover, by combining momentum-exchange and electrodynamic tether techniques, propellantless electrodynamic propulsion can be made useful for missions beyond LEO altitudes.

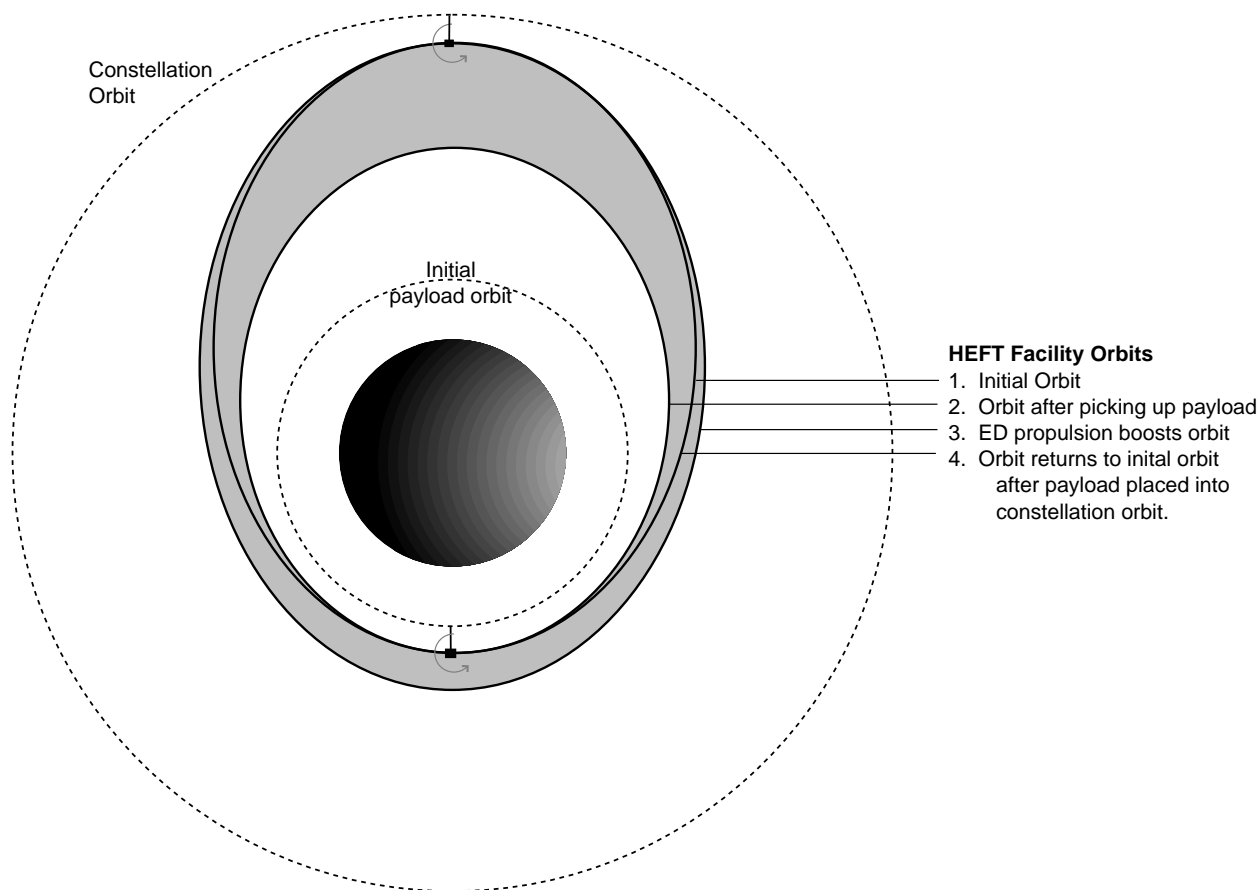


Figure 2. Overview of a momentum-exchange tether in elliptical LEO orbit designed to provide both boost and deorbit services to a constellation.

2.3 Cislunar Tether Transport System

To transport a payload from LEO to the surface of the moon and back requires a ΔV of greater than 10 km/s. Using storable chemical rockets, this ΔV would require a propellant mass of more than 16 times the payload mass; the cost of launching this propellant into orbit presents a prohibitive obstacle to significant commercial and scientific development of lunar resources. Using momentum-exchange tethers, however, it will be possible to create a system capable of exchanging payloads between LEO and the surface of the moon. A feasibility study funded by NASA's Institute for Advanced Concepts has developed a baseline design for a "Cislunar Tether Transport System," illustrated in Figure 3. Using currently available high-strength materials, this system will require a total mass on-orbit of less than 28 times the mass of the payloads it can handle; the system could thus "break-even" after just 2 round trips. By balancing the flow of mass to and from the Moon, this system could conserve its orbital energy and momentum, eliminating the need for transfer propellant for round trip travel. Because the system is fully reusable, and could have an operational lifetime measured in decades, it could potentially decrease the cost of frequent round-trip travel to the Moon by one to two orders of magnitude. A

simulation of a Cislunar Tether Transport System can be downloaded from <http://www.tethers.com/Cislunar.mov>.

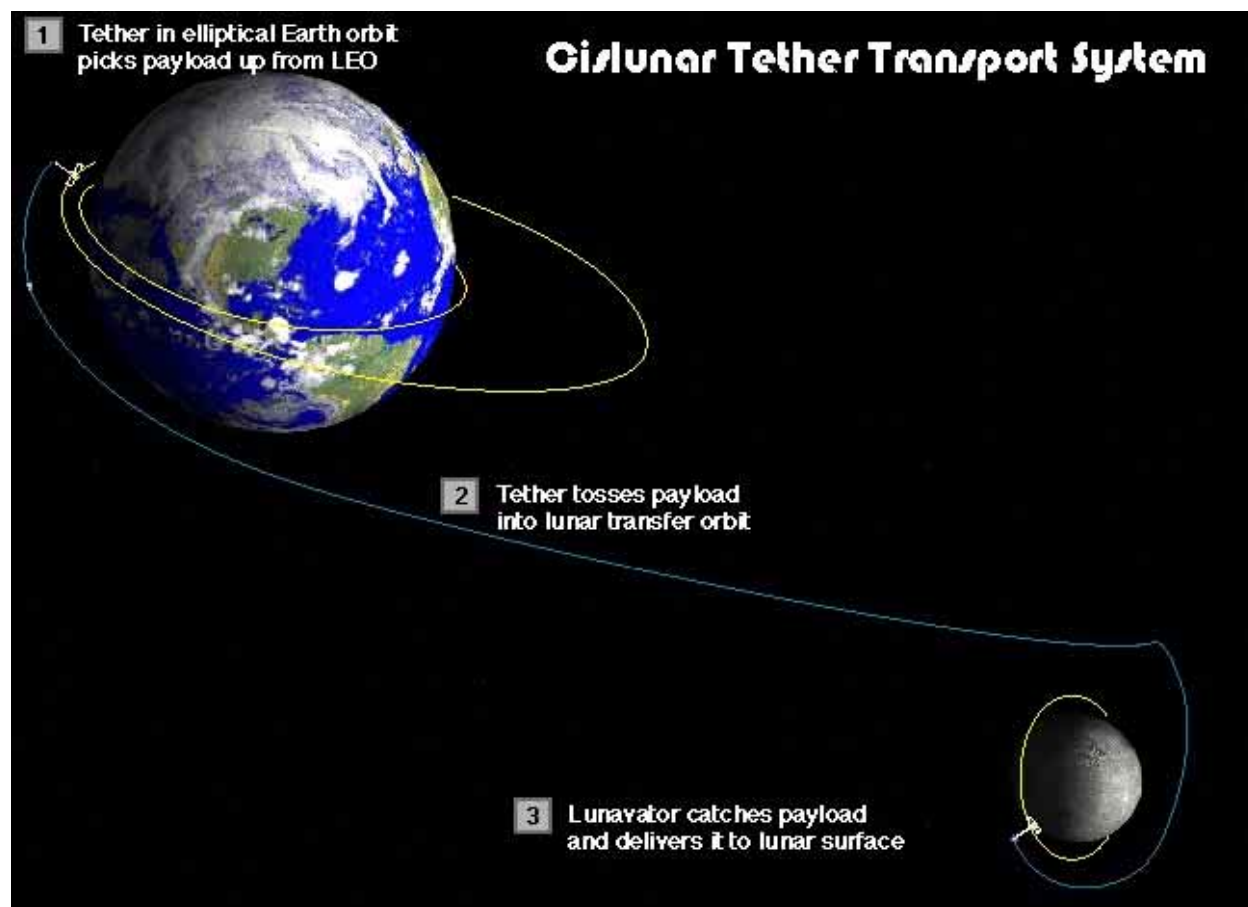


Figure 3. Overview of a Cislunar Tether Transport System that can exchange payloads between LEO and the surface of the Moon without needing transfer propellant by using momentum-exchange tethers.

2.4 Mars-Earth Rapid Interplanetary Tether Transport

Routine travel to and from Mars demands an efficient, rapid, low cost means of two-way transportation. The Mars-Earth Rapid Interplanetary Tether Transport (MERITT) system, illustrated in Figure 4, consists of two rotating momentum-exchange tethers in highly elliptical orbits; EarthWhip around Earth and MarsWhip around Mars. A payload capsule is launched out of the atmosphere of Earth into a suborbital trajectory. The payload is picked up by the EarthWhip tether as the tether nears perigee and is tossed a half-rotation later, slightly after perigee. The ΔV given the payload deep in the gravity well of Earth is sufficient to send the payload on a high-speed trajectory to Mars with no onboard propulsion needed except for midcourse guidance. At Mars, the incoming payload is caught by the MarsWhip tether in the vicinity of periapsis and the payload is released later at a velocity and altitude which will cause it to reenter the Martian atmosphere. The MERITT system works in both directions, is reusable, and the only major payload propellant requirement is that needed to raise the payload out of the planetary atmosphere and put it into the appropriate suborbital trajectory. Tethers with tip velocities of 2.5 km per second can send payloads to Mars in as little as 90 days if aerobraking is allowed to dissipate some of the high relative velocity on the Mars end. Tether-to-tether transfers without aerobraking may be accomplished in about 130 to 160 days. The mass of each tether system, using commercially available tether materials and reasonable safety factors,

including the mass of the two tether arms, grapple tips, and central facility, can be as little as 15 times the mass of the payload being handled. Unlike rocket propellant mass ratios, which can only launch one payload, the tether mass can be reused again and again to launch payload after payload. Such a tether system could reduce by orders-of-magnitude the propellant and other expendables required for round-trip travel to Mars, and thus they have the potential to make significant exploration and development of Mars affordable.

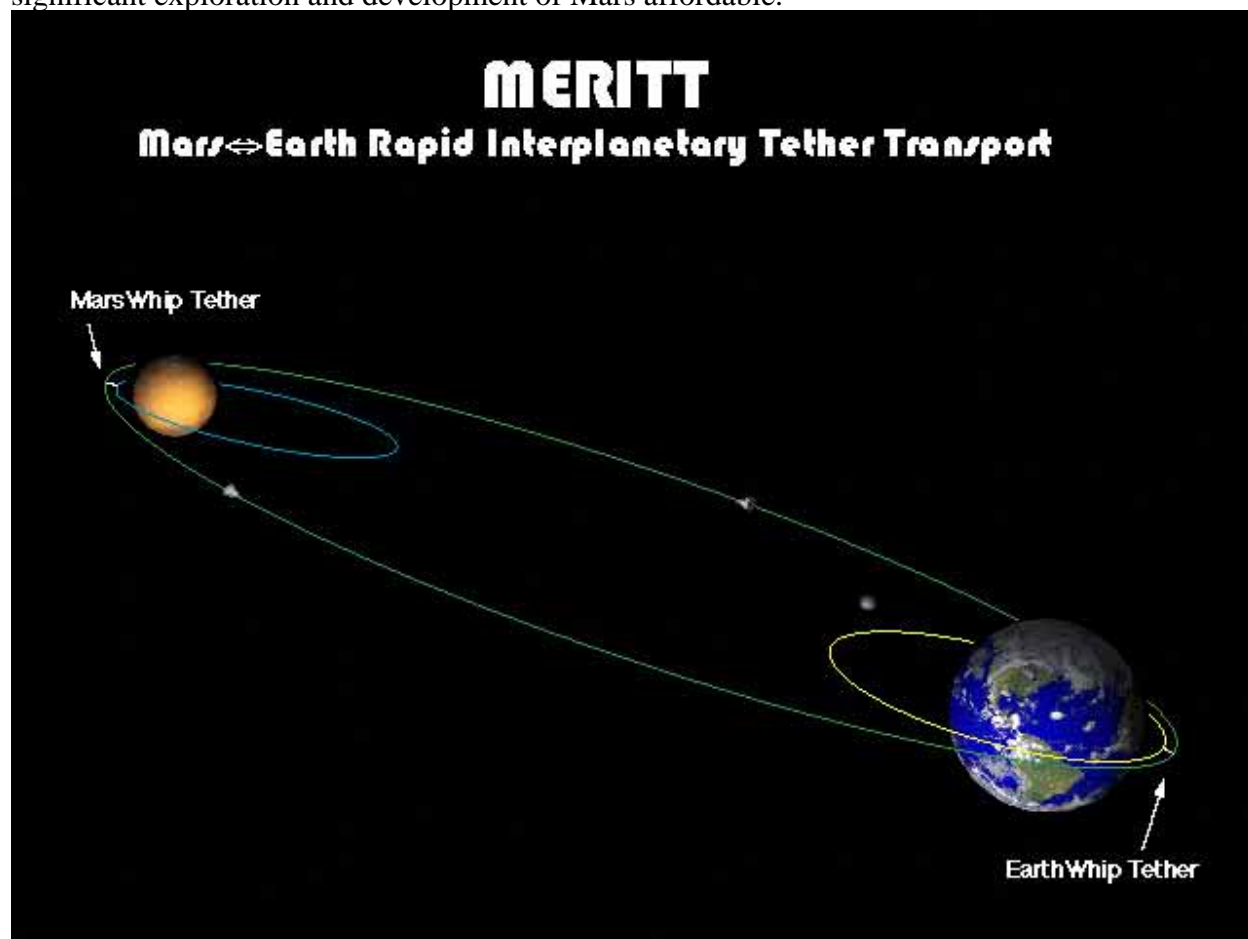


Figure 4. Overview of a system of two momentum-exchange tethers, one in Earth orbit, one in Mars orbit, that can repeatedly exchange payloads between Earth and Mars with rapid transit times with no transfer propellant needed.

3.0 POTENTIAL BENEFITS OF CANDIDATE TECHNOLOGY AGAINST BENEFIT (TECHNICAL) ATTRIBUTES

Affordable / Low Life Cycle Cost	
Min. Cost Impact on Launch System	Strength: Tether Transport System will reduce launch costs by eliminating the need to launch propellant for orbital transfer and interplanetary injection missions.
Low Recurring Cost	Strength: Tether Transport System will require no recovery, refueling, or refurbishment, thus allowing amortization of costs over multiple missions. Weakness: Each transfer mission will require command uplink and control.
Low Cost Sensitivity to Flight Growth	Strength: System becomes more advantageous with higher traffic rates.

Operation and Support	Strength: System eliminates need for ground support and command/control for upper stages. Weakness: Tether facilities will require periodic commands to maintain proper orbital parameters.
Initial Acquisition	Strength: The hardware is simple and should be relatively low-cost to develop.
Vehicle/System Replacement	Strength: Tether transfer systems could operate for <i>years</i> without replacement or refueling.
Dependable	Strength: Pre-commit commands/testing can check system health prior to use.
Highly Reliable	Strength: With the long-life tethers being developed by ProSEDS and SBIR, dependable tether systems will be possible for all applications.
Intact Vehicle Recovery	N/a
Mission Success	
Operate on Command	
Robustness	
Responsive	
Flexible	Strength: Tether system can accommodate various payloads and be used for several different transfer operations.
Capacity	Strength: Tether system can be used repeatedly, providing a total capacity many times that of upper stages.
Operable	Strength: The system is space based; there are only one-time launch site issues.
Environmental Compatibility	
Minimum Impact on Space Environment	Strength: There are no propulsive effluents. Weakness: Tether systems' lifetimes are currently limited by NASA Guidelines in terms of lifetime X area product.
Minimum Effect on Atmosphere	Strength: no impact
Minimum Environmental Impact all Sites	Strength: no impact
Public Support	
Benefit GNP	Strength: It is a new, low-cost space transfer technology that may enable commercial development of lunar and martian resources.
Social Perception	Strength: It is environmentally clean and low-cost Strength: The tether might be visible from the ground at dawn and dusk, allowing public viewing of an active space system.
Safety	Strength: There are no propellants or other caustic or harmful substances inherent to the system.

4.0 POTENTIAL BENEFITS OF CANDIDATE TECHNOLOGY AGAINST PROGRAMMATIC (CONSTRAINTS) ATTRIBUTES

Technology R&D Phase	
Affordable/Low Life Cycle Cost	
Cost to Develop and Mature the technology	Comment: see cost and roadmap section
Benefit Focused	Strength: The technology required directly benefits the attributes listed.
Schedule	
Risk	

Dual Use Potential	Strength: High (Potential use for deployment of commercial satellite constellations as well as deep-space applications)
Program Acquisition Phase	
Cost to acquire operational system	
Schedule	No known weaknesses
Risk	None significant
Technology Options	N/a
Investor Incentive	

I. Technology Roadmap and Cost

Technologies:

- Guidance and control systems for tether facilities (\$1M)
- Automated rendezvous & capture systems suitable for tether/payload rendezvous (\$10M)
- High-strength, survivable tether development (\$2M)
- Deployer with reeling capability (\$5M)

Demonstrations:

- Spinning Tether Orbital Transfer System (STOTS) Demonstration: (\$10M; not including launch costs)

5.0 OTHER RELEVANT INFORMATION

none

6.0 REFERENCES

- NASA/TP-1998-206959, Tether Transportation System Study
- AIAA-99-2690, "Cislunar Tether Transport System", (Cislunar.pdf)
- AIAA-99-2151 "Mars-Earth Rapid Interplanetary Tether Transport" (MERITT.pdf)
- "Space Tethers", Scientific American, Feb 1999, pp. 86-87. Downloadable from:
[http:// www.scientificamerican.com/1999/0299issue/0299beardsleybox3.html](http://www.scientificamerican.com/1999/0299issue/0299beardsleybox3.html)

7.0 POINTS OF CONTACT

Les Johnson

Principal Investigator, Propulsive Small Expendable Deployer System (ProSEDS)

National Aeronautics and Space Administration

George C. Marshall Space Flight Center

RA/10 Advanced Space Transportation Program

Huntsville, Alabama 35812

256-544-0614

Dr. Robert P. Hoyt

Tethers Unlimited, Inc.

1917 NE 143rd St.

Seattle, WA 98125-3236

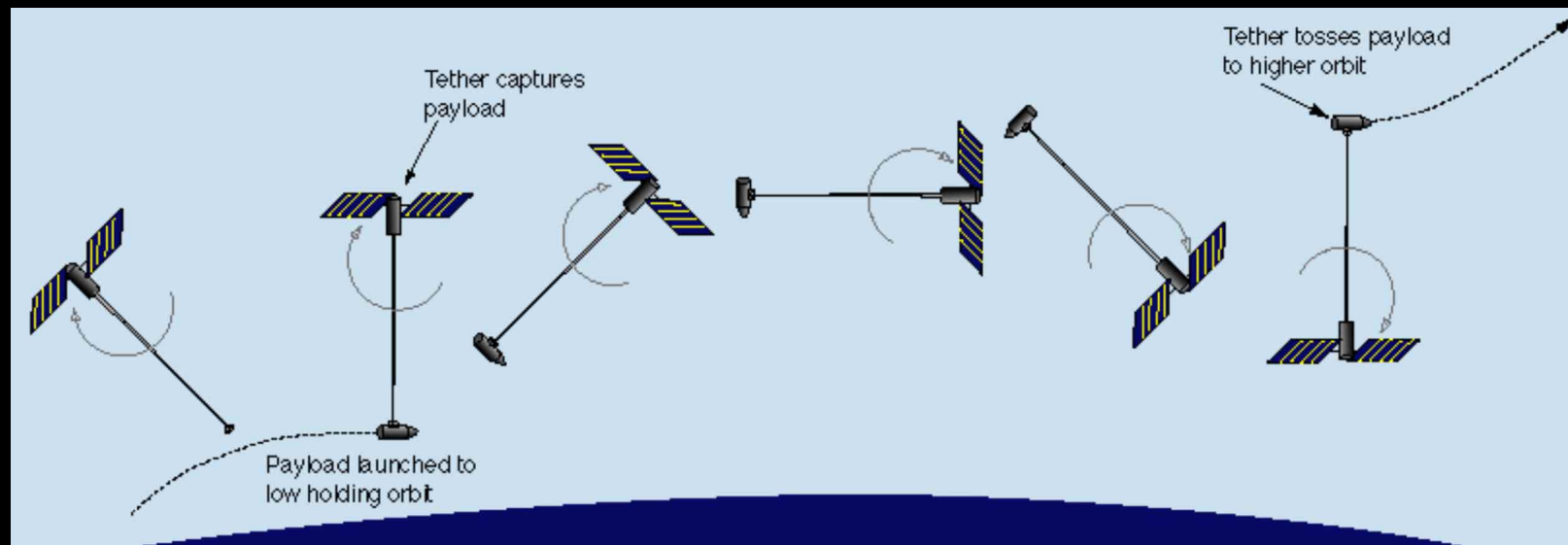
206-306-0400 hoyt@tethers.com www.tethers.com

Michael Bangham
Boeing/Huntsville
(205) 922-7261

Momentum-Exchange Tethers Can Provide Propellantless In-Space Propulsion



- Rotating tether in orbit can catch a payload in a lower orbit and “toss” it into a higher orbit
- Tether facility serves as a “battery” for orbital momentum and energy
- Tether “gives” some of its momentum & energy to payload
 - Use ED tether, SEP, or return traffic to restore orbit
- Provides rapid transfer times and minimal propellant expenditure



LEO-GEO Transfer



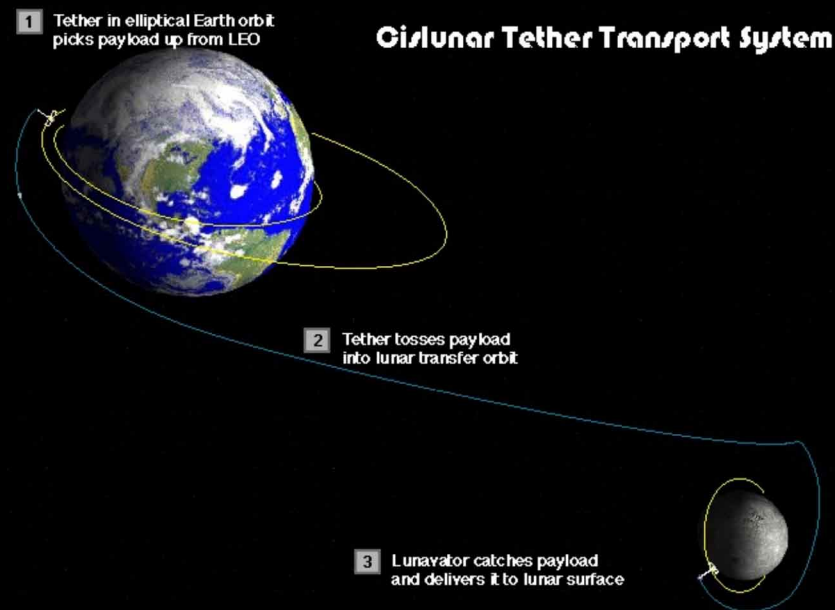
- NASA/MSFC/SAO/Boeing study designed a 2-tether system for deploying 4-ton payloads to GEO
- Tether system mass $< 6x$ payload mass
 - Inertial Upper Stage masses $> 3x$ payload
 - ↳ Tether competitive after just 2 missions
- System would deploy 12 payloads/year, 24 payloads before resupply
- System could reduce launch costs by 75% or more



Lunar Transport



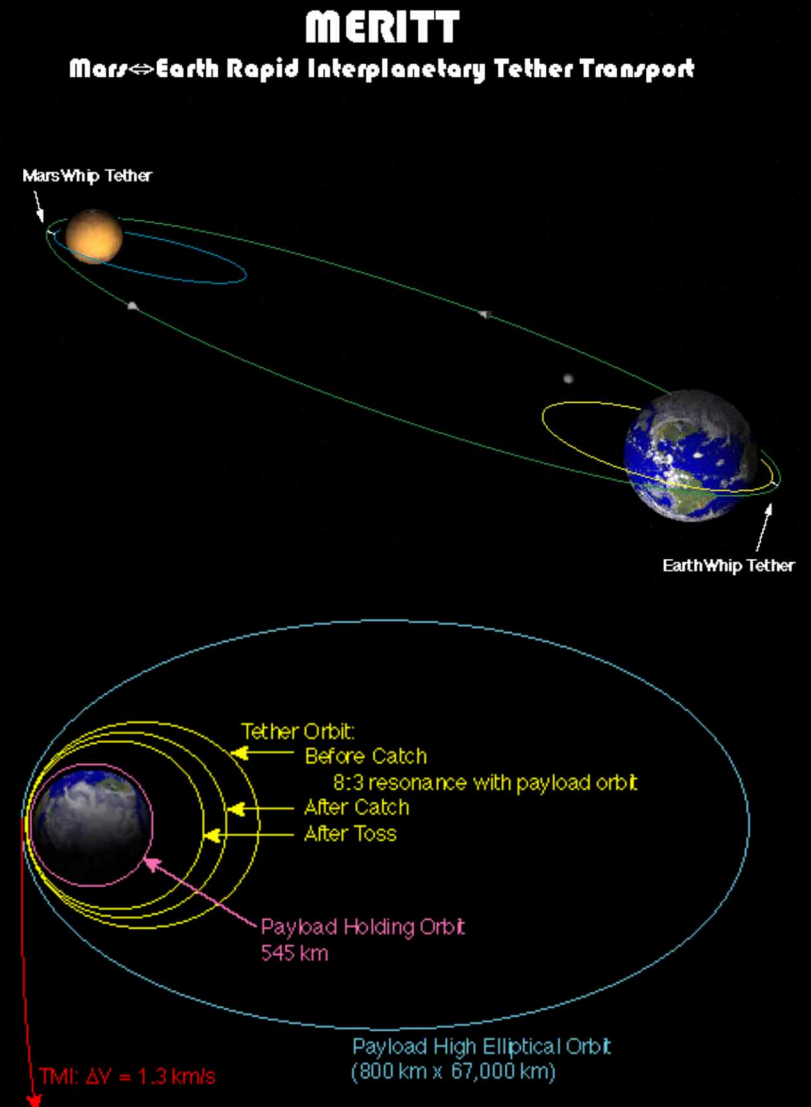
- **Momentum-Exchange Tethers can create a fully-reusable system for LEO \Leftrightarrow Lunar round-trip travel**
- **Rapid transit times**
- **Total system mass $< 28 \times$ payload mass**
- **Competitive w/ chemical rocket mass after only 2 round trip missions**



Transport to Mars



- **MERITT:**
 - System of two tethers, one in highly elliptical orbit around each planet
 - Provides rapid (140 day) transit to and from Mars
 - Each tether masses 15 x payload
- **MarsHEFT**
 - Tether massing 4.6 x payload
 - Boosts 85 ton Mars Cargo payloads to high-energy orbit
 - Boosts 15 ton payloads directly to Mars, 40 ton payloads to Moon, 100 ton payloads to GEO
 - Beats SEP in < 6 missions



Technologies Needed



- **Automated Rendezvous & Capture (AR&C) is a key technology**
 - **Rendezvous @ 1 gee relative acceleration - testable on ground**
 - **NASA/MSFC AR&C Team Believes Tether AR&C is “DOABLE”**
- **High Strength Survivable Tethers**
 - **Can use currently available material like Spectra 2000**
- **Electrodynamic Tether Propulsion**
 - **Combination of ME & ED Tether techniques enables propellantless propulsion from LEO->GEO+Moon+Mars**
- **Rotating tether systems**
 - **STOTS Mission**
 - **Use proven SEDS tether architecture, piggyback experiment**
 - **Demonstrate spin-up & control of tether**
 - **Throw a small payload into a resonant orbit**
 - **Catch payload when it returns**



AIAA 99-2690

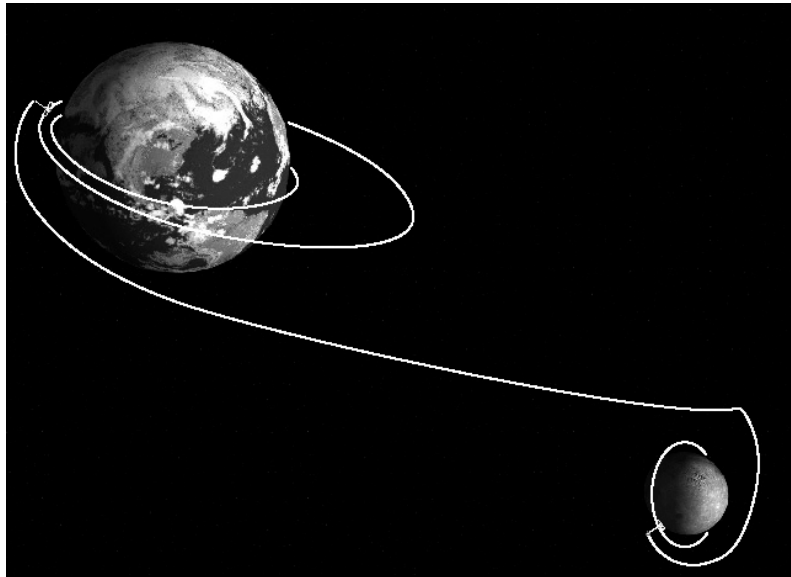
CISLUNAR TETHER TRANSPORT SYSTEM

R. Hoyt

Tethers Unlimited, Inc., Seattle, WA

C. Uphoff

Fortune Eight Aerospace, Niwot, CO



35th AIAA/ASME/SAE/ASEE
Joint Propulsion Conference & Exhibit
20-24 June 1999
Los Angeles, CA

For permission to copy or to republish, contact the American Institute of Aeronautics and Astronautics,
1801 Alexander Bell Drive, Suite 500, Reston, VA, 20191-4344.

CISLUNAR TETHER TRANSPORT SYSTEM

Robert P. Hoyt*

Tethers Unlimited, Inc., Seattle, Washington 98125-3236 USA

Chauncey Uphoff†

Fortune Eight Aerospace, Niwot, Colorado 80544 USA

Abstract

We describe a space systems architecture for repeatedly transporting payloads between low Earth orbit and the surface of the moon without significant use of propellant. This architecture consists of one rotating tether in elliptical, equatorial Earth orbit and a second rotating tether in a circular low lunar orbit. The Earth-orbit tether picks up a payload from a circular low Earth orbit and tosses it into a minimal-energy lunar transfer orbit. When the payload arrives at the Moon, the lunar tether catches it and deposits it on the surface of the Moon. Simultaneously, the lunar tether picks up a lunar payload to be sent down to the Earth orbit tether. By transporting equal masses to and from the Moon, the orbital energy and momentum of the system can be conserved, eliminating the need for transfer propellant. Using currently available high-strength tether materials, this system could be built with a total mass of less than 28 times the mass of the payloads it can transport. Using numerical simulations that incorporate the full three-dimensional orbital mechanics and tether dynamics, we have verified the feasibility of this system architecture and developed scenarios for transferring a payload from a low Earth orbit to the surface of the Moon that require less than 25 m/s of thrust for trajectory targeting corrections.

Nomenclature & Units

a	semimajor axis, m
C_3	orbital energy, $\equiv V^2 - 2\mu/r$, km^2/s^2
d	density, kg/m^3
e	ellipse eccentricity
E	orbital energy, J
F	safety factor
h	specific angular momentum, m^2/s
i	orbit inclination, degrees
J_2	2 nd geopotential coefficient
L	tether arm length, m
l	distance from facility to system's center of mass.
M	mass, kg
N	orbital resonance parameter
p	orbit semiparameter, $= a(1-e^2)$, m
r	radius, m
R_e	Earth radius, m
r_p	perigee radius, m
T	tensile strength, Pa
V	velocity, m/s
V_c	characteristic velocity, m/s
λ	argument of tether perigee w.r.t. Earth-Moon line
μ_e	Earth's gravitational parameter $= GM_e$, m^3/s^2
μ_m	Moon's gravitational parameter $= GM_m$, m^3/s^2
ω	angular velocity, radians/s
θ	true anomaly
$\dot{\omega}$	Apsidal precession/regression rate, rad/s
$\dot{\Omega}$	Nodal regression rate, radians/s

subscripts:

\subscript{a}	apoapse	\subscript{p}	periapse
\subscript{c}	critical	\subscript{m}	moon
\subscript{f}	facility	\subscript{g}	grapple
\subscript{p}	payload	\subscript{t}	tether

Introduction

A "Cislunar Tether Transport System" composed of one rotating momentum-exchange tether in elliptical, equatorial Earth orbit and a second rotating tether facility in a low lunar orbit can provide a means for repeatedly exchanging payloads between low Earth orbit (LEO) and the surface of the Moon, *with little or no propellant expenditure required*. In 1991, Forward¹ showed that such a system is theoretically possible from an energetics standpoint. A later study by Hoyt and Forward² developed a first-order design for such a system. These previous studies, however, utilized a number of simplifying assumptions regarding orbital and tether mechanics in the Earth-Moon system, including assumptions of coplanar orbits, ideal gravitational potentials, and infinite facility ballast masses. The purpose of this paper is to remove these assumptions and develop an architecture for such a system that takes into account the complexities of orbital mechanics in the Earth-Moon system.

The basic concept of the Cislunar Tether Transport System is to use a rotating tether in Earth orbit to pick payloads up from LEO orbits and toss them to the Moon, where a rotating tether in lunar orbit, called a "Lunavator™", could catch them and deliver them to the lunar

* President, CEO, & Chief Engineer, 1917 NE 143rd St., Seattle WA 98125-3236, Member AIAA.

† Chief Scientist, P.O. Box 307 Niwot, CO 80544.

surface. As the Lunavator™ delivers payloads to the Moon's surface, it can also pick up return payloads, such as water or aluminum processed from lunar resources, and send them down to LEO. By balancing the flow of mass to and from the Moon, the orbital momentum and energy of the system can be conserved, eliminating the need to expend large quantities of propellant to move the payloads back and forth. This system is illustrated in Figure 1.

Orbital Mechanics of the Earth-Moon System

Orbital mechanics in cislunar space are made quite complex by the different and varying orientations of the ecliptic plane, the Earth's equatorial plane, the Moon's orbital plane, and the Moon's equatorial plane. Figure 2 attempts to illustrate these different planes. The inclination of the Earth's equatorial plane (the "obliquity of the ecliptic"), is approximately 23.45° , but varies due to tidal forces exerted by the Sun and Moon. The angle i_m between the Moon's equatorial plane and a plane through the Moon's center that is parallel to the ecliptic plane is constant, about 1.58° . The inclination of the Moon's orbit relative to the ecliptic plane is also constant, about $\lambda_m = 5.15^\circ$.³ The line of nodes of the Moon's orbit regresses slowly, revolving once every 18.6 years. As a result, the inclination of the Moon's orbit relative to the Earth's equator varies between 18.3 - 28.6 degrees. The Moon's orbit also has a slight eccentricity, approximately $e_m = 0.0549$.

Tether Orbits

After considering many different options, including the three-tether systems proposed previously and various combinations of elliptical

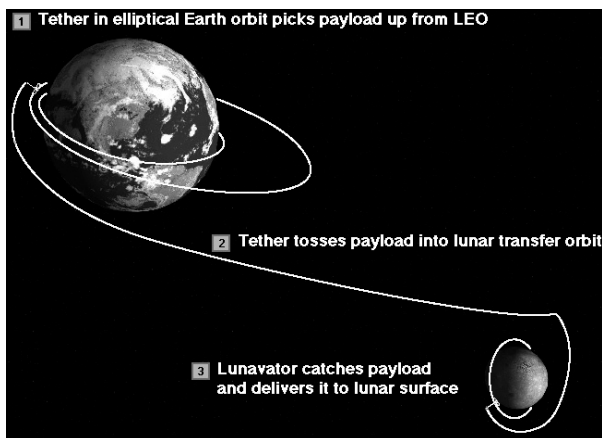


Figure 1. Conceptual illustration of the Cislunar Tether Transport System.

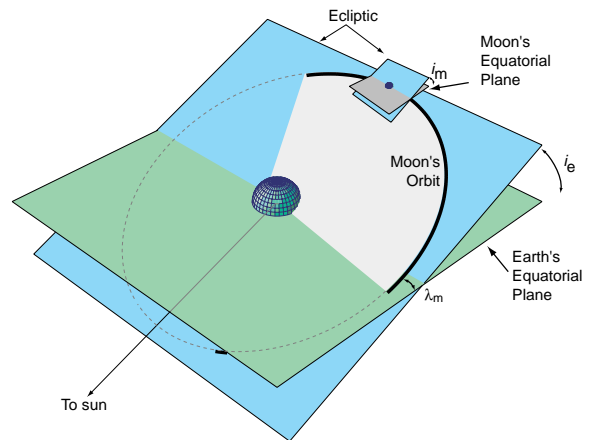


Figure 2. Schematic illustrating the geometry of the Earth-Moon system.

and circular orbits, we have determined that the optimum configuration for the Cislunar Tether system is to utilize one tether in an elliptical, equatorial Earth orbit and one tether in a polar, circular lunar orbit, as illustrated in Figure 1. This two-tether system will require the lowest total system mass, minimize the system complexity and provide the most frequent transfer opportunities. The Earth-orbit tether will pick payloads up from equatorial low-LEO orbits and throw them towards one of the two points where the Moon crosses the Earth's equatorial plane. As the payload approaches the Moon, it will need to perform a small ΔV maneuver to set it up into the proper approach trajectory; the size of this maneuver will vary depending upon the inclination of the Moon's orbit plane and launch dispersions, but under most conditions it will only require about 25 m/s of ΔV .

In the following sections, we will first develop a design for a tether facility for boosting payloads from low-LEO orbits to lunar transfer orbits (LTO). We will then develop a design for a "Lunavator™" capable of catching the payloads and delivering them to the surface of the Moon. We will then discuss the numerical simulations used to verify the feasibility of this system architecture.

Design of a Tether Boost Facility for Lunar Transfer Injection

The first stage of the Cislunar Tether Transport System will be a tether boost facility in elliptical Earth orbit capable of picking payloads up from low-LEO orbits and tossing

them to the Moon. In order to determine an optimum configuration for this facility, we must balance the need to minimize the required masses of the tethers and facilities with the need to make the orbital dynamics of the system as manageable as possible.

The mission of the Earth-orbit portion of the Cislunar Tether Transport System is to pick up a payload from low-Earth orbit and inject it into a near-minimum energy lunar transfer orbit. The desired lunar transfer trajectories have a C_3 of approximately -1.9 (km/s)^2 . A payload originating in a circular orbit at 350 km altitude has an initial velocity of 7.7 km/s and a C_3 of -60 (km/s)^2 . To impulsively inject the payload into a trajectory with a C_3 of -1.9 would require a ΔV of approximately 3.1 km/s.

Design Considerations

Tether System Staging

From an operational standpoint, the most convenient design for the Earth-orbit portion of a Cislunar Tether Transport System would be to start with a single tether facility in a circular low-Earth-orbit, with the tether retracted. The facility would rendezvous with the payload, deploy the payload at the end of the tether, and then use propellantless electrodynamic tether propulsion to spin up the tether until the tip speed reached 3.1 km/s and the tether could inject the payload into a LTO. However, because the tether transfers some of its orbital momentum and energy to the payload when it boosts it, a tether facility in circular orbit would require a very large ballast mass so that its orbit would not drop into the upper atmosphere after it boosts a payload. Furthermore, the strong dependence of the required tether mass on the tether tip speed will likely make this approach impractical with current material technologies. The required mass for a tapered tether depends upon the tip mass and the ratio of the tip velocity to the tether material's critical velocity according to the relation derived by Moravec:⁴

$$M_t = M_p \sqrt{\pi} \frac{\Delta V}{V_c} e^{\frac{\Delta V^2}{V_c^2}} \operatorname{erf} \left\{ \frac{\Delta V}{V_c} \right\}, \quad (1)$$

where $\operatorname{erf}()$ is the error function. The critical velocity of a tether material depends upon the tensile strength, the material density, and the design safety factor according to:

$$V_c = \sqrt{\frac{2T}{Fd}}. \quad (2)$$

The exponential dependence of the tether mass on the *square* of the velocity ratio results in a very rapid increase in tether mass with this ratio.

Currently, the best commercially-available tether material is Spectra® 2000, a form of highly oriented polyethylene manufactured by AlliedSignal. High-quality specimens of Spectra® 2000 have a room temperature tensile strength of 4 GPa, and a density of 0.97 g/cc. With a safety factor of 3, the material's critical velocity is 1.66 km/s. Using Equation (1), an optimally-tapered Spectra® tether capable of sustaining a tip velocity of 3.1 km/s would require a mass of over 100 times the payload mass. While this might be technically feasible for very small payloads, such a large tether mass probably would not be economically competitive with rocket technologies. In the future, very high strength materials such as "buckytube" yarns may become available with tensile strengths that will make a 3 km/s tether feasible; however, we will show that a different approach to the system architecture can utilize currently available materials to perform the mission with reasonable mass requirements.

The tether mass is reduced to reasonable levels if the $\Delta V/V_c$ ratio can be reduced to levels near unity or lower. In the Cislunar system, we can do this by placing the Earth-orbit tether into an elliptical orbit and arranging its rotation so that, at perigee, the tether tip can rendezvous with and capture the payload, imparting a 1.6 km/s ΔV to the payload. Then, when the tether returns to perigee, it can toss the payload ahead of it, giving it an additional 1.5 km/s ΔV . By breaking the 3.1 km/s ΔV up into two smaller boost operations with $\Delta V/V_c < 1$, we can reduce the required tether mass considerably. The drawback to this method is that it requires a challenging rendezvous between the payload and the tether tip; nonetheless, the mass advantages will likely outweigh that added risk.

Behavior of Elliptical Earth Orbits

One of the major challenges to designing a workable tether transportation system using elliptical orbits is motion of the orbit due to the oblateness of the Earth. The Earth's oblateness will cause the plane of an orbit to regress relative to the Earth's spin axis at a rate equal to:⁵

$$\dot{\Omega} = -\frac{3}{2} J_2 \frac{R_e^2}{p^2} \bar{n} \cos(i) \quad (3)$$

And the line of apsides (ie. the longitude of the perigee) to precess or regress relative to the orbit's nodes at a rate equal to:

$$\dot{\omega} = \frac{3}{4} J_2 \frac{R_e^2}{p^2} \bar{n} (5 \cos^2 i - 1) \quad (4)$$

In equations (3) and (4), \bar{n} is the "mean mean motion" of the orbit, defined as

$$\bar{n} = \sqrt{\frac{\mu_e}{a^3}} \left[1 - \frac{3}{4} J_2 \frac{R_e^2}{p^2} \sqrt{1-e^2} (1 - 3 \cos^2 i) \right]. \quad (5)$$

For an equatorial orbit, the nodes are undefined, but we can calculate the rate of apsidal precession relative to inertial space as the sum $\dot{\Omega} + \dot{\omega}$ of the nodal and apsidal rates given by Eqs. (3) and (4).

In order to make the orbital mechanics of the Cislunar Tether Transport System manageable, we place two constraints on our system design:

- First, the orbits of the tether facility will be equatorial, so that $i=0$ and the nodal regression given by Eq. (3) will not be an issue.
- Second, the tether system will throw the payload into a lunar transfer trajectory that is in the equatorial plane. This means that it can perform transfer operations when the Moon is crossing either the ascending or descending node of its orbit.

Nonetheless, we still have the problem of precession of the line of apsides of an orbit. If the

tether orbits are circular, this is not an issue, but it is an issue for systems that use elliptical orbits. In an elliptical orbit system we wish to perform all catch and throw operations at or near perigee. As illustrated in Figure 3, for the payload to reach the Moon's radius at the time when the Moon crosses the Earth's equatorial plane, the payload must be injected into an orbit that has a line of apsides at some small angle λ from the line through the Moon's nodes. If the orbit experiences apsidal precession, the angle λ will have the proper value only periodically. Consequently, in our designs we will seek to choose the orbital parameters such that the apsidal precession of the orbit will have a convenient resonance with the Moon's orbit.

Elliptical-Orbit Tether Boost Facility

In the Cislunar Tether Transport System, the transfer of payloads between a low-LEO and lunar transfer orbits is performed by a single rotating tether facility. This facility performs a catch and release maneuver to provide the payload with two boosts of approximately 1.5 km/s each. To enable the tether to perform two "separate" ΔV operations on the payload, the facility is placed into a highly elliptical orbit with its perigee in LEO. First, the tether rotation is arranged such that when the facility is at perigee, the tether is swinging vertically below the facility so that it can catch a payload moving more slowly than the facility. After it catches the payload, it waits for one orbit and adjusts its rotation slightly (by reeling the tether in or out) so that when it returns to perigee, the tether is swinging above the facility and it can release the payload into a trajectory moving faster than the facility.

HEFT Tether Boost Facility

In order to enable the Earth-orbit tether facility to boost materials to the Moon before a lunar base has been established and begins sending return payloads back to LEO, we propose to combine the principle of rotating momentum-exchange tethers with the techniques of electrodynamic tether propulsion to create a facility capable of reboosting its orbit after each payload transfer without requiring return traffic or propellant expenditure. This concept, the "High-strength Electrodynamic Force Tether" (HEFT) Facility,⁶ is illustrated in Figure 4. The HEFT Facility would include a central facility housing a power supply, ballast mass, plasma

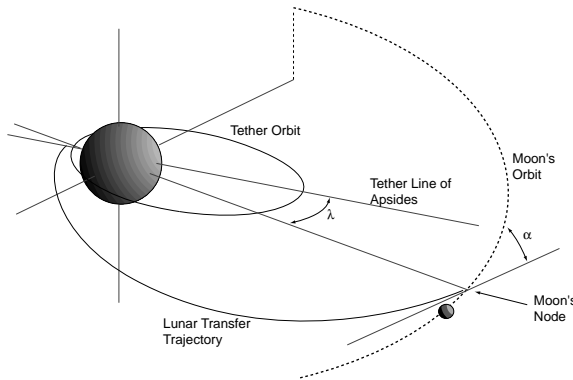


Figure 3. Geometry of the tether orbit and the Moon's orbit.

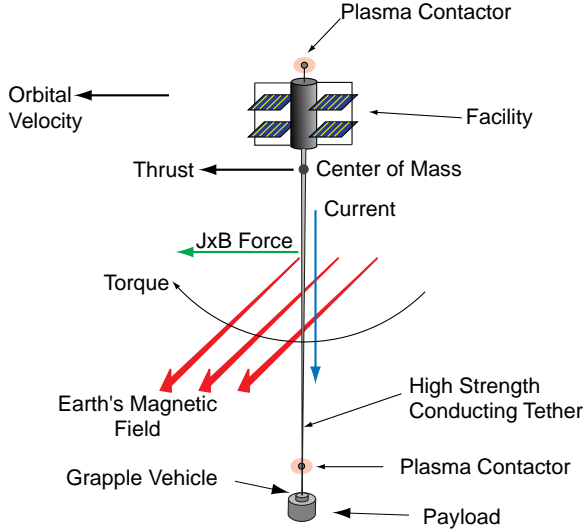


Figure 4. Schematic of the HEFT Facility design.

contactor, and tether deployer, which would extend a long, tapered, high-strength tether. A small grapple vehicle would reside at the tip of the tether to facilitate rendezvous and capture of the payloads. The tether would include a conducting core, and a second plasma contactor would be placed near the tether tip. By using the power supply to drive current along the tether, the HEFT Facility could generate electrodynamic forces on the tether. By properly varying the direction of the current as the tether rotates and orbits the Earth, the facility can use these electrodynamic forces to generate either a net torque on the system to increase its rotation rate, or a net thrust on the system to boost its orbit. The HEFT Facility thus could repeatedly boost payloads from LEO to the Moon, using propellantless electrodynamic propulsion to restore its orbit in between each payload boost operation.

Tether Design

In order to design the tether boost facility, we must determine the tether length, rotation rate, and orbit characteristics that will permit the tether to rendezvous with the payload and throw it into the desired lunar transfer trajectory.

In the baseline design, the payload begins in a circular Initial Payload Orbit (IPO) with a velocity of

$$V_{p,0} = \sqrt{\frac{\mu_e}{r_{IPO}}} \quad (6)$$

The facility is placed into an elliptical orbit with a perigee above the payload's orbit, with the difference between the facility's initial perigee and the payload orbital radius equal to the distance from the tether tip to the center of mass of the facility and tether:

$$r_{p,0} = r_{IPO} + (L - l_{cm,unloaded}), \quad (7)$$

where $l_{cm,unloaded}$ is the distance from the facility to the center of mass of the system before the payload arrives (this distance must be calculated numerically for a tapered tether).

The tether tip velocity is equal to the difference between the payload velocity and the facility's perigee velocity:

$$V_{t,0} = V_{p,0} + V_{IPO}. \quad (8)$$

In order to ensure that a payload will not be "lost" if it is not caught by the tether on its first opportunity, we choose the semimajor axis of the facility's orbit such that its orbital period will be some rational multiple N of the payload's orbital period:

$$P_{f,0} = NP_{IPO} \Rightarrow a_{f,0} = N^{2/3} r_{IPO} \quad (9)$$

For example, if $N=5/2$, this condition means that every two orbits the facility will have an opportunity to rendezvous with the payload, because in the time the facility completes two orbits, the payload will have completed exactly five orbits.

An additional consideration in the design of the system are the masses of the facility and tether. A significant facility mass is required to provide "ballast mass." This ballast mass serves as a "battery" for storing the orbital momentum and energy that the tether transfers to and from payloads. If all catch and throw operations are performed at perigee, the momentum exchange results primarily in a drop in the facility's apogee. A certain minimum facility mass is necessary to keep the post catch and throw orbit above the Earth's upper atmosphere. Some of the "ballast mass" will be provided by the mass of the tether deployer and winch, the facility power supply and power processing hardware, and the mass of the tether itself. If additional mass is required, it could be provided by available material in LEO, such as spent upper stage rockets and shuttle external tanks.

The tether mass required will depend upon the maximum tip velocity and the choices of tether material and design safety factor, as described by Eq. 1. For a tapered tether, the tether's center-of-mass will be closer to the facility end of the tether. This can be an important factor when the tether mass is significant compared to the payload and facility masses. In the calculations below, we have used a model of a tether tapered in a stepwise manner to calculate tether masses and the tether center-of-mass.

By conservation of momentum, the perigee velocity of the center of mass of the tether and payload after rendezvous is:

$$V_{p,1} = \frac{V_{p,0}(M_f + M_t) + V_{IPO}M_p}{(M_f + M_t) + M_p}. \quad (10)$$

When the tether catches the payload, the center-of-mass of the tether system shifts downward slightly as the payload mass is added at the bottom of the tether:

$$r_{p,1} = \frac{r_{p,0}(M_f + M_t) + V_{IPO}M_p}{(M_f + M_t) + M_p} \quad (11)$$

In addition, when the tether catches the payload, the angular velocity of the tether does not change, but because the center-of-mass shifts closer to the tip of the tether when the tether catches the payload, the tether tip velocity decreases. The new tether tip velocity can be calculated as

$$V_t' = V_t \left(\frac{L - l_{cm,loaded}}{L - l_{cm,unloaded}} \right) \quad (12)$$

At this point, it would be possible to specify the initial payload orbit, the payload/facility mass ratio, the facility/payload period ratio, and the desired LTO C_3 , and derive a system of equations from which one particular tether length and one tether tip velocity can be calculated that determine an "exact" system where the tether tip velocity need not be adjusted to provide the desired C_3 of the payload lunar trajectory. However, the resulting system design is rather restrictive, working optimally for only one particular value of the facility and tether masses, and results in rather short tether lengths that will require very high tip acceleration levels. Fortunately, we can provide an

additional flexibility to the system design by allowing the tether facility to adjust the tip velocity slightly by reeling the tether in or out a few percent. If, after catching the payload, the facility reels the tether in by an amount ΔL , the tip velocity will increase due to conservation of angular momentum:

$$V_t'' = \frac{V_t'(L - l_{cm,loaded})}{(L - l_{cm,loaded}) - \Delta L} \quad (13)$$

Then, when the facility returns to perigee, it can throw the payload into a lunar transfer trajectory with perigee characteristics:

$$\begin{aligned} r_{p,LTO} &= r_{p,1} + (L - l_{cm,loaded}) - \Delta L \\ V_{p,LTO} &= V_{p,1} + V_t' \end{aligned} \quad (14)$$

Using the equations above, standard Keplerian orbital equations, and equations describing the shift in the system's center-of-mass as the payload is caught and released, we have calculated a design for a single-tether system capable of picking up payloads from a circular LEO orbit and throwing them to a minimal-energy lunar trajectory. During its initial period of operation, while a lunar facility is under construction and no return traffic exists, the tether system will use electrodynamic tether propulsion to reboost itself after throwing each payload. Once a lunar facility exists and return traffic can be used to conserve the facility's orbital momentum, the orbit of the tether will be modified slightly to permit round trip traffic. The system parameters are listed below.

Initial System Design: Outbound Traffic Only Payload:

- mass $M_p = 2500$ kg
- altitude $h_{IPO} = 308$ km
- velocity $V_{IPO} = 7.72$ km/s

Tether Facility:

- tether length $L = 80$ km
- tether mass $M_t = 15,000$ kg
(Spectra® 2000 fiber, safety factor of 3.5)
- tether center-of-mass $L_{t,com} = 17.6$ km
(from facility)
- central facility mass $M_f = 11,000$ kg
- grapple mass $M_g = 250$ kg
(10% of payload mass)
- total system mass $M = 26,250$ kg
 $= 10.5 \times \text{payload mass}$
- facility power $P_{wr} = 11$ kW avg

- initial tip velocity: $V_{t,0} = 1530 \text{ m/s}$
- Pre-Catch Orbit:
 - perigee altitude $h_{p,0} = 378 \text{ km}$,
 - apogee altitude $h_{a,0} = 11,498 \text{ km}$
 - eccentricity $e_0 = 0.451$
 - period $P_0 = 5/2 P_{\text{IPO}}$
(rendezvous opportunity every 7.55 hrs)
- Post-Catch Orbit:
 - perigee altitude $h_{p,1} = 371 \text{ km}$,
 - apogee altitude $h_{a,1} = 9687 \text{ km}$
 - eccentricity $e_1 = 0.408$

After catching the payload, the facility reels in 2950 m of tether, increasing the tip velocity to 1607 m/s,

- Post-Throw Orbit:
 - perigee altitude $h_{p,2} = 365 \text{ km}$,
 - apogee altitude $h_{a,2} = 7941 \text{ km}$
 - eccentricity $e_2 = 0.36$

Lunar Transfer Trajectory:

- perigee altitude $h_{p,\text{lto}} = 438.7 \text{ km}$
- perigee velocity $V_{p,\text{lto}} = 10.73 \text{ km/s}$
- trajectory energy $C_3 = -1.9 \text{ km}^2/\text{s}^2$

Note that for a particular system design, the tether and facility mass will scale roughly linearly with the payload mass, so an equivalent system designed for sending 250 kg payloads to the Moon could be constructed with a tether mass of 1,500 kg and a facility mass of 1,100 kg. Note also that the tether mass is not dependent upon the tether length, so longer tethers can be used to provide lower tip acceleration levels with no mass penalty.

Electrodynamic Reboost of the Tether Orbit

After boosting the payload, the tether facility will be left in a lower energy elliptical orbit with a semimajor axis that is approximately 1780 km less than its original orbit. Once a lunar base and a lunar tether facility have been established and begin to send return traffic down to LEO, the tether facility can restore its orbit by catching and de-boosting these return payloads. In the period before a lunar base is established, however, the tether facility will use electrodynamic propulsion to reboost its apogee by driving current through the tether when the tether is near perigee. Because the tether is rotating, the direction of the current must be alternated as the tether rotates to produce a net thrust on the facility. Using a simulation of tether dynamics and electrodynamics, we have modeled reboost of a rotating tether system. Figure 5 shows the reboost of the tether's orbit over one day, assuming that the tether facility

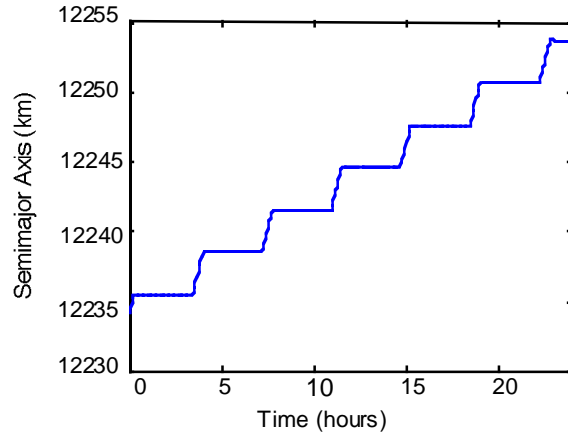


Figure 5. Electrodynamic propulsion reboost of the tether's orbit after the tether has boosted a payload into LTO.

has a power supply of 11 kW and is able to store up power during most of its orbit and expend it at a rate of 75 kW during the portion of the orbit when the tether is below 2000 km altitude. In one day, the facility can restore roughly 20 km to its orbit's semimajor axis; in roughly 85 days it could restore its orbit and be prepared to boost another payload to the Moon. More rapid reboost could be accomplished with a larger power supply.

Dealing with Apsidal Precession

As noted earlier, the oblateness of the Earth will cause the line of apsides of the tether facility's elliptical orbit to precess. In the Cislunar Tether Transport System, we can deal with this issue in two ways. First, we can utilize tether reeling maneuvers to counteract the apsidal precession.⁷ By simply reeling the tether in and out slightly once per orbit, the tether facility can exchange angular momentum between its rotation and its orbit, resulting in precession or regression of the line of apsides. With proper phasing and amplitude, tether reeling can hold the tether's orbit fixed so that it can send payloads to the Moon once per month.⁸

A second method is to choose the tether orbits such that their precession rates are nearly harmonic with the Moon's orbital rate, so that the line of apsides lines up with the Moon's nodes once every several months. Furthermore, we can use propellantless electrodynamic tether propulsion to "fine-tune" the precession rate, either by raising/lowering the orbit or by generating thrust perpendicular to the facility's velocity.

In the design given above, the mass and initial orbit of the tether facility was chosen

such that after throwing a payload to the Moon, the tether enters a lower energy elliptical orbit which will precess at a rate of 2.28 degrees per day. The initial, high-energy orbit has a slower precession rate of approximately 1.58 degrees per day. These orbits were chosen so that in the 95.6 days it takes the Moon to orbit 3.5 times around the Earth, the tether facility can reboost itself from its low-energy orbit to its high-energy orbit using propellantless electrodynamic propulsion, and, by properly varying the reboost rate, the apsidal precession can be adjusted so that the line of apsides will rotate exactly 180°, lining the tether orbit up properly to boost another payload to the Moon.

System Design for Round-Trip Traffic

Once a lunar base is established and begins to send payloads back down to LEO, the orbit of the tether system can be modified slightly to enable frequent opportunities for round-trip travel. First, the facility's orbit will be raised so that its high-energy orbit has a semimajor axis of 12577.572 km, and an eccentricity of 0.41515. The tether will then pick up a payload from a circular, 450 km orbit and toss it to the Moon so that it will reach the Moon as the Moon crosses its ascending node. The facility will then drop to a lower energy orbit. At approximately the same time, the return payload will be released by the lunar tether and begin its trajectory down to LEO. When the return payload reaches LEO, the Earth-orbit tether facility will catch it at perigee, carry it for one orbit, and then place it into the 450 km initial payload orbit. Upon dropping the return payload, the facility will place itself back into the high-energy orbit. The perigee of this orbit will precess at a rate such that after 4.5 lunar months (123 days) it will have rotated 180°, and the system will be ready to perform another payload exchange, this time as the Moon crosses its descending node. If more frequent round-trip traffic is desired, tether reeling could again be used to hold the orientation of the tether's orbit fixed, providing transfer opportunities once per sidereal month.

Design of a Lunavator™ Compatible with Minimal-Energy Lunar Transfers

The second stage of the Cislunar Tether Transport System is a lunar-orbit tether facility that catches the payloads sent by the Earth-

orbit tether and deposits them on the Moon with zero velocity relative to the surface.

Background: Moravec's Lunar Skyhook

In 1978, Moravec⁴ proposed that it would be possible to construct a tether rotating around the Moon that would periodically touch down on the lunar surface. Moravec's "Skyhook" would have a massive central facility with two tether arms, each with a length equal to the facility's orbital altitude. It would rotate in the same direction as its orbit with a tether tip velocity equal to the orbital velocity of the tether's center-of-mass so that the tether tips would periodically touch down on the Moon with zero velocity relative to the surface (to visualize this, imagine the tether as a spoke on a giant bicycle wheel rolling around the Moon).

As it rotates and orbits around the Moon, the tether could capture payloads from Earth as they passed perilune and then set them down on the surface of the Moon. Simultaneously, the tether could pick up payloads to be returned to Earth, and later throw them down to LEO.

Moravec found that the mass of the tether would be minimized if the tether had an arm length equal to one-sixth of the diameter of the Moon, rotating such that each of the two arms touched down on the surface of the Moon three times per orbit. Using data for the best material available in 1978, Kevlar, which has a density of 1.44 g/cc and a tensile strength of 2.8 GPa, Moravec found that a two-arm Skyhook with a design safety factor of $F=2$ would have to mass approximately 13 times the payload mass. Each arm of Moravec's tether would be 580 km long, for a total length of 1160 km, and the tether center-of-mass would orbit the Moon every 2.78 hours in a circular orbit with radius of 2,320 km. At that radius, the orbital velocity is 1.45 km/s, and so Moravec's Skyhook would rotate with a tip velocity of 1.45 km/s.

Using Moravec's minimal-mass solution, however, requires not only a very long tether but also requires that the payload have a very high velocity relative to the Moon at its perilune. Because the lunar tether in Moravec's design has an orbital velocity of 1.45 km/s and the tether tips have a velocity of 1.45 km/s relative to the center-of-mass, the payload's perilune velocity would need to be 2.9 km/s in order to match up with the tether tip at the top of their rotation. In order to achieve this high perilune velocity,

the outbound lunar transfer trajectory would have to be a high-energy hyperbolic trajectory. This presented several drawbacks, the most significant being that if the lunar tether failed to capture the payload at perilune, it would continue on and leave Earth orbit on a hyperbolic trajectory. Moreover, as Hoyt and Forward² found, a high lunar trajectory energy would also place larger ΔV demands on the Earth-orbit tethers, requiring two tethers in Earth orbit to keep the system mass reasonable.

Lunavator™ Design

In order to minimize the ΔV requirements placed upon the Earth-orbit portion of the Cislunar Tether Transport System and thereby permit the use of a single Earth-orbit tether with a reasonable mass, we have developed a method for a single lunar-orbit tether to capture a payload from a minimal-energy lunar transfer orbit and deposit it on the tether surface with zero velocity relative to the surface.

Moon-Relative Energy of a Minimum-Energy LTO

A payload that starts out in LEO and is injected into an elliptical, equatorial Earth-orbit with an apogee that just reaches the Moon's orbital radius will have a C_3 relative to the Moon of approximately $0.72 \text{ km}^2/\text{s}^2$. For a lunar transfer trajectory with a closest-approach altitude of several hundred kilometers, the payload will have a velocity of approximately 2.3 km/s at perilune. As a result, it would be moving too slowly to rendezvous with the upper

tip of Moravec lunar Skyhook, which will have a tip velocity of 2.9 km/s at the top of its rotation. Consequently, the design of the lunar tether system must be modified to permit a tether orbiting the Moon at approximately 1.5 km/s to catch a payload to at perilune when the payload's velocity is approximately 2.3 km/s , then increase both the tether length and the angular velocity so that the payload can be set down on the surface of the Moon with zero velocity relative to the surface. Simply reeling the tether in or out from a central facility will not suffice, because reeling out the tether will cause the rotation rate to decrease due to conservation of angular momentum.

A method that can enable the tether to catch a payload and then increase the tether rotation rate while lowering the payload is illustrated in Figure 6. The "Lunavator™" tether system is composed of a long tether, a counterbalance mass at one end, and a central facility that has the capability to climb up or down the tether. Initially, the facility would locate itself near the center of the tether, and the system would rotate slowly around the center-of-mass of the system, which would be located roughly halfway between the facility and the counterbalance mass. The facility could then capture an inbound payload at its perilune. The facility would then use energy from solar cells or other power supply to climb up the tether towards the counterbalance mass. The center-of-mass of the system will remain at the same altitude, but the distance

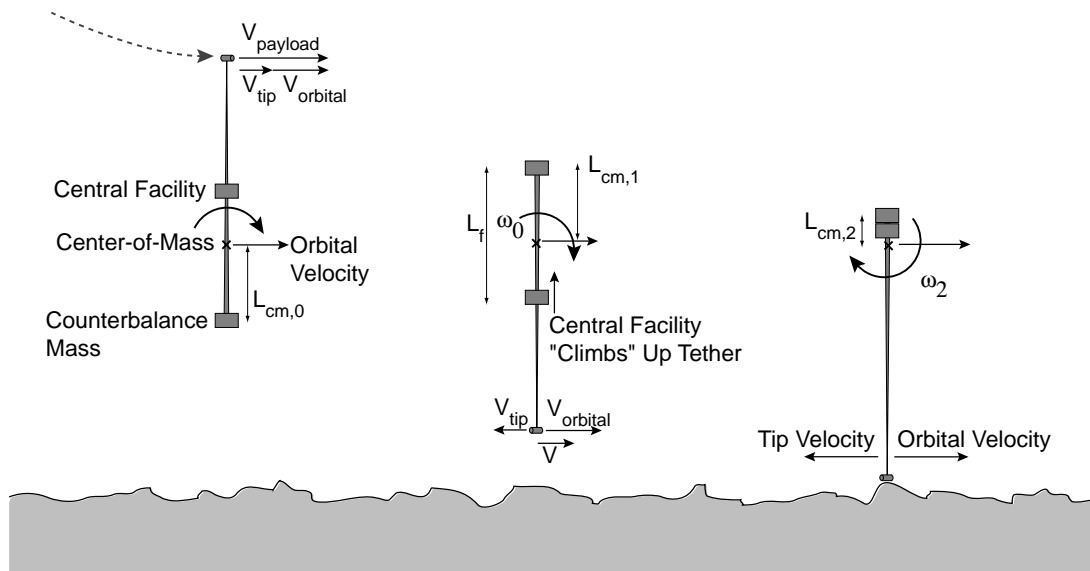


Figure 6. Method for a lunar tether to capture a payload from a minimal-energy LTO and deposit it on the Moon with zero velocity relative to the surface.

from the tether tip to the center-of-mass will increase, and conservation of angular momentum will cause the angular velocity of the system to increase as the facility mass moves closer to the center-of-mass.

Analysis

A first-order design for the Lunavator™ can be obtained by calculating the shift in the system's center-of-mass as the central facility changes its position along the tether. We begin by specifying the payload mass, the counterbalance mass, the facility mass, and the tether length. The required tether mass cannot be calculated simply by using Moravec's tapered tether mass equation, because that equation was derived for a free-space tether. The Lunavator™ must support not only the forces due to centripetal acceleration of the payload and tether masses, but also the tidal forces due to the Moon's gravity. The equations for the tether mass with gravity-gradient forces included are not analytically integrable, so the tether mass must be calculated numerically.

Prior to capture of the payload, the distance from the counterbalance mass to the center-of-mass of the tether system is

$$L_{cm,0} = \frac{M_f L_f + M_t L_{cm,t}}{M_c + M_f + M_t}, \quad (15)$$

where L_f is the distance from the counterbalance to the facility and $L_{cm,t}$ is the distance from the counterbalance to the center-of-mass of the tether. $L_{cm,t}$ must be calculated numerically for a tapered tether.

If the Lunavator™ is initially in a circular orbit with radius a_0 , it will have a center-of-mass velocity of

$$v_{cm,0} = \sqrt{\frac{\mu_m}{a_0}}. \quad (16)$$

At the top of the tether swing, it can capture a payload from a perilune radius of

$$r_p = a_0 + (L_t - L_{cm,0}). \quad (17)$$

A payload sent from Earth on a near-minimum energy transfer will have a $C_{3,m}$ of approximately $0.72 \text{ km}^2/\text{s}^2$. Its perilune velocity will thus be

$$v_p = \sqrt{\frac{2\mu_m}{a_0 + (L_t - L_{cm,0})} + C_{3,m}}. \quad (18)$$

In order for the tether tip's total velocity to match the payload velocity at rendezvous, the velocity of the tether tip relative to the center of mass must be

$$v_{t,0} = v_p - v_{cm,0}, \quad (19)$$

and the angular velocity of the tether system will be

$$\omega_{t,0} = \frac{v_{t,0}}{L_t - L_{cm,0}}. \quad (20)$$

When the tether captures the payload, the center of mass of the new system, including the payload, is at perigee of a new, slightly elliptical orbit, as illustrated in Figure 7 (it was in a circular orbit and caught a payload going faster than the center-of-mass). The perigee radius and velocity of the center-of-mass are

$$v_{p,1} = \frac{v_{cm,0}(M_c + M_f + M_t) + v_p M_p}{M_c + M_f + M_t + M_p}, \quad (21)$$

$$r_{p,1} = \frac{a_0(M_c + M_f + M_t) + r_p M_p}{M_c + M_f + M_t + M_p}, \quad (22)$$

and the new distance from the counterbalance mass to the system's center-of-mass of the system changes to

$$L_{cm,1} = \frac{M_f L_f + M_t L_{cm,t} + M_p L_t}{M_c + M_f + M_t + M_p}. \quad (23)$$

To increase the rotation rate of the tether system and increase the distance from the system's center of mass to the tether tip, the facility climbs up the tether to the counterbalance mass, reducing the distance from the counterbalance to the center-of-mass to

$$L_{cm,2} = \frac{M_t L_{cm,t} + M_p L_t}{M_c + M_f + M_t + M_p}. \quad (24)$$

By conservation of angular momentum, the angular velocity will increase to a new value of

$$\omega_2 = \omega_0 \frac{\left[L_{cm,1} M_c + (L_f - L_{cm,1}) M_f + (L_{cm,t} - L_{cm,1}) M_t + (L_t - L_{cm,1}) M_p \right]}{\left[L_{cm,2} M_f + (L_{cm,t} - L_{cm,2}) M_t + (L_t - L_{cm,2}) M_p \right]} \quad (25)$$

and the payload will then have a velocity relative to the center-of-mass of

$$v_{t,2} = \omega_2 (L_t - L_{cm,2}). \quad (26)$$

If the initial orbit parameters, tether lengths, and facility and tether masses are chosen properly, then $v_{t,2}$ can be made equal to the perigee velocity of the tether system and the distance from the center of mass to the payload can be made equal to the perigee altitude. When the tether returns to its perigee it can then deposit the payload on the surface of the Moon and simultaneously pick up a payload to be thrown back to Earth.

Lunavator™ Design

Using the equations given above, we have found the following first-order design for a Lunavator™ capable of catching payloads from minimal-energy lunar transfer orbits and depositing them on the surface of the Moon:

Payload Trajectory:

- mass $M_p = 2500 \text{ kg}$
- perigee altitude $h_p = 328.23 \text{ km}$
- Moon-relative energy $C_{3,M} = 0.719 \text{ km}^2/\text{s}^2$

Lunavator™:

- tether length $L = 200 \text{ km}$
- counterbalance mass $M_c = 15,000 \text{ kg}$
- facility mass $M_f = 15,000 \text{ kg}$
- tether mass $M_t = 11,765 \text{ kg}$
- Total Mass $M = 41,765 \text{ kg}$
 $= 16.7 \times \text{payload mass}$
- Orbit Before Catch:
 - central facility position $L_f = 155 \text{ km}$
 - tether tip velocity $V_{t,0} = 0.748 \text{ km/s}$
 - rotation rate $\omega_0 = 0.00566 \text{ rad/s}$
 - circular orbit altitude $h_{p,0} = 170.5 \text{ km}$
- Orbit After Catch:
 - perigee altitude $h_{p,0} = 178 \text{ km}$,
 - apogee altitude $h_{a,0} = 411.8 \text{ km}$
 - eccentricity $e_0 = 0.0575$

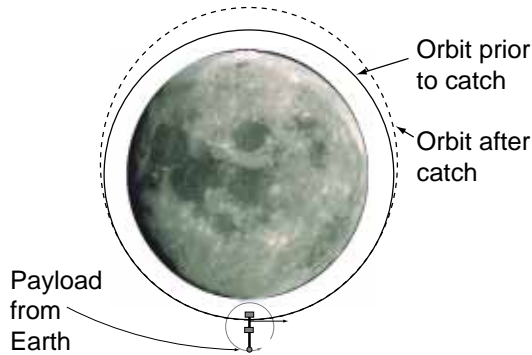


Figure 7. Lunavator™ orbits before and after payload capture.

After catching the payload, the central facility climbs up the tether to the counterbalance mass, changing the rotation rate to:

- adjusted rotation rate $\omega_0 = 0.00929 \text{ rad/s}$
- adjusted tip velocity $V_{t,2} = 1.645 \text{ km/s}$

Payload Delivery:

- drop-off altitude $h = 1 \text{ km}$
(top of a lunar mountain)
- velocity w.r.t. surface $v = 0 \text{ m/s}$

Lunavator™ Orbit: Polar vs. Equatorial

In order to provide the most consistent transfer scenarios, it is desirable to place the Lunavator™ into either a polar or equatorial lunar orbit. Each choice has relative advantages and drawbacks, but both are viable options.

Equatorial Lunar Orbit

The primary advantage of an equatorial orbit for the Lunavator™ is that equatorial lunar orbits are relatively stable. An equatorial Lunavator™, however, would only be able to service traffic to bases on the lunar equator. Because the lunar equatorial plane is tilted with respect to the Earth's equatorial plane, a payload boosted by the Earth-orbit tether facility will require a ΔV maneuver to bend its trajectory into the lunar equatorial plane. This ΔV can be provided either using a small rocket thrust or a lunar "slingshot" maneuver. These options will be discussed in more detail in a following section.

Polar Lunar Orbit

A polar orbit would be preferable for the Lunavator™ for several reasons. First, direct transfers to polar lunar trajectories are possible with little or no propellant expenditure required. Second, because a polar lunar orbit will remain oriented in the same direction while the Moon rotates inside of it, a polar Lunavator™ could service traffic to any point on the surface of the Moon, including the potentially ice-rich lunar poles. Polar lunar orbits, however, are unstable. The odd-harmonics of the Moon's potential cause a circular, low polar orbit to become eccentric, as illustrated in Figure 8. Eventually, the eccentricity becomes large enough that the perilune is at or below the lunar surface. For the 178 km circular orbit, the rate of eccentricity growth is approximately 0.00088 per day.

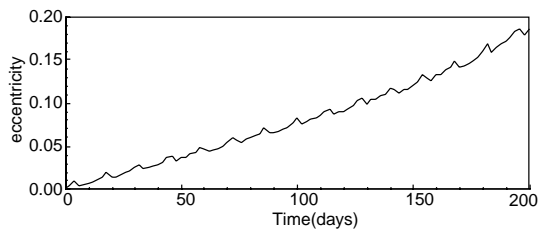


Figure 8. Evolution of the eccentricity of an initially circular 178 km polar lunar orbit, without tether reeling.

Fortunately, the techniques of orbital modification using tether reeling, proposed by Martínez-Sánchez and Gavit⁷ and by Landis⁹ may provide a means of stabilizing the orbit of the LunavatorTM without requiring expenditure of propellant. Tether reeling can add or remove energy from a tether's orbit by working against the non-linearity of a gravitational field. The basic concept of orbital modification using tether reeling is illustrated in Figure 9. When a tether is near the apoapsis of its orbit, the tidal forces on the tether are low. When it is near periapsis, the tidal forces on the tether are high. If it is desired to reduce the eccentricity of the tether's orbit, then the tether can be reeled in when it is near apoapsis, under low tension, and then allowed to unreel under higher tension when it is at periapsis. Since the tidal forces that cause the tether tension are, to first order, proportional to the inverse radial distance cubed, more energy is dissipated as the tether is unreeled at periapsis

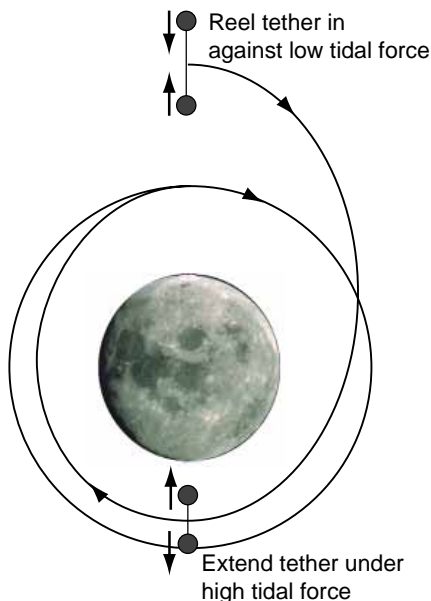


Figure 9. Schematic of tether reeling maneuver to reduce orbital eccentricity.

than is restored to the tether's orbit when it is reeled back in at apoapsis. Thus, energy is removed from the orbit. Conversely, energy can be added to the orbit by reeling in at periapsis and reeling out at apoapsis. Although energy is removed (or added) to the orbit by the reeling maneuvers, the orbital angular momentum of the orbit does not change. Thus the eccentricity of the orbit can be changed.

The theories developed in references 7 and 9 assumed that the tether is hanging (rotating once per orbit). Because the LunavatorTM will be rotating several times per orbit, we have extended the theory to apply to rapidly rotating tethers.⁸ Using a tether reeling scheme in which the tether is reeled in and out once per orbit as shown in Figure 9, we find that a reeling rate of 1 m/s will reduce the eccentricity of the LunavatorTM's orbit by 0.0011 per day, which should be more than enough to counteract the effects of lunar perturbations to the tether's orbit. Thus tether reeling may provide a means of stabilizing the orbit of a polar LunavatorTM without requiring propellant expenditure. This tether reeling, however, would add additional complexity to the system.

Cislunar System Simulations

Tether System Modeling

In order to verify the design of the orbital dynamics of the Cislunar Tether Transport System, we have developed a numerical simulation called "TetherSim" that includes:

- The 3D orbital mechanics of the tethers and payloads in the Earth-Moon system, including the effects of Earth oblateness, using Runge-Kutta integration of Cowell's method.
- Modeling of the dynamical behavior of the tethers, using a bead-and-spring model similar to that developed by Kim and Vadali.¹⁰
- Modeling of the electrodynamic interaction of the Earth-orbit tether with the ionosphere.

Using this simulation tool, we have developed a scenario for transferring a payload from a circular low-LEO orbit to the surface of the Moon using the tether system designs outlined above. We have found that for an average transfer scenario, mid-course trajectory corrections of approximately 25 m/s are necessary to target the payload into the desired polar lunar trajectory to enable rendezvous with the LunavatorTM. A

simulation of a transfer from LEO to the surface of the Moon can be viewed at www.tethers.com.

Targeting the Lunar Transfer

In addition to the modeling conducted with TetherSim, we have also conducted a study of the Earth-Moon transfer to verify that the payload can be targeted to arrive at the Moon in the proper plane to rendezvous with the Lunavator™. This study was performed with the MAESTRO code,¹¹ which includes the effects of luni-solar perturbations as well as the oblateness of the Earth. In this work we studied targeting to both equatorial and polar lunar trajectories.

Transfer to Equatorial Lunar Trajectories

Transfer of a payload from an equatorial Earth trajectory to an equatorial lunar trajectory can be achieved without propellant expenditure, but this requires use of a one-month “resonance hop” transfer, as illustrated in Figure 10. In a resonance hop maneuver, the payload is sent on a trajectory that passes the Moon in such a way that the lunar gravitational field slingshots the payload’s orbit into a one-month Earth orbit that returns to the Moon in the lunar equatorial plane. Using MAESTRO, we have developed a lunar transfer scenario that achieves this maneuver.

In order to avoid the one-month transfer time, we can instead use a small impulsive thrust as the payload crosses the lunar equator to bend its trajectory into the equatorial plane. A patched-conic analysis of such a transfer predicts that such a maneuver would require 98 to 135 m/s of ΔV . However, our numerical simulations of the transfer revealed that under most conditions, luni-solar perturbations of the payload’s trajectory will perform much of the needed

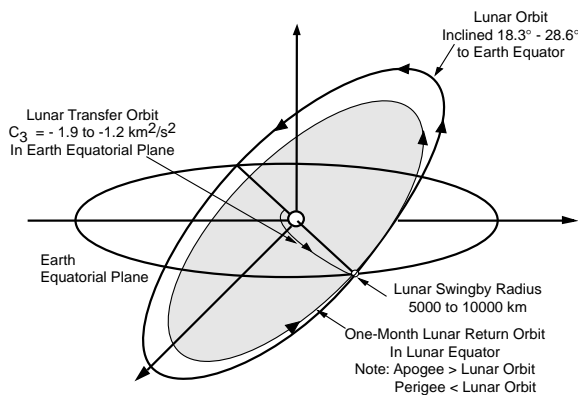


Figure 10. Schematic of one-month “resonance-hop” transfer to place payload in lunar equator without using propellant.

bending for us, and the velocity impulse needed to place the payload in a lunar equatorial trajectory is only about 25 m/s. Figure 11 shows the time-history of a transfer of a payload from the Earth-orbit tether boost facility to the Moon, projected onto the Earth’s equatorial plane.

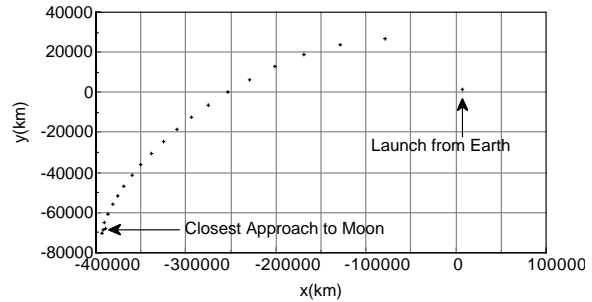


Figure 11. Transfer of payload to lunar equatorial trajectory, projected onto the True Earth Equator.

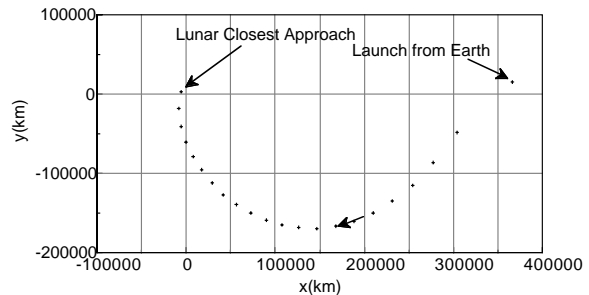


Figure 12. Projection of payload transfer onto Lunar Equatorial Plane (Moon centered frame).

Figure 12 shows this same transfer, projected onto the lunar equatorial plane in a Moon centered, rotating frame, with the x-axis pointing at the Earth. The motion of the payload relative to the lunar equator can be observed in Figure 13, which shows the trajectory projected onto the lunar x-z plane. The payload crosses the lunar equator approximately 10 hours before its closest approach to the Moon. Figure 14, which plots the Moon-relative velocity of the payload, shows that the payload’s velocity at the time of lunar equatorial crossing is about 925 m/s. However, a plot of the declination of the payload’s velocity with respect to the lunar equator, shown in Figure 15, reveals that that the declination of the Moon-relative velocity vector is only a few degrees, much less than the 18°-29° value predicted by a simple zero-patched conic analysis; the Moon’s (or Sun’s) gravity has bent the velocity vector closer to the lunar orbit plane.

At the time when the payload’s trajectory crosses the lunar equator, the declination of the

incoming velocity vector is only 1.52° . This dynamical situation permits us to bend the approach trajectory into the lunar equator with a very small amount of impulse supplied by the spacecraft propulsion system. In the case shown here, the amount of ΔV required is only 24.5 m/s, applied about 10 hours before closest approach to the Moon, as the spacecraft crosses the lunar equator.

Transfer to Polar Lunar Trajectories

Figure 16 shows a payload transfer targeted to a polar lunar trajectory with an ascending node (with respect to the lunar prime meridian) of -100.95° . This particular trajectory is a Type II transfer, with a central angle on the initial orbit

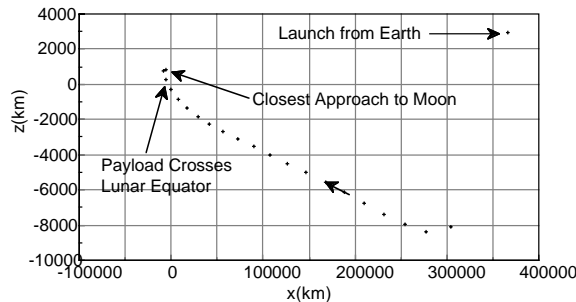


Figure 13. Projection of payload transfer onto Lunar x-z plane (Moon centered frame).

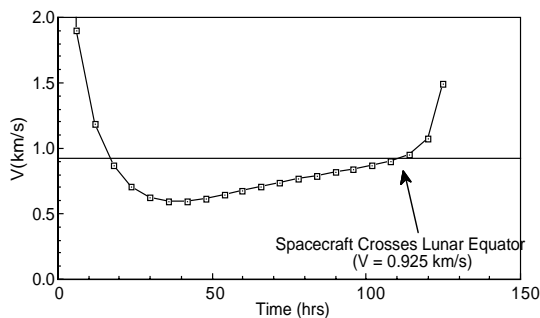


Figure 14. Moon-relative velocity of spacecraft.

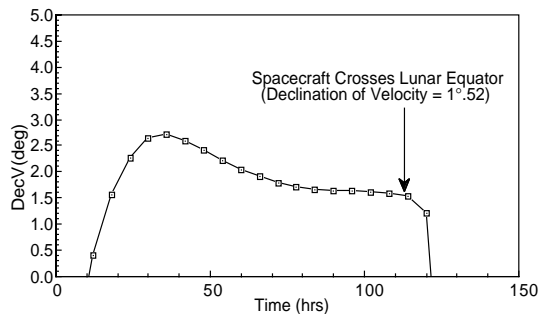


Figure 15. Declination of Moon-relative velocity vector with respect to Lunar Equator.

of greater than 180° . Similar transfers can be achieved with Type I trajectories (central angle of less than 180°). Essentially, these transfers are achieved by injecting the payload into an orbit that just reaches the Moon's orbit near the point where the Moon will cross the Earth's equatorial plane. When the payload reaches its apogee, it is moving only a few hundred meters per second. As the payload slowly drifts towards its apogee, the Moon approaches, moving at just over 1 km/s. The Moon then "captures" the payload, pulling it into a trajectory that is just barely hyperbolic relative to the Moon.

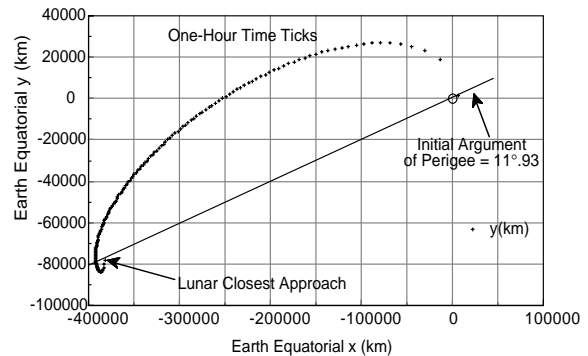


Figure 16. Time history of an Earth-Moon transfer targeted to a polar lunar trajectory.

We have found that by varying the energy of the translunar trajectory and adjusting the argument of perigee, it is possible to target the payload to rendezvous with a polar orbit Lunavator™ with a wide range of ascending node positions of the Lunavator™ orbit. Our simulations indicate that the viable nodal positions ranges at least $\pm 10^\circ$ from the normal to the Earth-Moon line.

Comparison to Rocket Transport

Travelling from LEO to the surface of the Moon and back requires a total ΔV of more than 10 km/s. To perform this mission using storable chemical rockets, which have an exhaust velocity of roughly 3.5 km/s, the standard rocket equation requires that a rocket system consume a propellant mass equal to 16 times the mass of the payload for each mission. The Cislunar Tether Transport System would require an on-orbit mass of less than 28 times the payload mass, but it would be able to transport many payloads. In practice, the tether system will require some propellant for trajectory corrections and rendezvous maneuvers, but the total ΔV for these maneuvers will likely be less than 100 m/s. Thus

a simple comparison of rocket propellant mass to tether system mass indicates that the fully reusable tether transport system could provide significant launch mass savings after only a few round trips. Although the development and deployment costs associated with a tether system would present a larger up-front expense than a rocket based system, for frequent, high-volume round trip traffic to the Moon, a tether system could achieve large reductions in transportation costs by eliminating the need to launch large quantities of propellant into Earth orbit.

Summary

Our analyses have concluded that the optimum architecture for a tether system designed to transfer payloads between LEO and the lunar surface will utilize one tether facility in an elliptical, equatorial Earth orbit and one tether in low lunar orbit. We have developed a preliminary design for a 80 km long Earth-orbit tether boost facility capable of picking payloads up from LEO and injecting them into a minimal-energy lunar transfer orbit. Using currently available tether materials, this facility would require a mass 10.5 times the mass of the payloads it can handle. After boosting a payload, the facility can use electrodynamic propulsion to reboost its orbit, enabling the system to repeatedly send payloads to the Moon without requiring propellant or return traffic. When the payload reaches the Moon, it will be caught and transferred to the surface by a 200 km long lunar tether. This tether facility will have the capability to reposition a significant portion of its "ballast" mass along the length of the tether, enabling it to catch the payload from a low-energy transfer trajectory and then "spin-up" so that it can deliver the payload to the Moon with zero velocity relative to the surface. This lunar tether facility would require a total mass of less than 17 times the payload mass. Both equatorial and polar lunar orbits are feasible for the LunavatorTM. Using two different numerical simulations, we have tested the feasibility of this design and developed scenarios for transferring payloads from a low-LEO orbit to the surface of the Moon, with only 25 m/s of ΔV needed for small trajectory corrections. Thus, it appears feasible to construct a Cislunar Tether Transport System with a total on-orbit mass requirement of less than 28 times the mass of the payloads it can handle, and this system could greatly reduce the cost of round-trip travel

between LEO and the surface of the Moon by minimizing the need for propellant expenditure.

Acknowledgments

This research was supported by a Phase I grant from NASA's Institute for Advanced Concepts, contract number NIAC-07600-011. Earlier work on this concept was supported by a subcontract on the NASA Grant NAG8-1303 to the Smithsonian Astrophysical Observatory. The authors would like to acknowledge Robert L. Forward of Tethers Unlimited, Inc., Enrico Lorenzini of SAO, and Joe Carroll of Tether Applications for helpful discussions.

References

1. Forward, R. L., "Tether Transport from LEO to the Lunar Surface," AIAA paper 91-2322, July 1991.
2. Hoyt, R. P., Forward, R. L., "Tether System for Exchanging Payloads Between Low Earth Orbit and the Lunar Surface", AIAA 97-2794, July 1997.
3. Danby, J.M.A., *Fundamentals of Celestial Mechanics*, 2nd Edition, Willmann-Bell, 1992, Ch. 14.
4. Moravec, H., "A Non-Synchronous Orbital Skyhook," *Journal of the Astronautical Sciences*, 25(4), Oct-Dec1977, pp. 307-322.
5. Battin, R. H., *An Introduction to the Mathematics and Methods of Astrodynamics*, AIAA, 1987, p. 504.
6. *Failure Resistant Multiline Tether*, Robert L. Forward and Robert P. Hoyt, PCT/US97/05840, filed 22 April 1997.
7. Martínez-Sánchez, M., Gavit, S.A., "Orbital Modifications using Forced Tether Length Variations", *Journal of Guidance, Control, and Dynamics*, 10(3) May-June 1987, pp 233-241.
8. Hoyt, R. P., "Maintenance Of Rotating Tether Orbits Using Tether Reeling", Appendix F in *Cislunar Tether Transport System*, Tethers Unlimited, Inc. Final Report on NASA Institute for Advanced Concepts Contract NIAC-07600-011.
9. Landis, G.A., "Reactionless Orbital Propulsion using Tether Deployment," *Acta Astronautica* 26(5), IAF Paper 90-254, 1992.
10. Kim, E., Vadali, S.R. "Modeling Issues related to Retrieval of Flexible Tethered Satellite Systems," *Journal of Guidance, Control, and Dynamics*, 18(5), 1995, pp 1169-76.
11. Uphoff, C., "Mission Analysis Evaluation and Space Trajectory Optimization Program", Final Report on NASA Contract NAS5-11900, March 1973.

REPORT DOCUMENTATION PAGE			Form Approved OMB No. 0704-0188	
Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503.				
1. AGENCY USE ONLY (Leave blank)	2. REPORT DATE 30 May 1999	83. REPORT TYPE AND DATES COVERED Final, November 1, 1998 – April 30, 1999		
4. TITLE AND SUBTITLE CISLUNAR TETHER TRANSPORT SYSTEM		5. FUNDING NUMBERS		
6. AUTHORS Robert P. Hoyt, Robert L. Forward, Chauncey W. Uphoff				
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Tethers Unlimited, Inc. 8114 Pebble Ct. Clinton, WA 98236		8. PERFORMING ORGANIZATION REPORT NUMBER		
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) NASA Institute for Advanced Concepts Universities Space Research Association 555 A Fourteenth Street, NW Atlanta, GA 30318		10. SPONSORING/MONITORING AGENCY REPORT NUMBER		
11. SUPPLEMENTARY NOTES				
12a. DISTRIBUTION/AVAILABILITY STATEMENT		12b. DISTRIBUTION CODE		
13. ABSTRACT (Maximum 200 words) The Phase I effort developed a design of a space systems architecture for repeatedly transporting payloads between low Earth orbit and the surface of the moon without significant use of propellant. This architecture consists of one rotating tether in elliptical, equatorial Earth orbit and a second rotating tether in a circular low lunar orbit. The Earth-orbit tether picks up a payload from a circular low Earth orbit and tosses it into a minimal-energy lunar transfer orbit. When the payload arrives at the Moon, the lunar tether catches it and deposits it on the surface of the Moon. Simultaneously, the lunar tether picks up a lunar payload to be sent down to the Earth orbit tether. By transporting equal masses to and from the Moon, the orbital energy and momentum of the system can be conserved, eliminating the need for transfer propellant. Using currently available high-strength tether materials, this system could be built with a total mass of less than 28 times the mass of the payloads it can transport. Using numerical simulations that incorporate the full three-dimensional orbital mechanics and tether dynamics, we have verified the feasibility of this system architecture and developed scenarios for transferring a payload from a low Earth orbit to the surface of the Moon that require less than 25 m/s of thrust for trajectory targeting corrections.				
14. SUBJECT TERMS tether, Moon, Mars, Hoytether™, electrodynamic, propellantless, cislunar, propulsion, orbit raising, momentum-exchange			15. NUMBER OF PAGES	
			16. PRICE CODE	
17. SECURITY CLASSIFICATION OF REPORT Unclassified	18. SECURITY CLASSIFICATION OF THIS PAGE Unclassified	19. SECURITY CLASSIFICATION OF ABSTRACT Unclassified	20. LIMITATION OF ABSTRACT none	